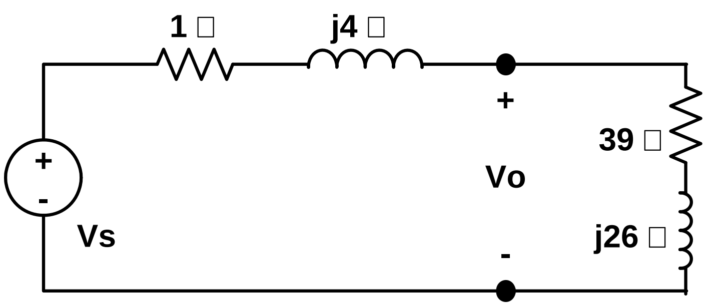
ECE 320
Homework
Due: Lesson 3 (Recite Lesson 2)

1. In the circuit shown in Figure 1, a load having an impedance of 39+j26 Ohms is fed from a voltage source through a line having an impedance of 1+j4 Ohms. The voltage source is 250V rms at 60 Hz.
   a. Calculate the load current and the load voltage.
   b. Calculate the real power and the reactive power delivered to the load.
   c. Calculate the real power losses in the line.
   d. Calculate the real power and the reactive power supplied by the source.



The impedance units are Ohms.

a. Calculate the load current and the load voltage.

$Z_L := (39 + j \cdot 26) \cdot \Omega$ $\quad Z_{line} := (1 + j \cdot 4) \cdot \Omega$ $\quad V_s := 250 \cdot V$ $\quad f_s := 60 \cdot Hz$ $\quad j := \sqrt{-1}$

The load current is found from a loop equation,

$I_L := \dfrac{V_s}{(Z_L + Z_{line})}$ $\quad I_L = (4 - 3i)\,A$ $\quad |I_L| = 5\,A$ $\quad \arg(I_L) = -36.87 \cdot deg$

Express ANSWERS in polar form.

Use Ohm's Law to find the load voltage.

$V_L := I_L \cdot Z_L$ $\quad V_L = (234 - 13i)\,V$ $\quad |V_L| = 234.361\,V$ $\quad \arg(V_L) = -3.18 \cdot deg$

Check with voltage division.

$V_L := \dfrac{Z_L}{Z_L + Z_{line}} \cdot V_s$ $\quad V_L = (234 - 13i)\,V$

b. Calculate the real power and the reactive power delivered to the load.

$VAr := V \cdot A$

Real Power $\quad P_L := (|I_L|)^2 \cdot Re(Z_L) = 975\,W$

Reactive Power $\quad Q_L := (|I_L|)^2 \cdot Im(Z_L) = 650 \cdot VAr$

Check:

$S_L := \dfrac{(|V_L|)^2}{\overline{Z_L}} = (975 + 650i)\,W$

c. Calculate the real power losses in the line.

$P_{loss} := \left(\left|I_L\right|\right)^2 \cdot \mathrm{Re}\left(Z_{line}\right)$ $\qquad P_{loss} = 25\,\mathrm{W}$

d. Calculate the real power and the reactive power supplied by the source.

$S_{source} := V_s \cdot \overline{I_L}$ $\qquad S_{source} = (1 + 0.75i)\cdot \mathrm{kV}\cdot \mathrm{A}$

$P_{source} := \mathrm{Re}\left(S_{source}\right) = 1\cdot \mathrm{kW}$

$Q_{source} := \mathrm{Im}\left(S_{source}\right) = 750\cdot \mathrm{VAr}$

Check

$P_{source} := P_L + P_{loss} = 1\cdot \mathrm{kW}$

$Q_{source} := \sqrt{\left(\left|S_{source}\right|\right)^2 - P_{source}{}^2} = 750\cdot \mathrm{VAr}$

2. For the same circuit shown in Figure 1, the source is a step voltage of 12.0u(t) where u(t) is a unit step function. The inductors are labeled with their 60Hz impedance. Find the voltage Vo as a function of time. Use of software is encouraged.

Restste the given.

$Z_L := (39 + j\cdot 26)\cdot \Omega$ $\quad Z_{line} := (1 + j\cdot 4)\cdot \Omega$ $\quad V_s := 250\cdot \mathrm{V}$ $\quad f_s := 60\cdot \mathrm{Hz}$ $\qquad j := \sqrt{-1}$

Calculate the inductances.

$L_L := \dfrac{\mathrm{Im}\left(Z_L\right)}{2\cdot \pi \cdot f_s} = 68.967\cdot \mathrm{mH}$ $\qquad L_s := \dfrac{\mathrm{Im}\left(Z_{line}\right)}{2\cdot \pi \cdot f_s} = 10.61\cdot \mathrm{mH}$

Restate the resistances.

$R_L := \mathrm{Re}\left(Z_L\right) = 39\,\Omega$ $\qquad R_s := \mathrm{Re}\left(Z_{line}\right) = 1\,\Omega$

By a loop equation, the current is

$$V_o(s) = \frac{V_s}{s}\cdot\left[\frac{L_L\cdot s + R_L}{\left(R_L + R_s\right) + \left(L_L + L_s\right)\cdot s}\right]$$

$R_L = 39\,\Omega$ $\qquad R_s = 1\,\Omega$ $\qquad L_L := 69\cdot \mathrm{mH}$ $\qquad L_s := 10.6\cdot \mathrm{mH}$ $\quad V_s = 250\,\mathrm{V}$

Solve for the time domain solution, an inverse LaPlace

$$\frac{V_s}{s}\cdot\left[\frac{L_L\cdot s + R_L}{\left(R_L + R_s\right) + \left(L_L + L_s\right)\cdot s}\right] \text{invlaplace}, s, \tau \;\rightarrow 281.19565217391304348\cdot \mathrm{V} - 31.195652173913043478\cdot \mathrm{V}\cdot e^{-\frac{0.50}{}}$$

This answer does not make sense. It does not agree with the initial or final voltage that can be easily calculated from the initial value theorem and the final value theorem.

$$V_{o\_init} := V_s \cdot \frac{L_L}{L_L + L_s} = 216.709\ \text{V} \qquad V_{o\_final} := V_s \cdot \frac{R_L}{R_L + R_s} = 243.75\ \text{V}$$

Let us use the linearity property of the LaPlace Transform and sum the two additive pieces of the expression that we have.

$$\frac{V_s}{s} \cdot \left[\frac{L_L \cdot s}{(R_L + R_s) + (L_L + L_s) \cdot s}\right] \text{invlaplace}, s, \tau \;\rightarrow 216.7085427135678392 \cdot V \cdot e^{-\frac{0.50251256281407035176 \cdot \tau \cdot \Omega}{mH}}$$

$$\frac{V_s}{s} \cdot \left[\frac{R_L}{(R_L + R_s) + (L_L + L_s) \cdot s}\right] \text{invlaplace}, s, \tau \;\rightarrow 243.75 \cdot V - 243.75 \cdot V \cdot e^{-\frac{0.50251256281407035176 \cdot \tau \cdot \Omega}{mH}}$$

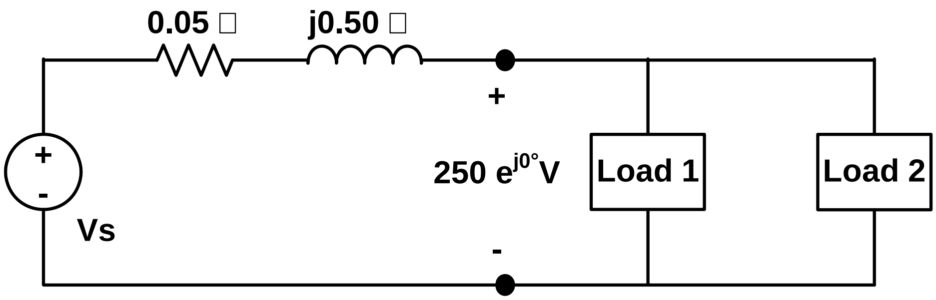
$$v_o(t) = \left(243.75 - 243.75 \cdot e^{-502.5 \cdot t} + 216.71 \cdot e^{-502.5 \cdot t}\right) \cdot V = \left(243.75 - 27.04 \cdot e^{-502.5 \cdot t}\right) \cdot V$$

This agrees with the initial and final conditions. I do not know why MathCAD does not get the right answer on the first try here.

The single time constant is

$$\tau := \left(\frac{L_s + L_L}{R_s + R_L}\right) = 1.99 \cdot ms$$

3. Two loads in the circuit shown in Figure 2 can be described as follows:
   Load 1 absorbs an average of 8kW at a leading power factor of 0.80.
   Load 2 absorbs 20kVA at a lagging power factor of 0.60.
   a. Determine the power factor of the combined two loads in parallel.
   b. Determine the source current.
   c. If the frequency is 60Hz, find the value of the capacitor which, if placed in parallel with the two loads, would correct the power factor to 1.00.



The impedance units are Ohms.

leading := 1

lagging := 1

kVA := kV·A

a. Determine the power factor of the combined two loads in parallel.

Load 1

$P_1 := 8\cdot kW$ $\quad pf_1 := 0.80\cdot leading$ $\quad S_1 := \dfrac{P_1}{pf_1} = 10\cdot kW$ $\quad Q_1 := -\sqrt{S_1\cdot S_1 - P_1\cdot P_1} = -6\cdot kW$

Load 2

$S_2 := 20\cdot kVA$ $\quad pf_2 := 0.60\cdot lagging$ $\quad P_2 := S_2\cdot pf_2 = 12\cdot kW$ $\quad Q_2 := \sqrt{S_2\cdot S_2 - P_2\cdot P_2} = 16\cdot kW$

Add up the orthogonal components

$P_T := P_1 + P_2 = 20\cdot kW$ $\quad Q_T := Q_1 + Q_2 = 10\cdot kW$ $\quad S_T := \sqrt{P_T\cdot P_T + Q_T\cdot Q_T} = 22.361\cdot kW$

Calculate the power factor from the definition.

$$pf_T := \frac{P_T}{S_T} = 0.894\cdot lagging$$

b. Determine the source current.

$V_0 := 250\cdot V$

By definition, the product of voltage and current magnitudes is apparent power.

$$I_S := \frac{S_T}{V_0} = 89.443\,A$$

The phase angle on apparent power is

$$\theta_T := -\operatorname{atan}\left(\frac{Q_T}{P_T}\right) = -26.565 \cdot deg$$

The phase angle on V is zero as given. Therefore, the current has the phase angle

$$\theta_I := \theta_T - 0 \cdot deg = -26.565 \cdot deg$$

c. If the frequency is 60Hz, find the value of the capacitor which, if placed in parallel with the two loads, would correct the power factor to 1.00.

$$kVAr := 10^3 \cdot V \cdot A \qquad \omega := 2 \cdot \pi \cdot 60 \cdot \frac{rad}{sec}$$

The capacitor proposed must exactly cancel out the reactive power of the combined load.

$$Q_C := -Q_T = -10 \cdot kVAr$$

The reactive power in a parallel circuit is

$$Q_C = \frac{\left(\left|V_0\right|\right)^2}{\left(\frac{-1}{\omega \cdot C_c}\right)}$$

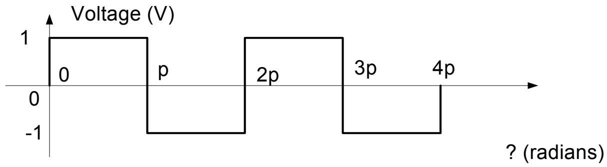
Solve for the capacitance.

$$C_c := \frac{-Q_C}{\left(\left|V_0\right|\right)^2 \cdot \omega} = 424.413 \cdot \mu F$$

4. Calculate the rms values of the voltages with the waveforms shown in Figure 3.

$$V_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T v(\theta)^2 \, d\theta}$$

where T is the period, here 2π.



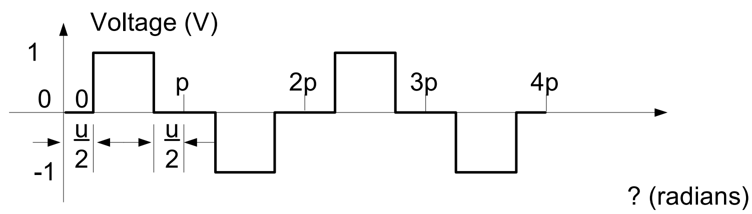
$$v(\theta) := \begin{vmatrix} 1 & \text{if } 0 \le \theta < \pi \\ (-1) & \text{if } \pi \le \theta \le 2\cdot\pi \end{vmatrix}$$

Symmetry applies here, so I could have just evaluated this integral over the first half cycle and then used the half cycle as the period.

$$V_{rms} := \sqrt{\frac{1}{2\cdot\pi} \cdot \int_0^{2\cdot\pi} v(\theta)^2 \, d\theta} \qquad V_{rms} = 1$$

For the second one, the same definition applies but the formulation is a little more work.

$$V_{rms} = \sqrt{\frac{1}{\pi} \cdot \int_{\frac{u}{2}}^{\pi - \frac{u}{2}} 1^2 \, d\theta}$$



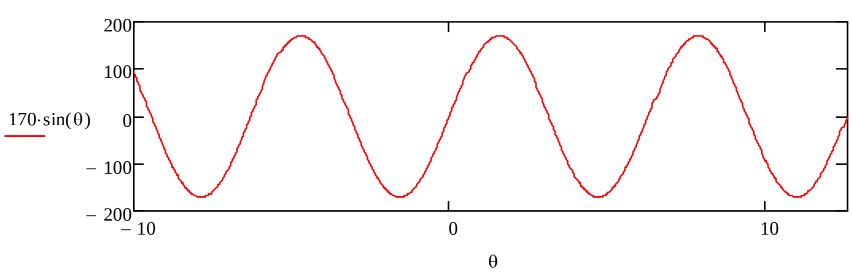
$$V_{rms} = \sqrt{\frac{1}{\pi} \cdot \left(\pi - \frac{u}{2} - \frac{u}{2}\right)}$$

Here, I used the symmetry to my advantage.

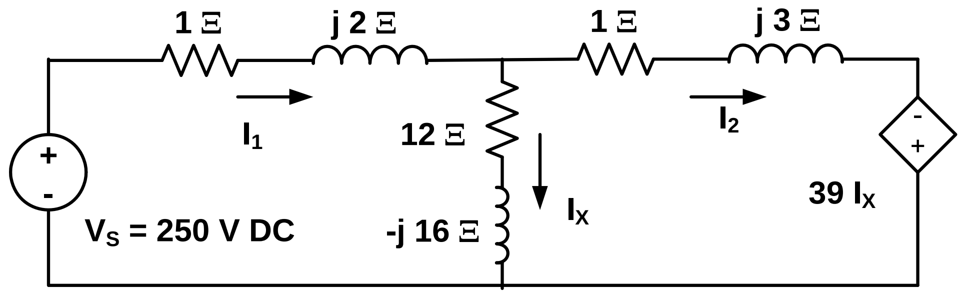
$$V_{rms} = \sqrt{\frac{\pi - u}{\pi}}$$

The last one is a sine wave of amplitude 170V as labelled. The rms value is

$$V_{rms} := \frac{170}{\sqrt{2}} \cdot V = 120.2 \text{ V}$$



5. Consider the circuit shown in Figure 4.
   a. Find the currents in each branch of this circuit.
   b. Find the real power and reactive power in each element (resistor, inductor, capacitor, and source).



Impedance units are Ohms.

A nodal solution:

$$\frac{V_x - V_S}{1 + j\cdot 2} + \frac{V_x}{12 - j\cdot 16} + \frac{V_S + 39\cdot I_x}{1 + j\cdot 3} = 0 \qquad I_x = \frac{V_x}{12 - j\cdot 16} \qquad V_S := 250$$

Rearrange the equations,

$$V_x\cdot\left(\frac{1}{1 + j\cdot 2} + \frac{1}{12 - j\cdot 16} + \frac{1}{1 + j\cdot 3}\right) + I_x\cdot\frac{39}{1 + j\cdot 3} = \frac{V_S}{1 + j\cdot 2} \qquad \frac{V_x}{12 - j\cdot 16} - I_x = 0$$

$$\begin{bmatrix} \frac{1}{1 + j\cdot 2} + \frac{1}{12 - j\cdot 16} + \frac{1}{1 + j\cdot 3} & \frac{39}{1 + j\cdot 3} \\ \frac{1}{(12 - j\cdot 16)} & -1 \end{bmatrix}\cdot\begin{pmatrix} V_x \\ I_x \end{pmatrix} = \begin{pmatrix} \frac{V_S}{1 + j\cdot 2} \\ 0 \end{pmatrix}$$

Solve

$$\begin{pmatrix} V_x \\ I_x \end{pmatrix} := \begin{bmatrix} \frac{1}{1 + j\cdot 2} + \frac{1}{12 - j\cdot 16} + \frac{1}{1 + j\cdot 3} & \frac{39}{1 + j\cdot 3} \\ \frac{1}{(12 - j\cdot 16)} & -1 \end{bmatrix}^{-1}\cdot\begin{pmatrix} \frac{V_S}{1 + j\cdot 2} \\ 0 \end{pmatrix} = \begin{pmatrix} 83.692 - 31.086i \\ 3.754 + 2.415i \end{pmatrix}$$

Analyze to find the other two currents,

$$I_1 := \frac{V_S - V_x}{(1 + j\cdot 2)} = 45.696 - 60.306i \qquad I_2 := \frac{V_x + 39\cdot I_x}{1 + j\cdot 3} = 41.942 - 62.721i$$

$$|I_1| = 75.663 \qquad |I_2| = 75.452 \qquad |I_x| = 4.464$$

$$\arg(I_1) = -52.848\cdot deg \qquad \arg(I_2) = -56.229\cdot deg \qquad \arg(I_x) = 32.754\cdot deg$$

A loop solution,

$$-V_S + (1 + j\cdot 2)\cdot I_1 + (12 - j\cdot 16)\cdot (I_1 - I_2) = 0$$

$$(12 - j\cdot 16)\cdot (I_2 - I_1) + (1 + j\cdot 3)\cdot I_2 - 39\cdot I_X = 0$$

$$I_x = I_1 - I_2$$

Arrange and solve,

$$\begin{bmatrix} 1 + j\cdot 2 + 12 - j\cdot 16 & -(12 - j\cdot 16) & 0 \\ -(12 - j\cdot 16) & 12 - j\cdot 16 + 1 + j\cdot 3 & -39 \\ 1 & -1 & -1 \end{bmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \\ I_x \end{pmatrix} = \begin{pmatrix} V_S \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} 1 + j\cdot 2 + 12 - j\cdot 16 & -(12 - j\cdot 16) & 0 \\ -(12 - j\cdot 16) & 12 - j\cdot 16 + 1 + j\cdot 3 & -39 \\ 1 & -1 & -1 \end{bmatrix}^{-1} \cdot \begin{pmatrix} V_S \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 45.696 - 60.306i \\ 41.942 - 62.721i \\ 3.754 + 2.415i \end{pmatrix}$$

$|I_1| = 75.663$ $\quad |I_2| = 75.452$ $\quad |I_x| = 4.464$

$\arg(I_1) = -52.848\cdot deg$ $\quad \arg(I_2) = -56.229\cdot deg$ $\quad \arg(I_x) = 32.754\cdot deg$

b. Find the real power and reactive power in each element (resistor, inductor, capacitor, and source).

Units of Real Power P are Watts;
Units for Reactive power Q are VArs.

$$P_{R1} := \left(|I_1|\right)^2\cdot(1) = 5.725 \times 10^3$$

$$P_{R2} := \left(|I_2|\right)^2\cdot(1) = 5.693 \times 10^3$$

$$P_{Rx} := \left(|I_x|\right)^2\cdot(12) = 239.12$$

$$P_S := \mathrm{Re}\left(V_S\cdot\overline{I_1}\right) = 1.142 \times 10^4$$

$$P_0 := \mathrm{Re}\left(39\cdot I_x\cdot\overline{I_2}\right) = 233.142$$

$$Q_1 := \left(|I_1|\right)^2\cdot(2) = 1.145 \times 10^4$$

$$Q_2 := \left(|I_2|\right)^2\cdot(3) = 1.708 \times 10^4$$

$$Q_C := \left(|I_x|\right)^2\cdot(-16) = -318.827$$

$$Q_0 := \mathrm{Im}\left(39\cdot I_x\cdot\overline{I_2}\right) = 1.313 \times 10^4$$

$$Q_S := \mathrm{Im}\left(V_S\cdot\overline{I_1}\right) = 1.508 \times 10^4$$

$$\frac{251256281407035176 \cdot \tau \cdot \Omega}{mH}$$