ECE 320
HW4 Per unit

1. 2.1 b on page 144

2.1 b A 20kVA, 8000/480V distribution transformer has the resistance and reactances shown in the upper figure. Find the per unit equivalent circuit.

$$kVA := kV \cdot A$$

On the left side of the transformer,

$$Z_{baseH} := \frac{(8000 \cdot V)^2}{100 \cdot kVA} \qquad Z_{baseH} = 640\,\Omega$$

On the right side of the transformer,

$$Z_{baseL} := \frac{(480 \cdot V)^2}{100 \cdot kVA} \qquad Z_{baseL} = 2.304\,\Omega$$

Converting each element to per unit by dividing its ohmic value by the Zbase on its side of the transformer:

$$R_P := 5 \cdot \Omega \qquad R_S := 0.05 \cdot \Omega$$

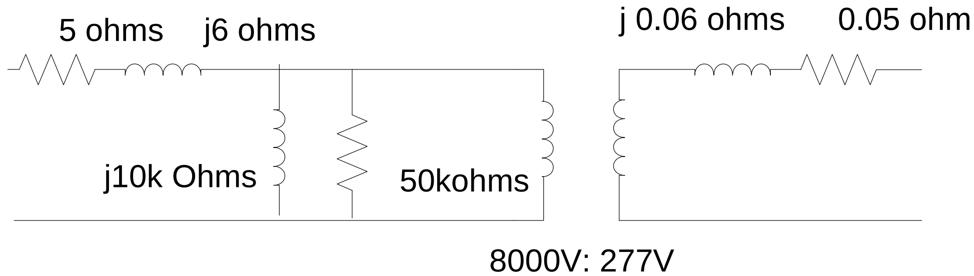
$$X_P := 6 \cdot \Omega \qquad X_S := 0.06 \cdot \Omega$$

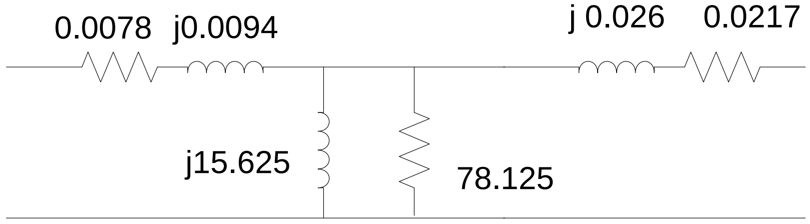
$$R_C := 50 \cdot k\Omega \qquad X_M := 10 \cdot k\Omega$$

$$R_{Ppu} := \frac{R_P}{Z_{baseH}} \qquad R_{Ppu} = 7.813 \times 10^{-3} \qquad R_{Spu} := \frac{R_S}{Z_{baseL}} \qquad R_{Spu} = 0.0217$$

$$X_{Ppu} := \frac{X_P}{Z_{baseH}} \qquad X_{Ppu} = 9.375 \times 10^{-3} \qquad X_{Spu} := \frac{X_S}{Z_{baseL}} \qquad X_{Spu} = 0.02604$$

$$R_{Cpu} := \frac{R_C}{Z_{baseH}} \qquad R_{Cpu} = 78.125 \qquad X_{Mpu} := \frac{X_M}{Z_{baseH}} \qquad X_{Mpu} = 15.625$$





2. 2.1 c on page 144. Work the problem in per unit. Leave the answer in per unit.

2. Problem 2.1c on page 144 of the textbook

Asume that this transformer is supplying rated load at 277 V and 0.85pf lagging. What is this transformer's input voltage? What is its voltage regulation?

$lagging := 1 \qquad j := \sqrt{-1}$

Restate the circuit parameters.

$V_{Lpu} := 1.0 \qquad pf_L := 0.85 \cdot lagging \qquad I_{Lpu} := 1.0 \quad \theta_L := acos(pf_L) = 31.788\,deg$

$R_{Spu} = 0.022 \qquad R_{Ppu} = 7.813 \times 10^{-3} \qquad X_{Spu} = 0.026 \qquad X_{Ppu} = 9.375 \times 10^{-3}$

$R_{Cpu} = 78.125 \qquad X_{Mpu} = 15.625$

Use circuit analysis to work toward the input voltage.

$$V_{Mpu} := V_{Lpu} + \left(R_{Spu} + j \cdot X_{Spu}\right) \cdot I_{Lpu} \cdot e^{-j \cdot \theta_L} = 1.032 + 0.011i$$

$$I_{Ppu} := I_{Lpu} \cdot e^{-j \cdot \theta_L} + \frac{V_{Mpu}}{j \cdot X_{Mpu}} + \frac{V_{Mpu}}{R_{Cpu}} = 0.864 - 0.593i$$

$$V_{inpu} := V_{Mpu} + I_{Ppu} \cdot \left(R_{Ppu} + j \cdot X_{Ppu}\right) = 1.044 + 0.014i$$

$$\left|V_{inpu}\right| = 1.045 \qquad arg\left(V_{inpu}\right) = 0.777\,deg$$

Calculate the voltage regulation.

$$V_{regulation} := \frac{\left|V_{inpu}\right| - \left|V_{Lpu}\right|}{\left|V_{inpu}\right|} = 4.266\,\%$$

3. Problem 2.24 on page 150-151 of the textbook. Work the problem and do the calculations in per unit.

Figure P2-24 shows a one-line diagram of a power system consisting of a three phase 480V, 60 Hz generator supplying two loads through a transmission line with a pair of transformers at either end.

a. Sketch the per phase equivalent circuit of this power system.

$R_{T1} := 0.010$ $\quad Z_{line} := (1.5 + j\cdot 10)\cdot\Omega$ $\quad R_{T2} := 0.020$ $\quad Z_{Load1} := 0.45\cdot e^{j\cdot 36.87\cdot deg}\cdot\Omega$

$X_{T1} := 0.040$ $\quad X_{T2} := 0.085$ $\quad Z_{Load2} := -j\cdot 0.8\cdot\Omega$

$S_{T1} := 10^{6}\cdot V\cdot A$ $\quad S_{T2} := 500\cdot kV\cdot A$

Define the system base for this power system.

$S_{base} := 10^{6} V\cdot A$ $\quad V_{baseg} := 480\cdot V$ $\quad V_{baseline} := 14.4\cdot kV$ $\quad V_{baseL} := 480\cdot V$

Define base impedances as necessary.

$$Z_{baseline} := \frac{V_{baseline}^{2}}{S_{base}} = 207.36\,\Omega \qquad Z_{baseT2} := \frac{V_{baseline}^{2}}{S_{T2}} = 414.72\,\Omega \qquad Z_{baseL} := \frac{V_{baseL}^{2}}{S_{base}} = 0.23\,\Omega$$

Convert the transformer T1 to the system base.

$$R'_{T2} := R_{T2}\cdot\frac{Z_{baseT2}}{Z_{baseline}} = 0.04 \qquad X'_{T2} := X_{T2}\cdot\frac{Z_{baseT2}}{Z_{baseline}} = 0.17$$

Calculate the line impedance in per unit on the system base.

$$Z'_{line} := \frac{Z_{line}}{Z_{baseline}} = 7.234 \times 10^{-3} + 0.048i$$

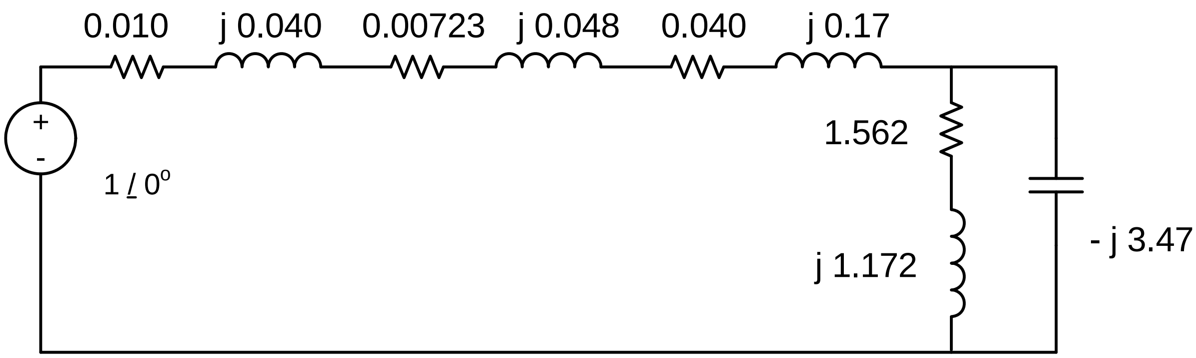
Calculate the load impedances in per unit on the system base.

$$Z'_{Load1} := \frac{Z_{Load1}}{Z_{baseL}} = 1.562 + 1.172i \qquad Z'_{Load2} := \frac{Z_{Load2}}{Z_{baseL}} = -3.472i$$

The input voltage is 480V, which is 1.0 per unit. The transformer T1 is already on the system base.

$V_{in} := 1.0$ $\quad R'_{T1} := 0.010$ $\quad X'_{T1} := 0.040$

Draw the result



b. With the switch open (only Load 1 connected), find the real power P, reactive power Q, and apparent power S supplied by the generator. What is the power factor of the generator?

Find the current with a loop equation.

$$I_{gen1} := \frac{V_{in}}{R'_{T1} + j \cdot X'_{T1} + Z'_{line} + R'_{T2} + j \cdot X'_{T2} + Z'_{Load1}} = 0.347 - 0.306i \qquad \left|I_{gen1}\right| = 0.463$$

$$\arg\left(I_{gen1}\right) = -41.442 \cdot deg$$

Calculate the complex power in and then sort out the P, Q, and S of the generator.

$$S_{in} := V_{in} \cdot \overline{I_{gen1}} = 0.347 + 0.306i \qquad \left|S_{in}\right| = 0.463$$

$$P_{gen1} := \mathrm{Re}\left(S_{in}\right) = 0.347 \qquad Q_{gen1} := \mathrm{Im}\left(S_{in}\right) = 0.306 \qquad S_{gen1} := \left|S_{in}\right| = 0.463$$

Power factor of the generator is

$$pf_{gen1} := \frac{P_{gen1}}{S_{gen1}} = 0.75 \cdot lagging$$

c. With the switch closed, find the real power P, reactive power Q, apparent power S of the generator. What is the power factor of the generator?

Calculate the new load impedance.

$$Z'_{Load} := \frac{1}{\frac{1}{Z'_{Load1}} + \frac{1}{Z'_{Load2}}} = 2.436 + 0.114i$$

Find the current in a similar manner as before.

$$I_{gen2} := \frac{V_{in}}{R'_{T1} + j\cdot X'_{T1} + Z'_{line} + R'_{T2} + j\cdot X'_{T2} + Z'_{Load}} = 0.392 - 0.059i \qquad \left|I_{gen2}\right| = 0.397$$

$$\arg\left(I_{gen2}\right) = -8.495\cdot deg$$

Calculate the complex power in and then sort out the P, Q, and S of the generator.

$$S_{in} := V_{in}\cdot\overline{I_{gen2}} = 0.392 + 0.059i \qquad \left|S_{in}\right| = 0.397$$

$$P_{gen2} := \mathrm{Re}\left(S_{in}\right) = 0.392 \qquad Q_{gen2} := \mathrm{Im}\left(S_{in}\right) = 0.059 \qquad S_{gen2} := \left|S_{in}\right| = 0.397$$

Power factor of the generator is

$$pf_{gen2} := \frac{P_{gen2}}{S_{gen2}} = 0.989\cdot lagging$$

d. What are the transmission losses (line losses plus transformer losses) in this system with the switch open? With the switch closed? what is the effect of adding Load 2 to the system?

$$P_{tr1} := \left(\left|I_{gen1}\right|\right)^2\cdot\left(R'_{T1} + \mathrm{Re}\left(Z'_{line}\right) + R'_{T2}\right) = 0.012$$

$$P_{tr2} := \left(\left|I_{gen2}\right|\right)^2\cdot\left(R'_{T1} + \mathrm{Re}\left(Z'_{line}\right) + R'_{T2}\right) = 9.006 \times 10^{-3}$$

Adding capacitance reduces the line current and line losses. That's why the power company charges industrial customers according to their power factor as well as per Watt consumed.

4. 2.14 on page 148. Work the problem in per unit this time, both without the effect of the transformers in the circuit (circuit (a)) and then with the effect of the transformers (circuit (b)).

2.14 A 13.2kV single phase generator supplies power to a load through a transmission line. The load's impedance is Z_{load}=500 ohms at 36.87 degrees, and the transmission line impedance is Z_{line}=60 ohms at 53.1 degrees.

a. If the generator is connected to the load, what is the ratio of the load voltage to the generated voltage? What are the transmission losses of the system?

b. If a 1:10 step up transformer is placed at the output of the generator and a 10:1 transformer is placed at the load end of the transmission line, what is the new ratio of the load voltage to the generated voltage? What are the transmission line losses of the system now? Assume the transformers to be ideal.

Do the problem in per unit.

$V_G := 13.2 \cdot kV$ $\qquad Z_{line} := 60 \cdot e^{j \cdot 53.1 \cdot deg} \cdot \Omega$ $\qquad Z_{load} := 500 \cdot e^{j \cdot 36.87 \cdot deg} \cdot \Omega$

For the first part of the problem, assume the following bases:

$V_{base} := 13.2 \cdot kV$ $\qquad Z_{base} := 500 \cdot \Omega$ $\qquad S_{base} := \frac{V_{base}^2}{Z_{base}}$ $\qquad S_{base} = 348.48 \cdot kW$

Convert the circuit elements to per unit.

$V_G := \frac{V_G}{V_{base}}$ $\qquad Z_{line} := \frac{Z_{line}}{Z_{base}}$ $\qquad Z_{load} := \frac{Z_{load}}{Z_{base}}$

$V_G = 1$ $\qquad Z_{line} = 0.072 + 0.096i$ $\qquad Z_{load} = 0.8 + 0.6i$

Use voltage division to get the load voltage:

$V_{load} := V_G \cdot \frac{Z_{load}}{Z_{line} + Z_{load}}$ $\qquad V_{load} = 0.896 - 0.027i$ $\qquad \left|V_{load}\right| = 0.896$ $\qquad \arg\left(V_{load}\right) = -1.723 \cdot deg$

Calculate the voltage ratio of input to output as requested.

$$\frac{\left|V_{load}\right|}{\left|V_G\right|} = 0.896$$

To find the losses, calculate the line current first.

$I_{line} := \frac{V_G}{Z_{line} + Z_{load}}$ $\qquad I_{line} = 0.701 - 0.559i$

Then use I^2R to find the losses.

$P_{loss} := \left(\left|I_{line}\right|\right)^2 \cdot Re\left(Z_{line}\right)$ $\qquad P_{loss} = 0.058$

Checking, the losses in watts are,

$P_{loss} \cdot S_{base} = 20.17 \cdot kW$

Now find the same items with the transformers in place. The base voltage and impedance are the same for load and generator, but different for the line.

$V_{base} = 13.2 \cdot kV$ $\quad V_{baseline} := V_{base} \cdot 10$ $\quad Z_{base} = 500\,\Omega$ $\quad Z_{baseline} := Z_{base} \cdot 10^2$

$V_{baseline} = 132 \cdot kV$ $\quad Z_{baseline} = 50 \cdot k\Omega$

Convert the line impedance to per unit.

$$Z_{line} := 60 \cdot e^{j \cdot 53.1 \cdot deg} \cdot \Omega$$

$$Z_{line} := \frac{Z_{line}}{Z_{baseline}}$$

$$Z_{line} = 7.205 \times 10^{-4} + 9.596i \times 10^{-4}$$

Use voltage division to get the load voltage:

$$V_{load} := V_G \cdot \frac{Z_{load}}{Z_{line} + Z_{load}} \qquad V_{load} = 0.999 - 3.346i \times \qquad \left|V_{load}\right| = 0.999 \qquad \arg\left(V_{load}\right) = -0.019 \cdot deg$$

Calculate the voltage ratio of input to output as requested.

$$\frac{\left|V_{load}\right|}{\left|V_G\right|} = 0.9988$$

To find the losses, calculate the line current first.

$$I_{line} := \frac{V_G}{Z_{line} + Z_{load}} \qquad I_{line} = 0.799 - 0.6i$$

Then use I2R to find the losses.

$$P_{loss} := \left(\left|I_{line}\right|\right)^2 \cdot Re\left(Z_{line}\right) \qquad P_{loss} = 7.188 \times 10^{-4}$$

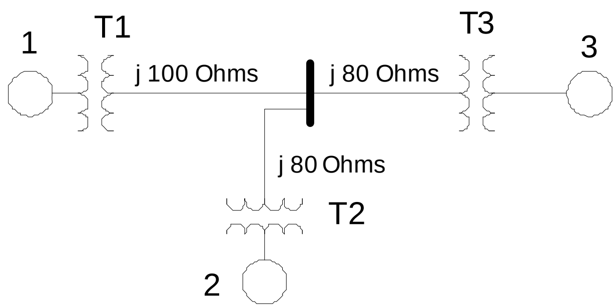
Checking, the losses in watts are,

$P_{loss} \cdot S_{base} = 0.251 \cdot kW$

5. A one-line diagram of an unloaded power system is shown below. Reactances of the two sections of transmission line are shown on the diagram. The generators and transformers are specified as follows:

Generator 1	20 MVA	6.9kV	X=0.15 per unit
Generator 2	10 MVA	6.9kV	X=0.15 per unit
Generator 3	30 MVA	13.8kV	X=0.15 per unit
Transformer 1	25 MVA	6.9kV : 115kV	X=10%
Transformer 2	12 MVA	6.9kV : 115kV	X=10%
Transformer 3	30 MVA	10:1	X=10%

Draw a circuit diagram for this system with all reactances labelled in per unit. Choose a base of 30 MVA and 6.9kV at generator 1.



$MVA := 10^6 \cdot V \cdot A$

Select a base of 30 MVA and 6.9kV.

$S_b := 30 \cdot 10^6 \cdot V \cdot A \quad V_{b1} := 6.9 \cdot kV \quad V_{b2} := 115 \cdot kV \quad V_{b3} := \frac{115 \cdot kV}{10} = 11.5\,kV \quad V_{b4} := 6.9 \cdot kV$

$Z_{b1} := \frac{V_{b1}^2}{S_b} = 1.587\,\Omega \quad Z_{b2} := \frac{V_{b2}^2}{S_b} = 440.833\,\Omega$

$Z_{b3} := \frac{V_{b3}^2}{S_b} = 4.408\,\Omega \quad Z_{b4} := \frac{V_{b4}^2}{S_b} = 1.587\,\Omega$

Compute the bases of the equipment.

$Z_{bG1} := \frac{(6.9 \cdot kV)^2}{20 \cdot MVA} = 2.381\,\Omega \quad Z_{bG2} := \frac{(6.9 \cdot kV)^2}{10 \cdot MVA} = 4.761\,\Omega \quad Z_{bG3} := \frac{(13.8 \cdot kV)^2}{30 \cdot MVA} = 6.348\,\Omega$

$Z_{bT1} := \frac{(6.9 \cdot kV)^2}{25 \cdot MVA} = 1.904\,\Omega \quad Z_{bT2} := \frac{(6.9 \cdot kV)^2}{12 \cdot MVA} = 3.967\,\Omega \quad Z_{bT3} := \frac{(11.5 \cdot kV)^2}{30 \cdot MVA} = 4.408\,\Omega$

List the impedances of the equipment.

$X_{G1} := 0.15 \quad X_{G2} := 0.15 \quad X_{G3} := 0.15$

$X_{T1} := 0.10 \quad X_{T2} := 0.10 \quad X_{T3} := 0.10$

Compute the impedances on the system base.

$$X_{g1} := \frac{X_{G1} \cdot Z_{bG1}}{Z_{b1}} = 0.225$$

$$X_{t1} := \frac{X_{T1} \cdot Z_{bT1}}{Z_{b1}} = 0.12$$

$$X_{g2} := \frac{X_{G2} \cdot Z_{bG2}}{Z_{b4}} = 0.45$$

$$X_{t2} := \frac{X_{T2} \cdot Z_{bT2}}{Z_{b4}} = 0.25$$

$$X_{g3} := \frac{X_{G3} \cdot Z_{bG3}}{Z_{b3}} = 0.216$$

$$X_{t3} := \frac{X_{T3} \cdot Z_{bT3}}{Z_{b3}} = 0.1$$

Compute the line impedances.

$$X_{line1} := \frac{(j \cdot 100 \cdot \Omega)}{Z_{b2}} = 0.227i$$

$$X_{line2} := \frac{(j \cdot 80 \cdot \Omega)}{Z_{b2}} = 0.181i$$

$$X_{line3} := \frac{(j \cdot 80 \cdot \Omega)}{Z_{b2}} = 0.181i$$