

Effect of inductor resistance

For boost converter, recall

$$V_o = \frac{V_s}{1 - D}$$

converters are not ideal. Some effects are possible to analyze. for example, inductors have resistance. It is a problem that we are trying to get rid of, but have not yet done so at a reasonable price. So we find,

$$P_{out} + P_{loss} = P_{source}$$

$$V_o \cdot I_o + I_{Lrms}^2 \cdot r_L = V_s \cdot I_{Lave}$$

We know from our work with the boost converter that

$$I_o = I_{Lave} \cdot (1 - D)$$

$$I_o = \frac{V_o}{R}$$

Combining these two equations,

$$\frac{V_o}{R} = I_{Lave} \cdot (1 - D)$$

$$I_{Lave} = \frac{V_o}{R \cdot (1 - D)}$$

Now go back to the power balance and substitute for the average inductor current.

$$V_o \cdot I_o + I_{Lrms}^2 \cdot r_L = V_s \cdot I_{Lave}$$

Because we assume a small ripple,

$$I_{Lave} = I_{Lrms}$$

$$V_o \cdot \frac{V_o}{R} + \left[\frac{V_o}{R \cdot (1 - D)} \right]^2 \cdot r_L = V_s \cdot \frac{V_o}{R \cdot (1 - D)}$$

Multiply through by $\frac{R}{V_o^2}$

$$1 + \frac{r_L}{[R \cdot (1 - D)]^2} = \frac{V_s}{V_o} \cdot \frac{1}{1 - D}$$

Simplify

$$\frac{V_s}{V_o} = (1 - D) + \frac{r_L}{R \cdot (1 - D)}$$

$$\frac{V_o}{V_s} = \frac{1}{\left[(1 - D) + \frac{r_L}{R \cdot (1 - D)} \right]} = \frac{1 - D}{(1 - D)^2 + \frac{r_L}{R}}$$

Efficiency

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}}$$

$$\eta = \frac{\frac{V_o^2}{R}}{\frac{V_o^2}{R} + I_{Lrms}^2 \cdot r_L} = \frac{\frac{V_o^2}{R}}{\frac{V_o^2}{R} + \left[\frac{V_o}{R \cdot (1 - D)} \right]^2 \cdot r_L}$$

Divide out the $\frac{V_o^2}{R}$

$$\eta = \frac{1}{1 + \frac{r_L}{R \cdot (1 - D)^2}}$$

