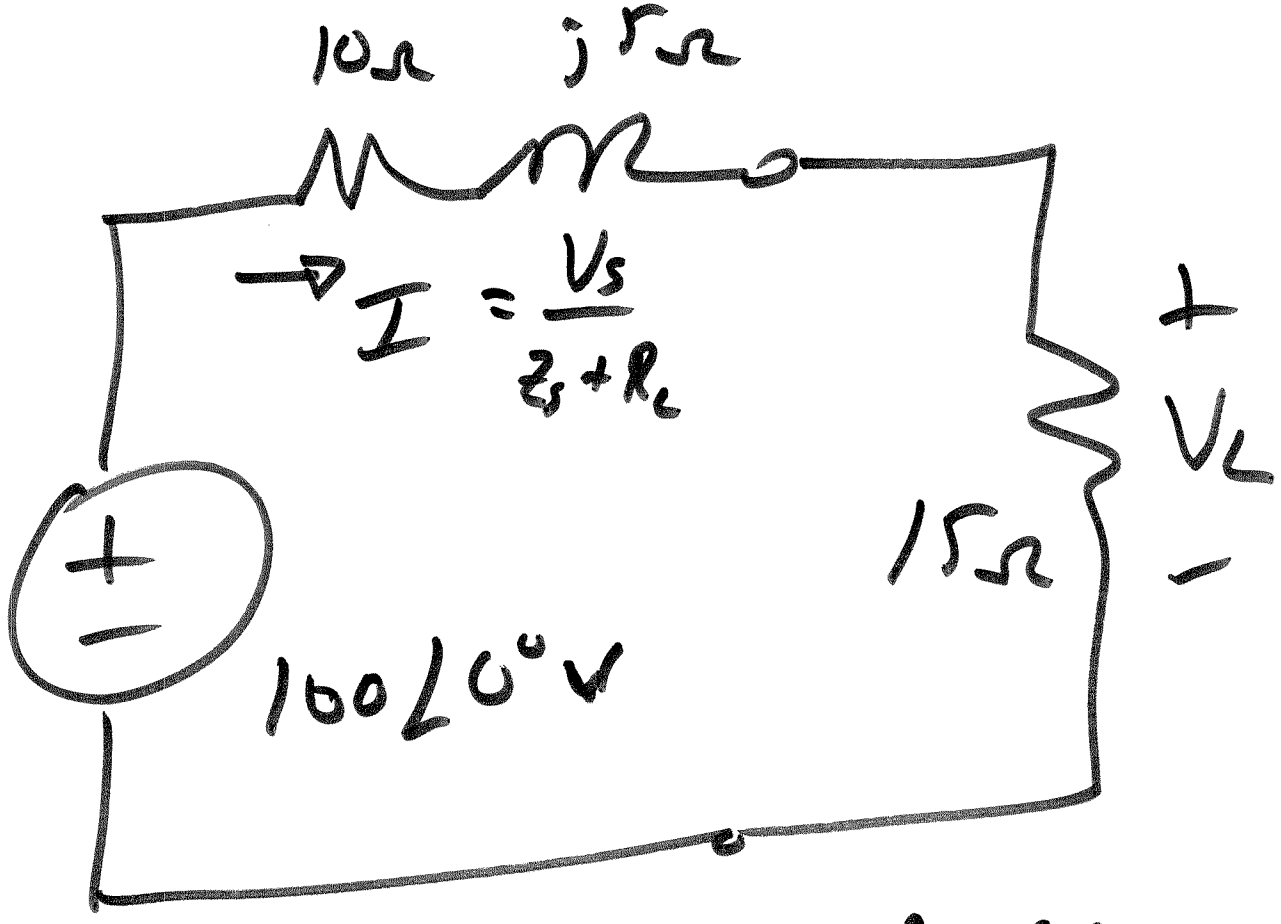
ECE 320 & ECE 329

ENERGY SYSTEMS I

BACKGROUND STUDY IN ENERGY SYSTEMS

SESSION no. 5



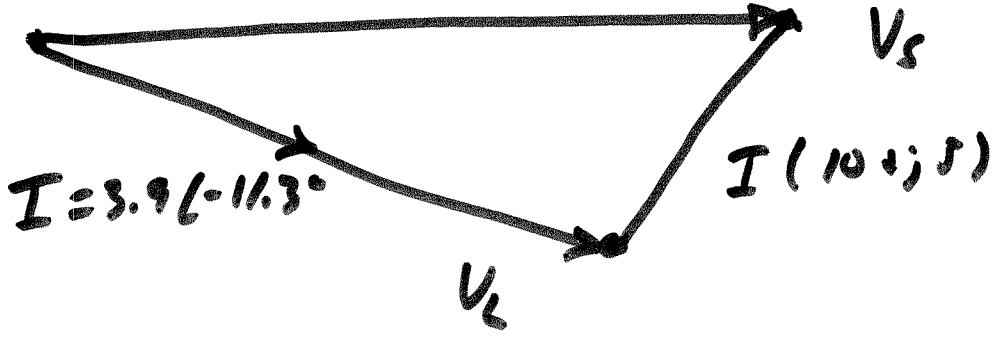
FIND OUTPUT POWER

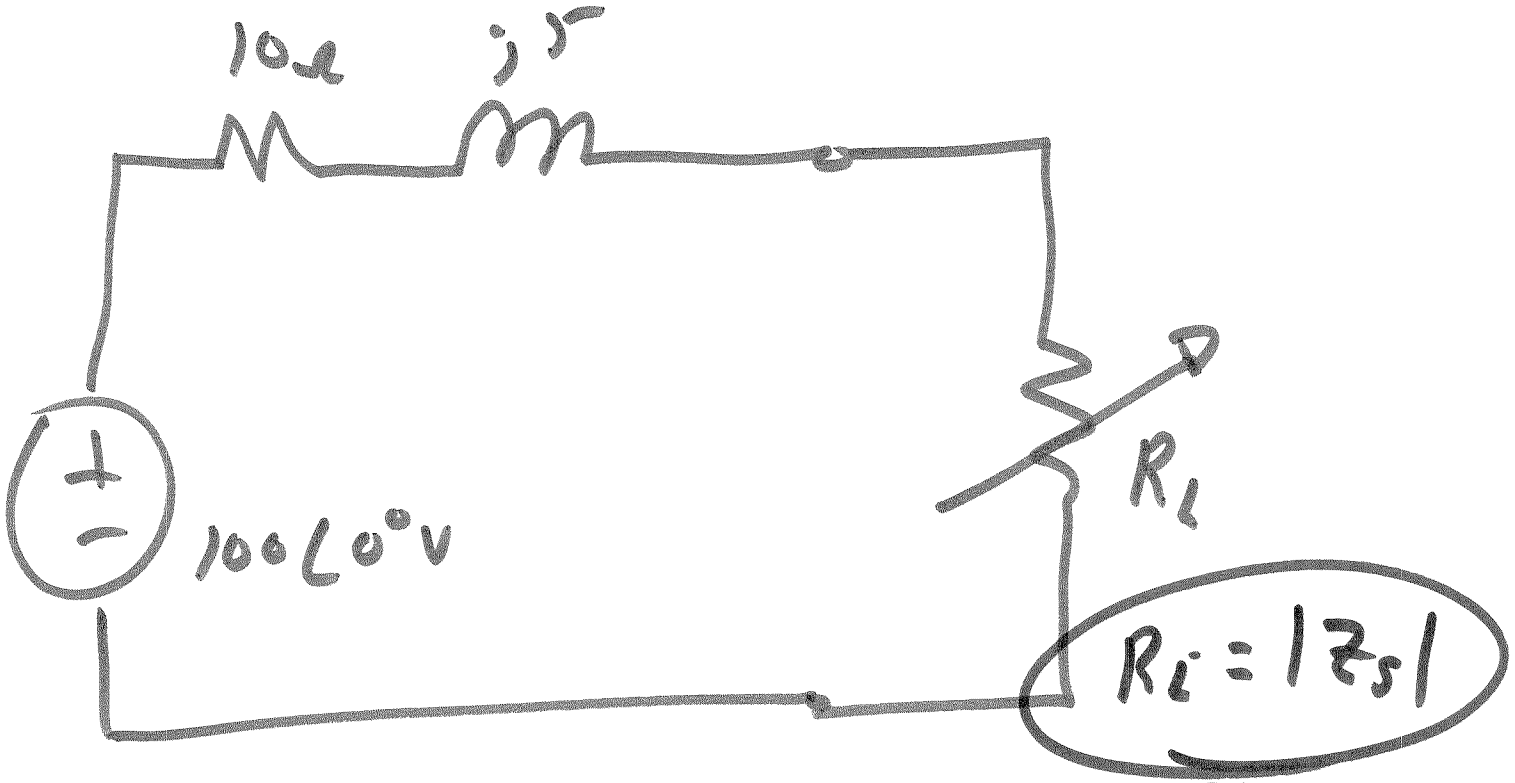
$$P = R_e(V I^*)$$

$$P = R_e(I R \cdot I^*)$$

$$\Rightarrow I I^* = |I|^2$$

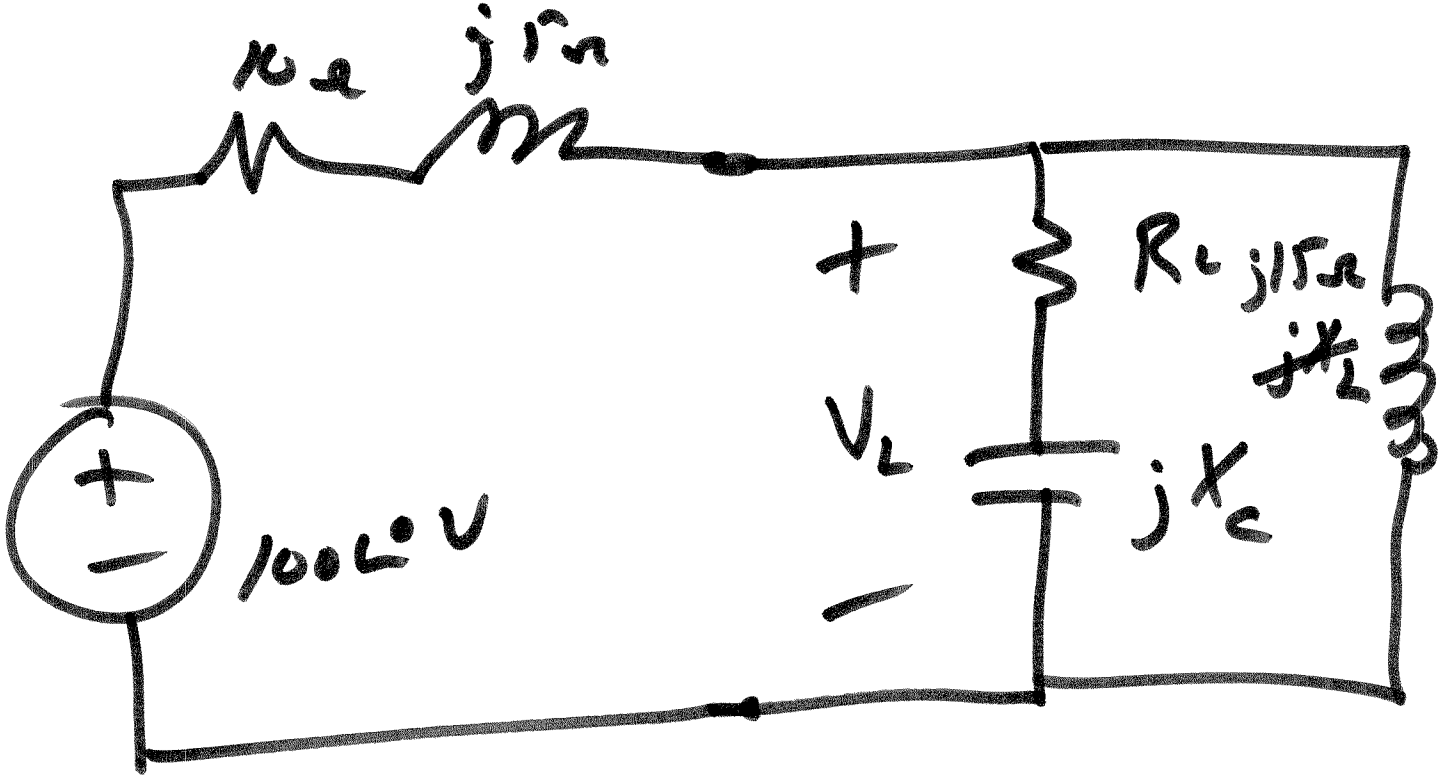
$$P = |I|^2 R$$


V_s
$I = 3.9\angle -11.3°$
$I(10+j8)$
V_L



FIND MAX POWER OUT.

FOR ANY R_L



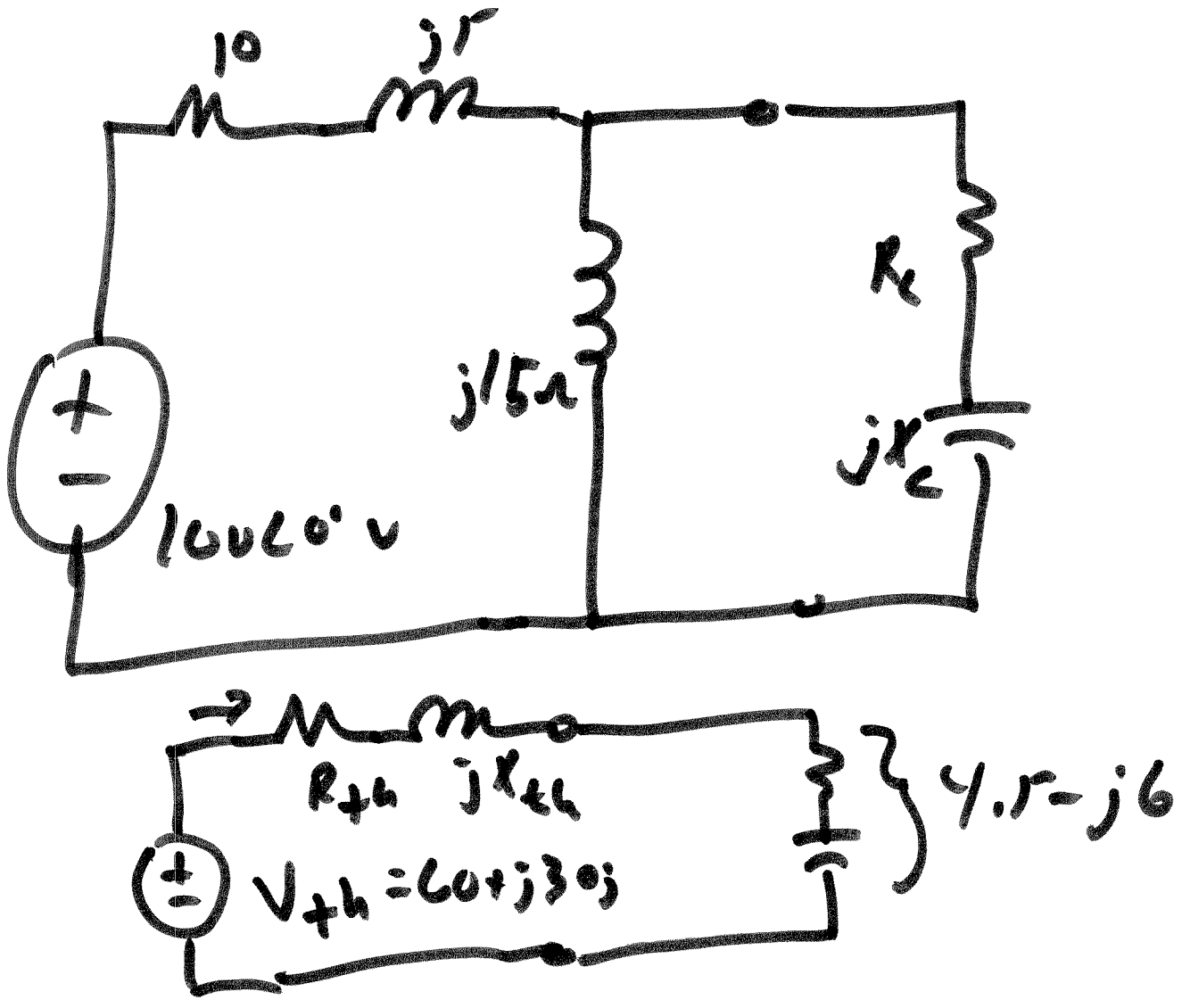
$$I = \frac{100\angle 0^\circ V}{10 + R_L}$$

$X_C = -5\Omega$

$R_L = 10\Omega$

$I = 5\angle 0^\circ A$

$P_0 = |I|^2 R_L = 250W$


10
j5
j15Ω
R_L
jX_C
100∠0° V
R_{th}
jX_{th}
$V_{th} = 60 + j30$ V
4.5 − j6

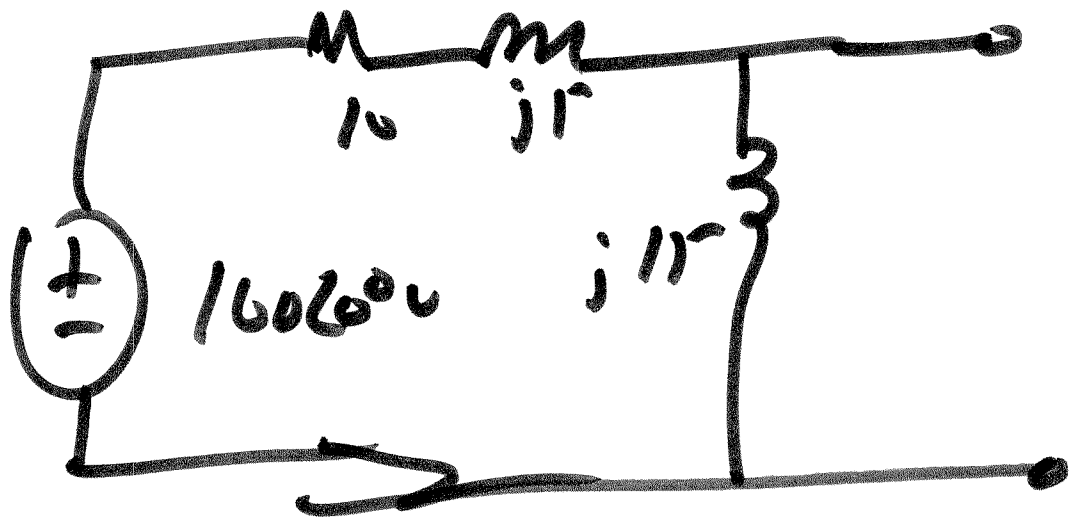
$$V_{th} = (100\angle 0^\circ)\left(\frac{j15}{10+j5+j15}\right)$$

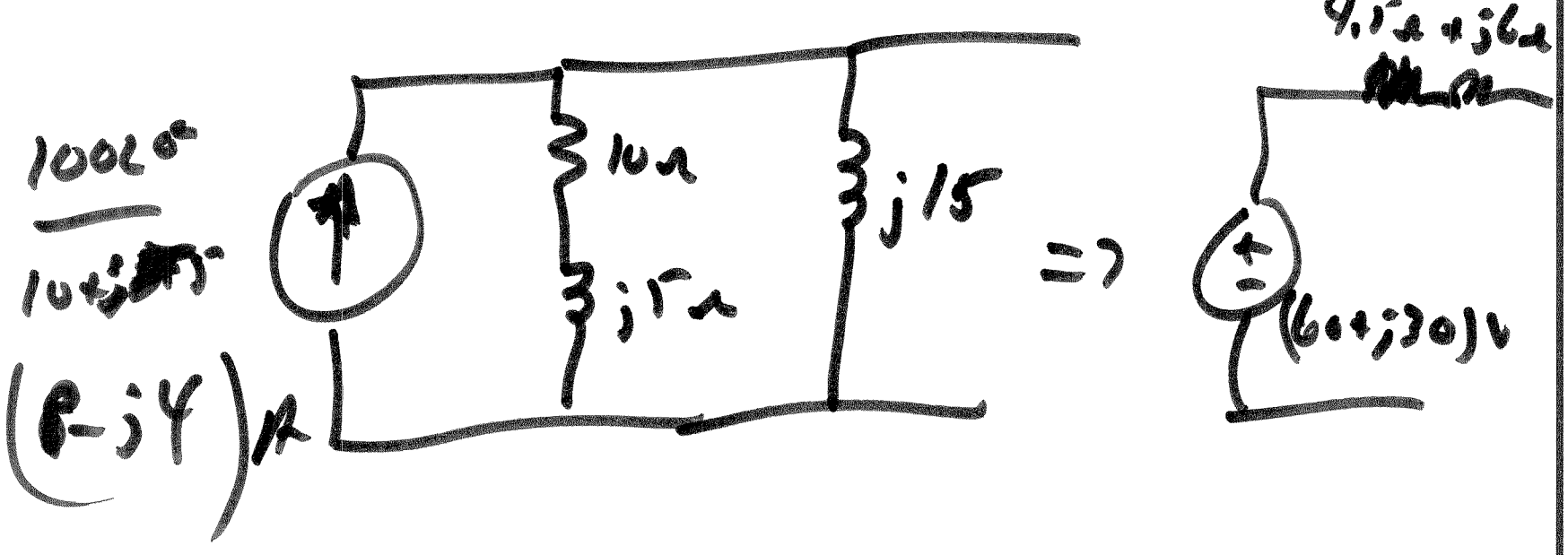
$$V_{th} = (60 + j30)\,V$$

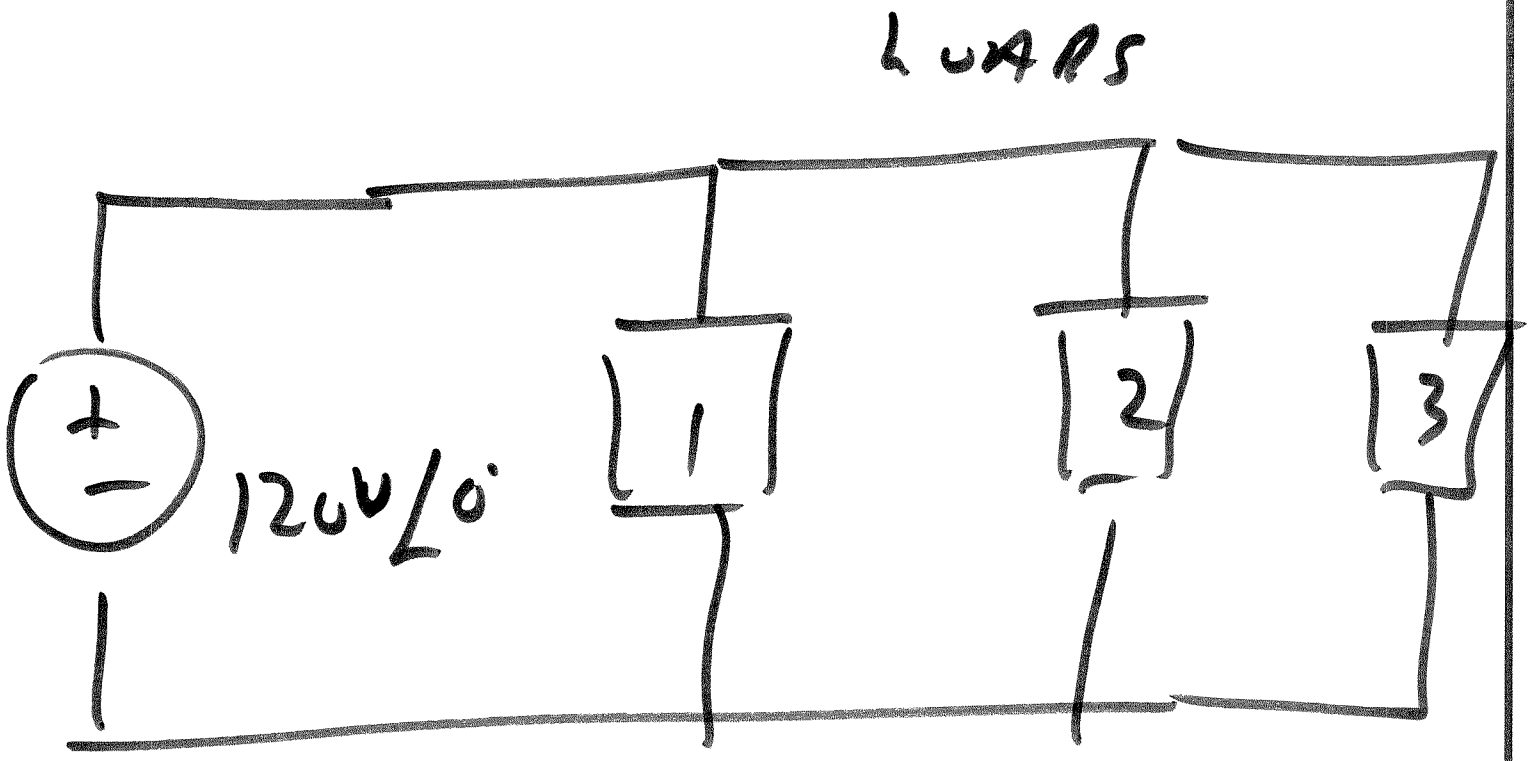
$$Z_{th} = \frac{(j15)(10+j5)}{(j15)+(10+j5)} = 4.5 + j6$$

$$I_{max\,p} = \frac{60+j30}{9} = 6.67 + j3.33$$

$$P_{max} = |I_{max\,p}|^2 (4.5) = 250\,W$$

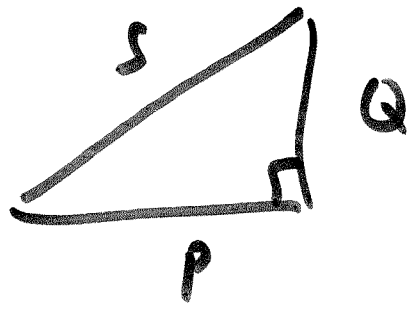

10
j15
160∠0° V
j15


100∠0°
10+j5 Ω
10 Ω
j15 Ω
j15
(8−j4) Ω
=>
6.15 Ω + j6 Ω
(60+j30) V



FIND P, Q, S, I FOR EACH LOAD

GIVEN
1: 10 KVA; 0.9 pf LAGGING
2: 4 kW + j 3 kVAR
3: 5 A ∠ 10° INDUCTIVE (PARTLY)



$$P_1 = (10\,kVA)(0.9) = 9\,kW$$

$$Q_1 = \sqrt{S_1^2 - P_1^2} = \sqrt{(10\,kVA)^2 - (9\,kW)^2}$$

$$Q_1 = 4.36\,kVAr$$

$$\theta_1 = \cos^{-1} 0.9$$

$$I_1 = \frac{S_1}{V_1} = \frac{10\,kVA}{120V} = 83.4\angle -25.9°$$

$P_2 = 4\ kW$

$Q_2 = 3\ kVAr$

$S_2 = \sqrt{P_2^2 + Q_2^2} = 5\ kVA$

$\theta_2 = \cos^{-1} 0.8$

$I_2 = \left(\frac{S_2}{V_2}\right)^* = \frac{5\ kVA}{120\ V} = 41.67 \angle -36.9°\ A$

$I_3 = 5 \angle -10°\ A$

$pf_3 = \cos 16°$

$S_3 = V I_3^* = (120 \angle 0°)(5 \angle -16°) = 600 \angle -16°\ VA$

$P_3 = 600 \cos 16° = 576.7\ W$

$Q_3 = 600 \sin 16° = 165.3\ VAr$

Power output example. For a circuit with a 100V AC source and a (10+j5)Ohm source impedance, find the output power to a load resistance of 15 Ohms.

$V_S := 100 \cdot V \qquad Z_S := (10 + j \cdot 5) \cdot \Omega \qquad j := \sqrt{-1}$

$|Z_S| = 11.18\,\Omega \qquad R_L := 15 \cdot \Omega$

Find current and then power.

$$I := \frac{V_S}{Z_S + R_L} = (3.846 - 0.769i)\,A$$

$|I| = 3.922\,A \qquad \arg(I) = -11.31\,deg$

$P_o := (|I|)^2 \cdot R_L = 230.769\,W$

Checking, find voltage at the load.

$$V_L := V_S \cdot \frac{R_L}{Z_S + R_L} = (57.692 - 11.538i)\,V$$

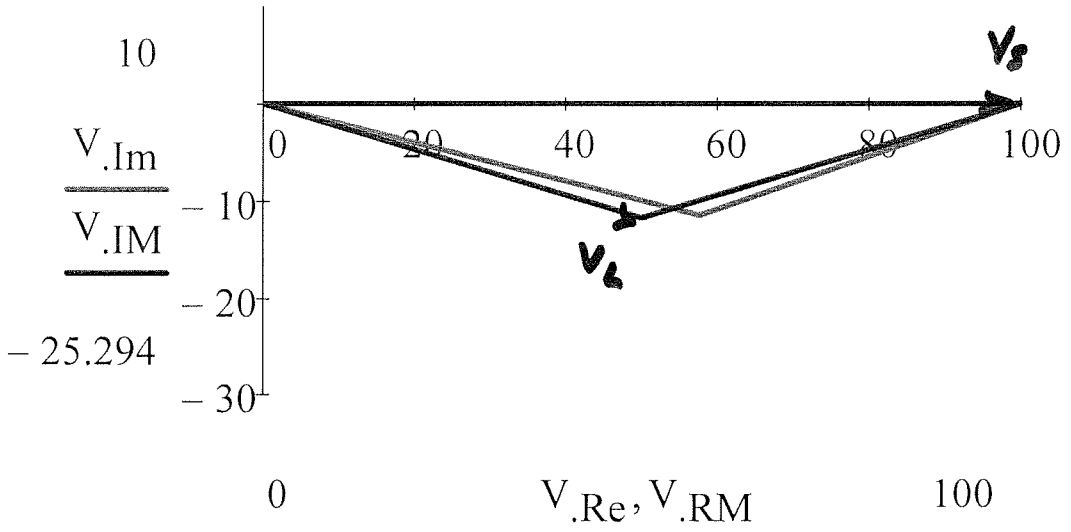
$|V_L| = 58.835\,V \qquad \arg(V_L) = -11.31\,deg$

Or, checking,

$V_L := I \cdot R_L = (57.692 - 11.538i)\,V$

Plot

$$V_{Re} := \begin{pmatrix} 0 \\ V_S \\ Re(V_L) \\ 0 \end{pmatrix} \qquad V_{Im} := \begin{pmatrix} 0 \\ 0 \\ Im(V_L) \\ 0 \end{pmatrix}$$


10
V.Im
V.IM
– 10
– 20
– 25.294
– 30
0
20
40
60
80
100
V_s
V_L
0
V.Re, V.RM
100

Max power transfer condition:

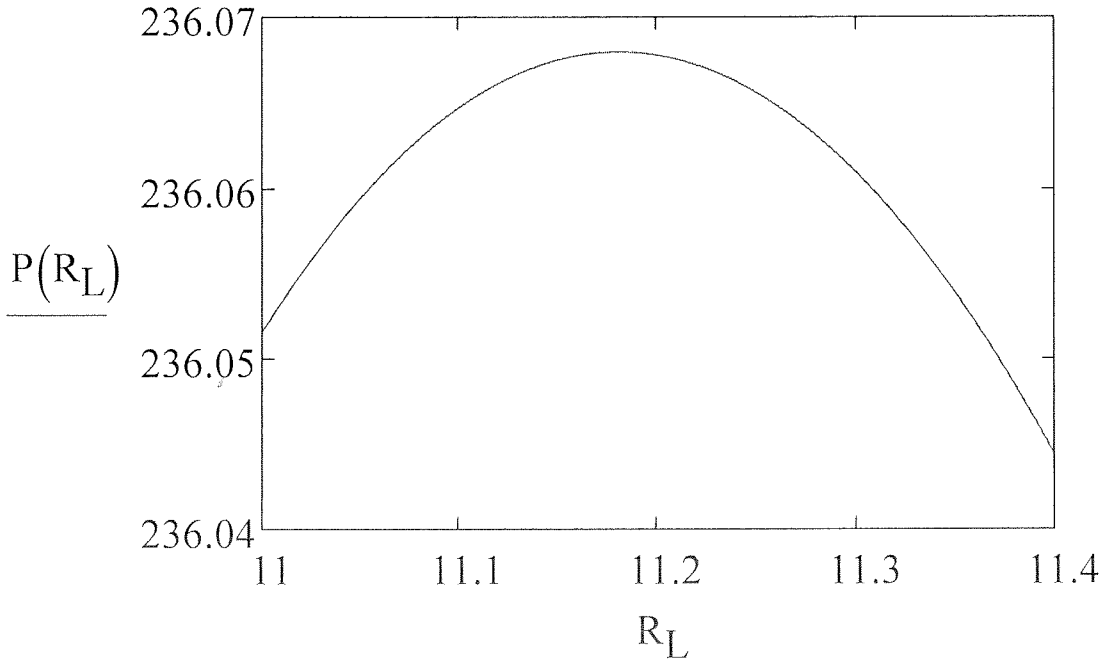
$$R_{LM} := \left|Z_S\right| = 11.18\,\Omega$$

$$V_{LM} := V_S \cdot \frac{R_{LM}}{Z_S + R_{LM}} = (50 - 11.803i)\,V$$

$$\left|V_{LM}\right| = 51.374\,V \quad \arg\left(V_{LM}\right) = -13.283\,deg$$

Plot

$$V_{RM} := \begin{pmatrix} 0 \\ V_S \\ Re\left(V_{LM}\right) \\ 0 \end{pmatrix} \qquad V_{IM} := \begin{pmatrix} 0 \\ 0 \\ Im\left(V_{LM}\right) \\ 0 \end{pmatrix}$$


236.07
236.06
236.05
236.04
P(R_L)
11
11.1
11.2
11.3
11.4
R_L

Max Power Transfer Example. For a circuit with a 100V AC source and a (10+j5)Ohm source impedance, find the output power as a function of load resistance for a resistance only load.

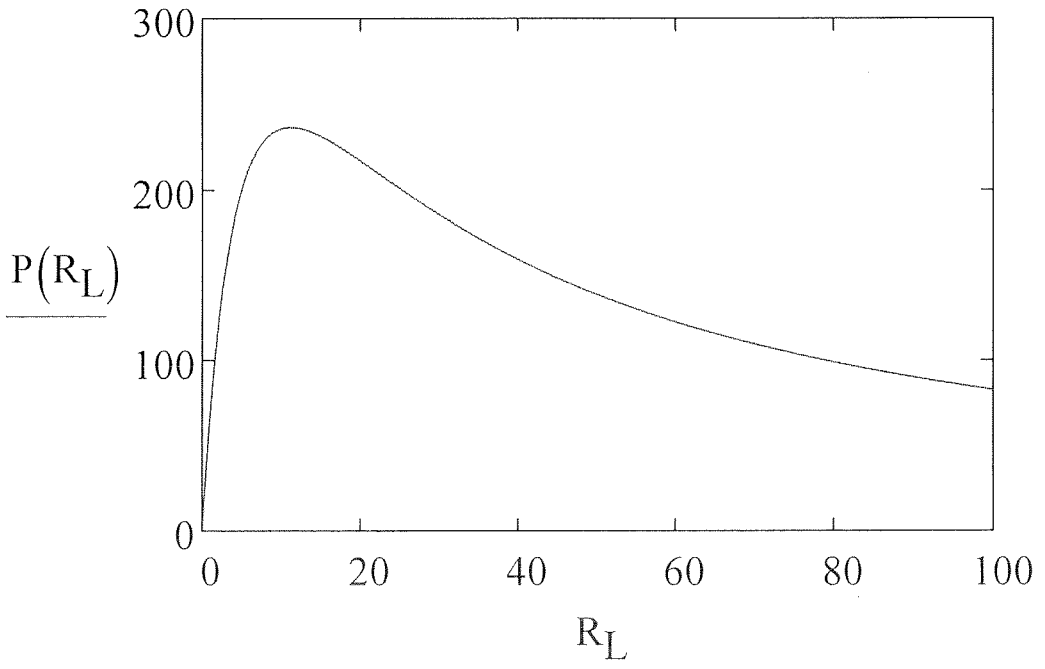
$$V_S := 100 \cdot V \qquad Z_S := (10 + j \cdot 5) \cdot \Omega \qquad j := \sqrt{-1}$$

Find current and then power.

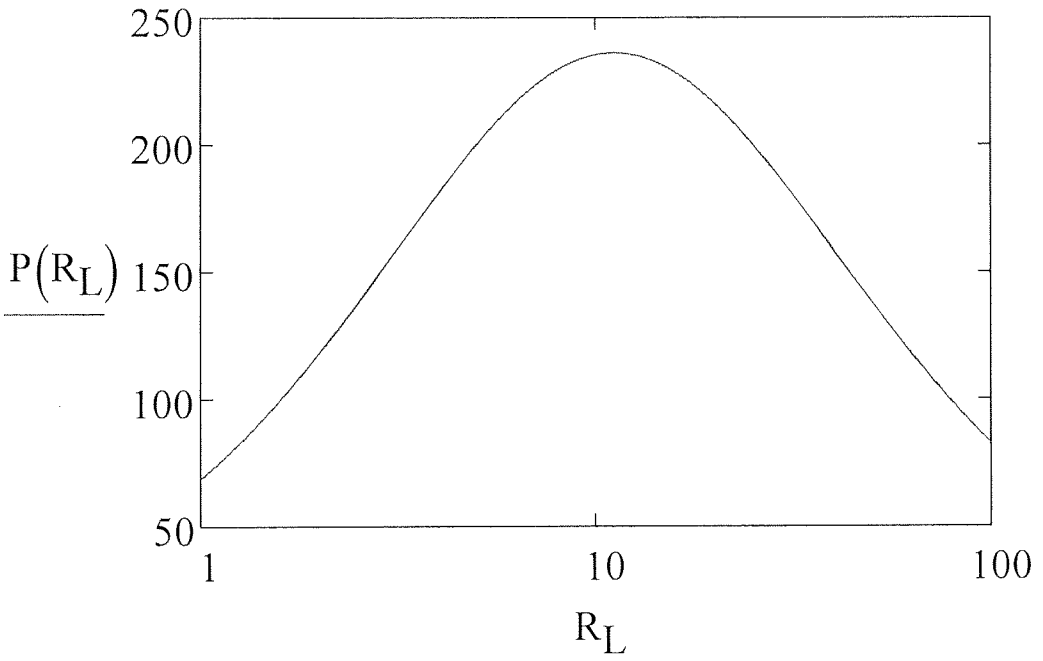
$$I(R_L) := \frac{V_S}{Z_S + R_L}$$

$$P(R_L) := \left(\left|I(R_L)\right|\right)^2 \cdot R_L$$

Plot, first on a linear scale and then on a semilog scale.



And on the semilog scale



ECE 320

Energy Systems I

Lesson 5

AC Power