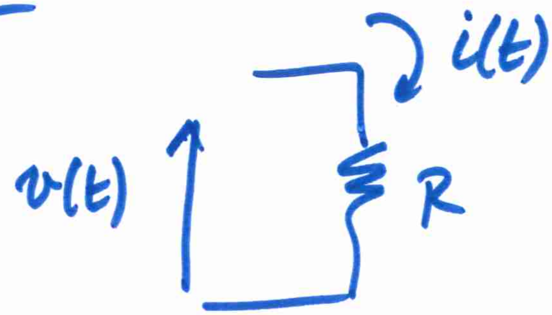
01/17/24

S3

$p(t)$? **Example**: resistor.

$$v(t) = R\, i(t)$$



$$i(t) = I_{max} \cos(\omega t)$$
$$v(t) = V_{max} \cos(\omega t)$$

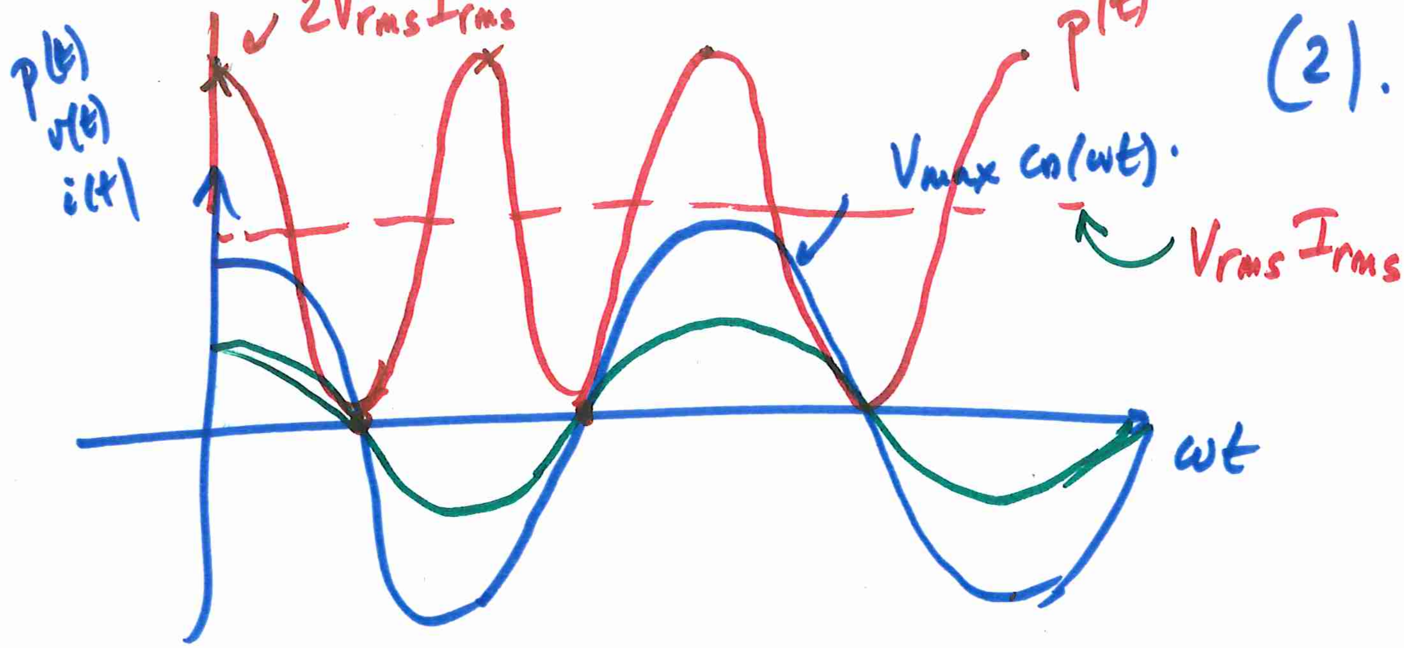
$$p(t) = v(t)\, i(t) = V_{max} I_{max} \underline{\cos(\overset{A}{\omega t}) \cos(\overset{A}{\omega t})}$$

MATH:

$$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$
$$\cos(A)\cos(A) = \frac{1}{2}\left[\cos(2A) + \underbrace{\cos(0)}_{1}\right].$$

$$p(t) = \frac{V_{max} I_{max}}{2}\left[\cos(2\omega t) + 1\right]$$
$$= \frac{V_{max}}{\sqrt{2}} \frac{I_{max}}{\sqrt{2}}\left[1 + \cos(2\omega t)\right].$$
$$p(t) = V_{rms} I_{rms} \left(1 + \cos(2\omega t)\right].$$

(2).

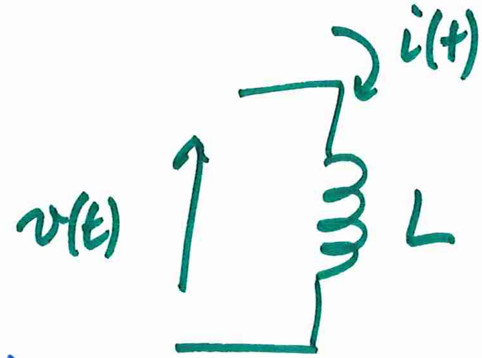


power is oscillating at twice the frequency (ω)

if v(t) is at 60 HZ, p(t) oscillates at <u>120 HZ</u>.

$$P_{AVE} = V_{rms} I_{rms}.$$

<u>Inductor</u>:

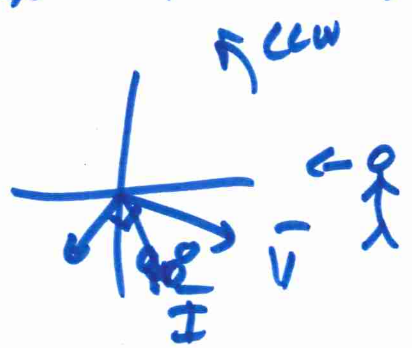


$$\begin{cases} v(t) = V_{max} \cos(\omega t) \\ i(t) = I_{max} \cos(\omega t - 90^\circ) \end{cases}$$

$$v(t) = L \frac{di(t)}{dt}$$

$$I e^{j(\omega t - 90^\circ)}$$

Current lags voltage by 90°.



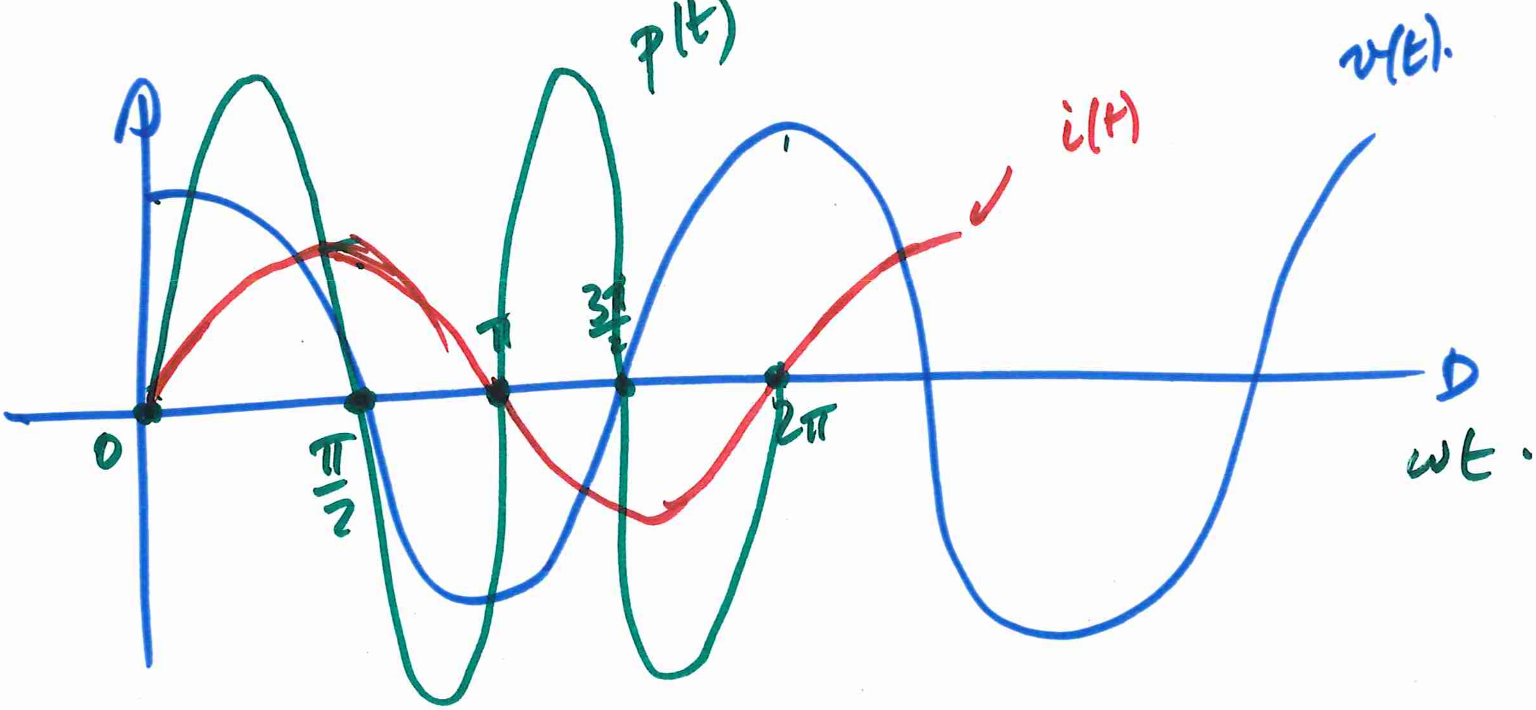
(3).

$$p(t) = V_{max} \cos(\omega t)\, I_{max} \cos(\omega t - 90^\circ)$$

$$p(t) = \frac{V_{max} I_{max}}{2} \left(\cos(2\omega t - 90^\circ) \right).$$

$$p(t) = \frac{V_{max} I_{max}}{2} \sin(2\omega t)$$

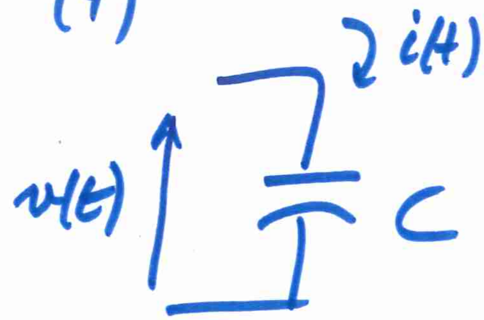
$$p(t) = V_{rms} I_{rms} \sin(2\omega t)$$ ✓



$P_{AVE} = 0\,W$

(4)

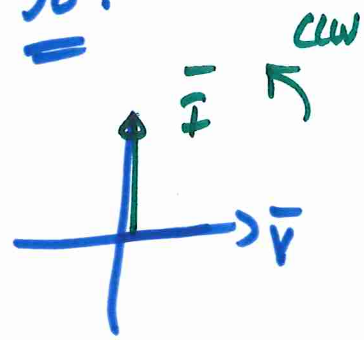
Capacitor:



$$v(t) = V_{max} \cos(\omega t)$$

$$i(t) = I_{max} \cos(\omega t + 90^\circ) \qquad i(t) = C\frac{dv}{dt}.$$

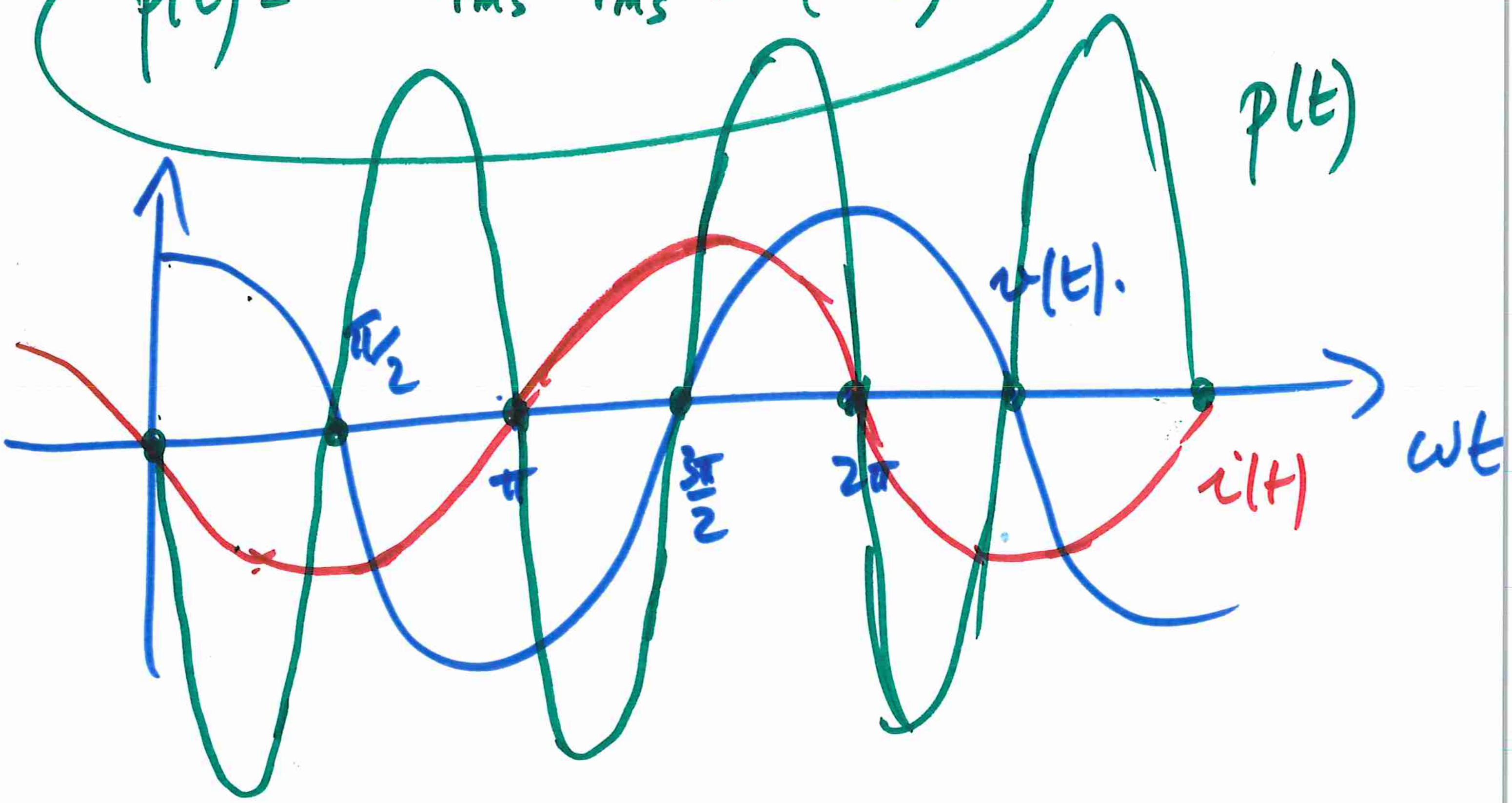
Current leads the voltage by <u>90°</u>.



$$p(t) = \frac{V_{max} I_{max}}{2}\left[\cos(2\omega t + 90^\circ) + \cos(-90^\circ)\right]$$

($\cos(-90^\circ) = 0$)

$$p(t) = -V_{rms} I_{rms} \sin(2\omega t)$$



In general:

Real power: $P_{real} = P_{AVE} = VI\cos(\theta_V - \theta_I)$

Reactive power: $Q = VI\sin(\theta_V - \theta_I)$.

$$\begin{cases} v(t) = V_{max}\cos(\omega t) \\ i(t) = I_{max}\cos(\omega t - \theta) \end{cases}$$

$$p(t) = \underbrace{V_{rms} I_{rms} \cos\theta}_{P_{real}} \left[1 + \cos(2\omega t) \right] + \underbrace{\left(V_{rms} I_{rms} \sin\theta \right)}_{Q} \sin(2\omega t).$$

Resistor: $P = V_{rms} I_{rms}$, [W]. $Q = 0$ VARs.

Inductor: $P = 0$, $Q = V_{rms} I_{rms}$ VARs.

Capacitor: $P = 0$, $Q = -V_{rms} I_{rms}$ VARs.