

1. Problem 2.2 A single phase power system is shown in Figure P2-1. The power source feeds a 100 kVA, 14kV / 2.4 kV transformer through a feeder impedance of $38.2 + j140$ ohm. The transformer's equivalent series impedance referred to its low voltage side is $0.10 + j 0.4$ ohm. the load on the transformer is 90kW at 0.80 pf lagging and 2300 V.

- What is the voltage at the power source of the system?
- What is the voltage regulation of the transformer?
- How efficient is the overall power system?

$$j := \sqrt{-1}$$

- What is the voltage at the power source of the system?

Rewrite the given.

$$S_T := 100 \cdot \text{kV} \cdot \text{A} \quad Z_F := (38.2 + j \cdot 140) \cdot \text{ohm} \quad Z_T := (0.10 + j \cdot 0.40) \cdot \text{ohm}$$

$$P_{\text{Load}} := 90 \cdot \text{kW} \quad \text{pf}_{\text{Load}} := 0.80 \quad \text{lagging} \quad V_{\text{Load}} := 2300 \text{ V} \quad N_T := \frac{14.4 \text{ kV}}{2400 \text{ V}} \quad N_T = 6$$

$$\text{pf}_{\text{Load}} := 0.85$$

Calculate the load current from the given data.

$$I_{\text{Load}} := \frac{P_{\text{Load}}}{V_{\text{Load}} \cdot \text{pf}_{\text{Load}}} \quad I_{\text{Load}} = 46.036 \text{ A} \quad \theta_{\text{Load}} := \arccos(\text{pf}_{\text{Load}}) \quad \theta_{\text{Load}} = 31.788 \cdot \text{deg}$$

$$I_{\text{Load}} := I_{\text{Load}} \cdot e^{-j \cdot \theta_{\text{Load}}} \quad I_{\text{Load}} = (39.13 - 24.251i) \text{ A}$$

$$Z_T = (0.1 + 0.4i) \Omega$$

Calculate the currents and voltages in the system in sequence through the load.

$$V_2 := V_{\text{Load}} + I_{\text{Load}} \cdot Z_T \quad V_2 = (2.314 + 0.013i) \cdot \text{kV} \quad |V_2| = 2.314 \times 10^3 \text{ V} \quad \arg(V_2) = 0.328 \text{ deg}$$

$$V_1 := N_T \cdot V_2 \quad V_1 = (13.882 + 0.079i) \cdot \text{kV}$$

$$I_1 := \frac{I_{\text{Load}}}{N_T} \quad I_1 = (6.522 - 4.042i) \text{ A}$$

$$V_{\text{source}} := V_1 + I_1 \cdot Z_F \quad V_{\text{source}} = (14.697 + 0.838i) \cdot \text{kV} \quad |V_{\text{source}}| = 14.721 \cdot \text{kV}$$

$$\arg(V_{\text{source}}) = 3.263 \cdot \text{deg}$$

b. What is the voltage regulation of the transformer?

Voltage regulation is found by comparing the no load output voltage to the loaded output voltage. Find the no load output voltage first. At no load, there is no current in the model given. Therefore, the no load voltage is the reflected primary voltage.

$$V_{NL} := \frac{V_1}{N_T} \quad V_{NL} = (2.314 + 0.013i) \cdot \text{kV} \quad |V_{NL}| = 2.314 \cdot \text{kV}$$

$$\arg(V_{NL}) = 0.328 \cdot \text{deg}$$

We already have the loaded output voltage. Calculate the voltage regulation.

$$V_{\text{regulation}} := \frac{|V_{NL}| - |V_{Load}|}{|V_{Load}|} \quad V_{\text{regulation}} = 0.594 \cdot \%$$

c. How efficient is the overall power system?

$$V_{Load} = 2.3 \cdot \text{kV}$$

Operating data is given different than rated data. Efficiency is power out divided by power in.

$$P_{out} := \text{Re}(V_{Load} \cdot \overline{I_{Load}}) \quad P_{out} = 90 \cdot \text{kW}$$

$$P_{in} := \text{Re}(V_{source} \cdot \overline{I_1}) \quad P_{in} = 92.461 \cdot \text{kW}$$

$$\eta := \frac{P_{out}}{P_{in}} \quad \eta = 97.339 \cdot \%$$

2. Problem 2.3 The secondary winding of a transformer has a terminal voltage of $v_s(t) = 282.8 \sin 377t$ V. The turns ratio of the transformer is 100:200 ($a=0.50$)

If the secondary current of the transformer is $i_s(t) = 7.07 \sin(377t - 36.87 \text{ degrees})$ A, what is the primary current of this transformer?

$$j_w := \sqrt{-1}$$

What are its voltage regulation and efficiency?

Restate the given information:

$$\omega_s := 377 \cdot \frac{\text{rad}}{\text{sec}} \quad V_{spk} := 282.8 \cdot \text{V} \quad I_{spk} := 7.07 \cdot \text{A} \quad \theta_s := -36.87 \cdot \text{deg} \quad N_t := \frac{100}{200} = 0.5$$

Convert the peak values to rms. Set the phase angle on the secondary voltage to zero and apply the phase angle on the current.

$$V_s := \frac{V_{spk}}{\sqrt{2}} = 199.97 \text{ V} \quad I_s := \frac{I_{spk}}{\sqrt{2}} \cdot e^{j \cdot \theta_s} = (3.999 - 3i) \text{ A} \quad |I_s| = 4.999 \text{ A} \quad \arg(I_s) = -36.87 \cdot \text{deg}$$

Using the turns ratio, calculate the voltage and current on the primary side of the internal "ideal" transformer. The phase angles are zero and θ_s , respectively.

$$V_P := V_s \cdot N_t = 99.985 \text{ V} \quad I_P := \frac{I_s}{N_t} = (7.999 - 5.999i) \text{ A} \quad |I_P| = 9.998 \text{ A} \quad \arg(I_P) = -36.87 \cdot \text{deg}$$

To put the answer in the same form as the question,

$$i_p(t) = 10 \cdot \sqrt{2} \cdot \sin(377 \cdot t - 36.87 \cdot \text{deg}) \cdot \text{A}$$

It is an ideal transformer. Its voltage regulation is ZERO and its efficiency is 1.00.

3. Problem 2.6 A 1000 VA 230 V / 115 V transformer has been tested to determine its equivalent circuit. The results of the tests are shown below.

a. Find the equivalent circuit for this transformer referred to the low voltage side of the transformer.

b. Find the transformer's voltage regulation at rated conditions and at (1) 0.8 pf lag, (2) 1.0 pf, and (3) 0.8 pf lead.

**Open circuit test
secondary LV side**

**Short circuit test
primary HV side**

$$j_w := \sqrt{-1}$$

$$V_{OC} := 115 \cdot \text{V}$$

$$V_{SC} := 17.1 \cdot \text{V}$$

$$I_{OC} := 0.11 \cdot \text{A}$$

$$I_{SC} := 8.7 \cdot \text{A}$$

$$P_{OC} := 3.9 \cdot \text{W}$$

$$P_{SC} := 38.1 \cdot \text{W}$$

a. Find the equivalent circuit for this transformer referred to the low voltage side of the transformer.

For the open circuit data,

$$Y_E := \frac{I_{OC}}{V_{OC}} = 9.565 \times 10^{-4} \frac{1}{\Omega}$$

$$G_C := \frac{P_{OC}}{V_{OC}^2} = 2.949 \times 10^{-4} \frac{1}{\Omega}$$

$$B_M := \sqrt{Y_E \cdot Y_E - G_C \cdot G_C} = 9.099 \times 10^{-4} \frac{1}{\Omega}$$

$$R_C := \frac{1}{G_C} = 3.391 \text{ k}\Omega$$

$$X_M := \frac{1}{B_M} = 1.099 \text{ k}\Omega$$

For the short circuit data,

$$Z_{SE} := \frac{V_{SC}}{I_{SC}} = 1.966 \Omega$$

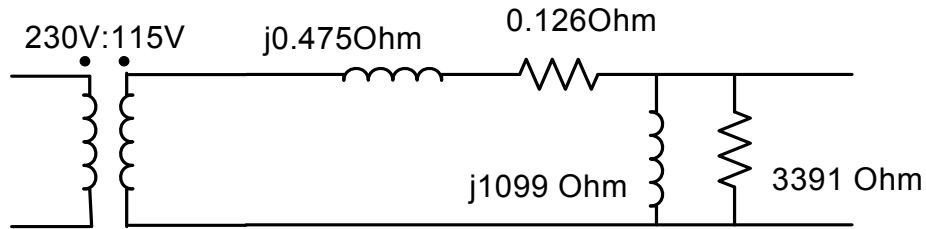
$$R_{eq} := \frac{P_{SC}}{I_{SC}^2} = 0.503 \Omega$$

$$X_{eq} := \sqrt{Z_{SE} \cdot Z_{SE} - R_{eq} \cdot R_{eq}} = 1.9 \Omega$$

Short circuit data is on the HV side. To convert to the LV side,

$$N_T := \frac{230 \cdot \text{V}}{115 \cdot \text{V}} = 2$$

$$R'_{eq} := \frac{R_{eq}}{N_T^2} = 0.126 \Omega \quad X'_{eq} := \frac{X_{eq}}{N_T^2} = 0.475 \Omega$$



b. Find the transformer's voltage regulation at rated conditions and at (1) 0.8 pf lag, (2) 1.0 pf, and (3) 0.8 pf lead.

Voltage regulation related no load output voltage to loaded output voltage. Rewriting the given,

$$\underline{V}_P := 230 \cdot V \quad \underline{V}_S := 115 \cdot V \quad \underline{S}_T := 1000 \cdot V \cdot A \quad \underline{N}_T := \frac{V_P}{V_S} \quad N_T = 2$$

At pf=0.80 lagging $\underline{pf}_{Load} := 0.8$ lagging

Find the load current, referred to the low voltage (load) side.

$$\underline{V}_S = 115 \text{ V} \quad \underline{I}_{Load} := \frac{\underline{S}_T}{\underline{V}_S} \quad I_{Load} = 8.696 \text{ A} \quad \theta_{Load} := \arccos(\underline{pf}_{Load}) \quad \theta_{Load} = 36.87 \cdot \text{deg}$$

$$\underline{I}_{Load} := I_{Load} \cdot e^{-j \cdot \theta_{Load}} \quad I_{Load} = (6.957 - 5.217i) \text{ A}$$

Find the loaded source voltage, referred to the load side,.

$$\underline{I}_S := \underline{I}_{Load} + \frac{\underline{V}_S}{R_C} + \frac{\underline{V}_S}{j \cdot X_M} = (6.99 - 5.322i) \text{ A}$$

$$\underline{V}'_{Source} := \underline{V}_S + \underline{I}_S \cdot (R'_{eq} + j \cdot X'_{eq}) = (118.408 + 2.651i) \text{ V}$$

$$|V'_{Source}| = 118.437 \text{ V}$$

$$\arg(V'_{Source}) = 1.282 \cdot \text{deg}$$

At no load, output voltage is essentially equal to reflected input voltage. Finding the voltage regulation,

$$\underline{V}_{NL} := \underline{V}'_{Source} \quad \underline{V}_{NL} = (118.408 + 2.651i) \text{ V} \quad |V_{NL}| = 118.437 \text{ V}$$

$$V_{Regulation} := \frac{|V_{NL}| - |V_S|}{|V_S|} \quad V_{Regulation} = 2.989 \cdot \%$$

Repeat for 1.0 power factor.

At pf=1.0 $\text{pf}_{\text{Load}} := 1.0$

Find the load current, referred to the low voltage (load) side.

$$V_S = 115 \text{ V} \quad \text{I}_{\text{Load}} := \frac{S_T}{V_S} \quad \text{I}_{\text{Load}} = 8.696 \text{ A} \quad \theta_{\text{Load}} := \text{acos}(\text{pf}_{\text{Load}}) \quad \theta_{\text{Load}} = 0 \cdot \text{deg}$$

$$\text{I}_{\text{Load}} := \text{I}_{\text{Load}} \cdot e^{-j \cdot \theta_{\text{Load}}} \quad \text{I}_{\text{Load}} = 8.696 \text{ A}$$

Find the loaded source voltage, referred to the load side,.

$$\text{I}_S := \text{I}_{\text{Load}} + \frac{V_S}{R_C} + \frac{V_S}{j \cdot X_M} = (8.73 - 0.105i) \text{ A}$$

$$\text{V}'_{\text{Source}} := V_S + \text{I}_S \cdot (R'_{\text{eq}} + j \cdot X'_{\text{eq}}) = (116.148 + 4.133i) \text{ V} \quad |V'_{\text{Source}}| = 116.222 \text{ V}$$

$$\arg(V'_{\text{Source}}) = 2.038 \cdot \text{deg}$$

At no load, output voltage is essentially equal to reflected input voltage. Finding the voltage regulation,

$$\text{V}_{\text{NL}} := \text{V}'_{\text{Source}} \quad \text{V}_{\text{NL}} = (116.148 + 4.133i) \text{ V} \quad |V_{\text{NL}}| = 116.222 \text{ V}$$

$$\text{V}_{\text{Regulation}} := \frac{|V_{\text{NL}}| - |V_S|}{|V_S|} \quad \text{V}_{\text{Regulation}} = 1.062 \cdot \%$$

Repeat for 0.8 power factor leading.

At pf=0.80 lagging $\text{pf}_{\text{Load}} := 0.8$ leading

Find the load current, referred to the low voltage (load) side.

$$V_S = 115 \text{ V} \quad \text{I}_{\text{Load}} := \frac{S_T}{V_S} \quad \text{I}_{\text{Load}} = 8.696 \text{ A} \quad \theta_{\text{Load}} := -\text{acos}(\text{pf}_{\text{Load}}) \quad \theta_{\text{Load}} = -36.87 \cdot \text{deg}$$

$$\text{I}_{\text{Load}} := \text{I}_{\text{Load}} \cdot e^{-j \cdot \theta_{\text{Load}}} \quad \text{I}_{\text{Load}} = (6.957 + 5.217i) \text{ A}$$

Find the loaded source voltage, referred to the load side,.

$$\text{I}_S := \text{I}_{\text{Load}} + \frac{V_S}{R_C} + \frac{V_S}{j \cdot X_M} = (6.99 + 5.113i) \text{ A}$$

$$\text{V}'_{\text{Source}} := V_S + \text{I}_S \cdot (R'_{\text{eq}} + j \cdot X'_{\text{eq}}) = (113.451 + 3.964i) \text{ V} \quad |V'_{\text{Source}}| = 113.52 \text{ V}$$

$$\arg(V'_{\text{Source}}) = 2.001 \cdot \text{deg}$$

At no load, output voltage is essentially equal to reflected input voltage. Finding the voltage regulation,

$$V_{\text{NL}} := V'_{\text{Source}} \quad V_{\text{NL}} = (113.451 + 3.964i) \text{ V} \quad |V_{\text{NL}}| = 113.52 \text{ V}$$

$$V_{\text{Regulation}} := \frac{|V_{\text{NL}}| - |V_{\text{S}}|}{|V_{\text{S}}|} \quad V_{\text{Regulation}} = -1.287 \cdot \%$$

This is quite common. Capacitive loads often cause a voltage rise across the lines of an AC system.

c. Determine the efficiency at rated conditions and 0.80 power factor lagging.

$$S_{\text{T}} = 1 \cdot \text{kV} \cdot \text{A} \quad V_{\text{P}} = 230 \text{ V} \quad V_{\text{S}} = 115 \text{ V} \quad \text{pf}_{\text{Load}} := 0.80 \text{ lagging}$$

$$R'_{\text{eq}} = 0.126 \Omega \quad R_{\text{C}} = 3.391 \cdot \text{K}\Omega$$

Recall resistances and current, refer to the high voltage (input) side.

$$I_{\text{Load}} := \frac{S_{\text{T}}}{V_{\text{S}}} \cdot e^{-j \cdot \arccos(\text{pf}_{\text{Load}})} \quad I_{\text{Load}} = (6.957 - 5.217i) \text{ A} \quad |I_{\text{Load}}| = 8.696 \text{ A} \quad \arg(I_{\text{Load}}) = -36.87 \text{ deg}$$

$$I_{\text{S}} := I_{\text{Load}} + \frac{V_{\text{S}}}{R_{\text{C}}} + \frac{V_{\text{S}}}{j \cdot X_{\text{M}}} = (6.99 - 5.322i) \text{ A}$$

$$V'_{\text{Source}} := V_{\text{S}} + I_{\text{S}} \cdot (R'_{\text{eq}} + j \cdot X'_{\text{eq}}) = (118.408 + 2.651i) \text{ V} \quad |V'_{\text{Source}}| = 118.437 \text{ V}$$

Find the losses; then calculate efficiency.

$$\arg(V'_{\text{Source}}) = 1.282 \cdot \text{deg}$$

$$P_{\text{LossS}} := (|I_{\text{S}}|)^2 \cdot R'_{\text{eq}} \quad P_{\text{LossS}} = 9.714 \text{ W}$$

$$P_{\text{LossC}} := \frac{(|V_{\text{S}}|)^2}{R_{\text{C}}} \quad P_{\text{LossC}} = 3.9 \text{ W} \quad |V_{\text{S}}| = 115 \text{ V}$$

$$P_{\text{out}} := \text{Re}(V_{\text{S}} \cdot \overline{I_{\text{Load}}}) \quad P_{\text{out}} = 800 \text{ W}$$

$$\eta := \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{LossS}} + P_{\text{LossC}}} \quad \eta = 98.3 \cdot \%$$

Check

$$P_{\text{in}} := \operatorname{Re}(V_{\text{Source}} \cdot \overline{I_S}) = 813.614 \text{ W} \quad \frac{P_{\text{out}}}{P_{\text{in}}} = 0.983$$

Problem 2.6 A 30 kVA, 8000/230V distribution transformer has an impedance referred to the primary of $20+j100 \text{ Ohms}$. The components of the excitation branch referred to the primary side are $R_C=100\text{k}\Omega$ and $X_M=20\text{k}\Omega$.

- a. If the primary voltage is 7967V and the load impedance is $Z_L=2.0+j0.7\Omega$, what is the secondary voltage of the transformer? What is the voltage regulation of the transformer?
 b. If the load is disconnected and a capacitor of $-j3\Omega$ is connected in its place, what is the secondary voltage of the transformer? What is its voltage regulation under these conditions?

Restate the given.

$$j := \sqrt{-1}$$

$$S_T := 30 \cdot \text{kV} \cdot \text{A} \quad V_D := 8 \cdot \text{kV} \quad V_S := 230 \cdot \text{V} \quad R_{\text{eq}} := 20 \cdot \Omega \quad X_{\text{eq}} := 100 \cdot \Omega \quad R_C := 100 \cdot \text{k}\Omega \quad X_M := 20 \cdot \text{k}\Omega$$

Calculate the turns ratio from the given voltage ratio.

$$N_T := \frac{V_P}{V_S} \quad N_T = 34.783$$

Initial conditions on the terminal voltage input and the load impedance.

$$V_D := 7967 \cdot \text{V} \quad Z_L := (2.0 + j \cdot 0.7) \cdot \Omega$$

Convert all impedances to the primary side. All but the load are already there.

$$Z_{LP} := N_T^2 \cdot Z_L \quad Z_{LP} = (2.42 + 0.847i) \cdot \text{k}\Omega$$

Write a node equation on the primary side, including the reflected load, and solve for the node voltage.

$$\frac{V_M - V_P}{R_{\text{eq}} + j \cdot X_{\text{eq}}} + \frac{V_M}{R_C} + \frac{V_M}{j \cdot X_M} + \frac{V_M}{Z_{LP}} = 0$$

$$V_M \cdot \left(\frac{1}{R_{\text{eq}} + j \cdot X_{\text{eq}}} + \frac{1}{R_C} + \frac{1}{j \cdot X_M} + \frac{1}{Z_{LP}} \right) = \frac{V_P}{R_{\text{eq}} + j \cdot X_{\text{eq}}}$$

$$V_M := \frac{V_P}{(R_{\text{eq}} + j \cdot X_{\text{eq}}) \cdot \left(\frac{1}{R_{\text{eq}} + j \cdot X_{\text{eq}}} + \frac{1}{R_C} + \frac{1}{j \cdot X_M} + \frac{1}{Z_{LP}} \right)} = (7.761 \times 10^3 - 259.134i) \text{ V}$$

$$|V_M| = 7.765 \cdot \text{kV}$$

Reflect this to the secondary to get the load voltage.

$$\arg(V_M) = -1.912 \cdot \text{deg}$$

$$V_L := \frac{V_M}{N_T} \quad V_L = (223.118 - 7.45i) \text{ V} \quad |V_L| = 223.242 \text{ V}$$

$$\arg(V_L) = -1.912 \cdot \text{deg}$$

To get the voltage regulation, first find the open circuit voltage. We do this by voltage division.

$$Z_M := \frac{R_C \cdot j \cdot X_M}{R_C + j \cdot X_M} \quad Z_M = (3.846 \times 10^3 + 1.923i \times 10^4) \Omega$$

$$V_{MOC} := \frac{V_P \cdot Z_M}{Z_M + R_{eq} + j \cdot X_{eq}} \quad V_{MOC} = 7.926 \times 10^3 \text{ V} \quad |V_{MOC}| = 7.926 \cdot \text{kV}$$

Calculate voltage regulation.

$$V_{reg} := \frac{|V_{MOC}| - |V_M|}{|V_M|} \cdot 100\% \quad V_{reg} = 2.071\%$$

Part b.

Initial conditions on the terminal voltage input and the load impedance.

$$V_D := 7967 \cdot \text{V} \quad Z_L := (-j \cdot 3) \cdot \Omega$$

Convert all impedances to the primary side. All but the load are already there.

$$Z_{LP} := N_T^2 \cdot Z_L \quad Z_{LP} = -3.629i \cdot \text{k}\Omega$$

Write a node equation on the primary side, including the reflected load, and solve for the node voltage.

$$\frac{V_M - V_P}{R_{eq} + j \cdot X_{eq}} + \frac{V_M}{R_C} + \frac{V_M}{j \cdot X_M} + \frac{V_M}{Z_{LP}} = 0$$

$$V_M \cdot \left(\frac{1}{R_{eq} + j \cdot X_{eq}} + \frac{1}{R_C} + \frac{1}{j \cdot X_M} + \frac{1}{Z_{LP}} \right) = \frac{V_P}{R_{eq} + j \cdot X_{eq}}$$

$$V_M := \frac{V_P}{(R_{eq} + j \cdot X_{eq}) \cdot \left(\frac{1}{R_{eq} + j \cdot X_{eq}} + \frac{1}{R_C} + \frac{1}{j \cdot X_M} + \frac{1}{Z_{LP}} \right)} = (8.149 \times 10^3 - 45.93i) \text{ V}$$

$$|V_M| = 8.149 \cdot \text{kV}$$

Reflect this to the secondary to get the load voltage.

$$\arg(V_M) = -0.323 \cdot \text{deg}$$

$$V_L := \frac{V_M}{N_T} \quad V_L = (234.281 - 1.32i) \text{ V} \quad |V_L| = 234.284 \text{ V}$$

$$\arg(V_L) = -0.323 \cdot \text{deg}$$

To get the voltage regulation, first find the open circuit voltage. We do this by voltage division.

$$Z_M := \frac{R_C \cdot j \cdot X_M}{R_C + j \cdot X_M} \quad Z_M = (3.846 + 19.231i) \cdot \text{k}\Omega$$

$$V_{\text{MOC}} := \frac{V_P \cdot Z_M}{Z_M + R_{\text{eq}} + j \cdot X_{\text{eq}}} \quad V_{\text{MOC}} = 7.926 \times 10^3 \text{ V} \quad |V_{\text{MOC}}| = 7.926 \cdot \text{kV}$$

Calculate voltage regulation.

$$V_{\text{reg}} := \frac{|V_{\text{MOC}}| - |V_M|}{|V_M|} \cdot 100 \cdot \% \quad V_{\text{reg}} = -2.739 \cdot \%$$

The capacitor gives us a voltage rise across the transformer.

Problem 2.14 A 13.8 kV single phase generator supplies power to a load through a transmission line. The load's impedance is 500 ohms at an inductive angle of 36.87 degrees. The transmission line's impedance is 60 ohms at an inductive angle of 60 degrees.

a. If the generator is directly connected to the load as shown in Figure 2.3a, what is the ratio of the load voltage to the generated voltage? What are the transmission losses of the system?

b. If a 1:10 step up transformer is placed at the output of the generator and a 10:1 transformer is placed at the load end of the transmission line, what is the new ratio of load voltage to generated voltage? What are transmission losses of the system now? Assume transformers to be ideal.

$$V_G := 13.8 \cdot \text{kV} \quad Z_{\text{Load}} := 500 \cdot e^{j \cdot 36.87 \cdot \text{deg}} \cdot \text{ohm} \quad Z_{\text{line}} := 60 \cdot e^{j \cdot 60 \cdot \text{deg}} \cdot \text{ohm} \quad N_T := 10$$

$$Z_{\text{Load}} = (399.999 + 300.001i) \Omega \quad Z_{\text{line}} = (30 + 51.962i) \Omega$$

a. If the generator is directly connected to the load as shown in Figure 2.3a, what is the ratio of the load voltage to the generated voltage? What are the transmission losses of the system?

A voltage division finds the load voltage.

$$V_{\text{Load}} := V_G \cdot \frac{Z_{\text{Load}}}{Z_{\text{Load}} + Z_{\text{line}}} \quad V_{\text{Load}} = (12.406 - 0.527i) \cdot \text{kV} \quad \frac{|V_{\text{Load}}|}{|V_G|} = 0.9$$

For line losses, find the current first and then calculate losses.

$$I_{\text{line}} := \frac{V_G - V_{\text{Load}}}{Z_{\text{line}}} \quad I_{\text{line}} = (19.218 - 15.73i) \text{ A} \quad |I_{\text{line}}| = 24.835 \text{ A}$$

$$P_{\text{loss}} := (|I_{\text{line}}|)^2 \cdot \text{Re}(Z_{\text{line}}) \quad P_{\text{loss}} = 18.503 \cdot \text{kW}$$

b. What percentage of the power supplied by the source reaches the load?

$$P_{\text{Load}} := \text{Re}(V_{\text{Load}} \cdot \overline{I_{\text{line}}}) = 246.702 \text{ kW}$$

$$\eta_b := \frac{P_{\text{Load}}}{P_{\text{Load}} + P_{\text{loss}}} = 0.93$$

c. If a 1:10 step up transformer is placed at the output of the generator and a 10:1 transformer is placed at the load end of the transmission line, what is the new ratio of load voltage to generated voltage? What are transmission losses of the system now? Assume transformers to be ideal.

Reflect everything to the generator side for reference.

$$V_G = 13.8 \cdot \text{kV} \quad Z_{\text{Load}} := Z_{\text{Load}} \cdot N_T^2 \cdot \frac{1}{N_T^2} \quad Z_{\text{Load}} = (399.999 + 300.001i) \Omega$$

$$Z_{\text{line}} := Z_{\text{line}} \cdot \frac{1}{N_T^2} \quad Z_{\text{line}} = (0.3 + 0.52i) \Omega$$

Do a voltage division to find load voltage.

$$V_{\text{Load}} := V_G \cdot \frac{Z_{\text{Load}}}{Z_{\text{Load}} + Z_{\text{line}}} \quad V_{\text{Load}} = (13.785 - 6.491i \times 10^{-3}) \cdot \text{kV} \quad |V_{\text{Load}}| = 13.785 \cdot \text{kV} \quad \frac{|V_{\text{Load}}|}{|V_G|} = 0.999$$

Because the two transformers have the same turns ratio, this value is equal to the load voltage.

Now find the losses.

$$I_{\text{line}} := \frac{V_G - V_{\text{Load}}}{Z_{\text{line}}} \quad I_{\text{line}} = (22.048 - 16.552i) \text{ A}$$

$$P_{\text{loss}} := (|I_{\text{line}}|)^2 \cdot \text{Re}(Z_{\text{line}}) \quad P_{\text{loss}} = 228.024 \cdot \text{W}$$

d. What percentage of the power supplied by the source reaches the load?

$$P_{\text{Load}} := \text{Re}(V_{\text{Load}} \cdot \overline{I_{\text{line}}}) = 304.032 \text{ kW}$$

$$\eta_{\text{max}} := \frac{P_{\text{Load}}}{P_{\text{Load}} + P_{\text{loss}}} = 0.999$$

e. Compare the efficiencies of the transmission system with and without transformers.

Dramatically improved by the use of transformers.

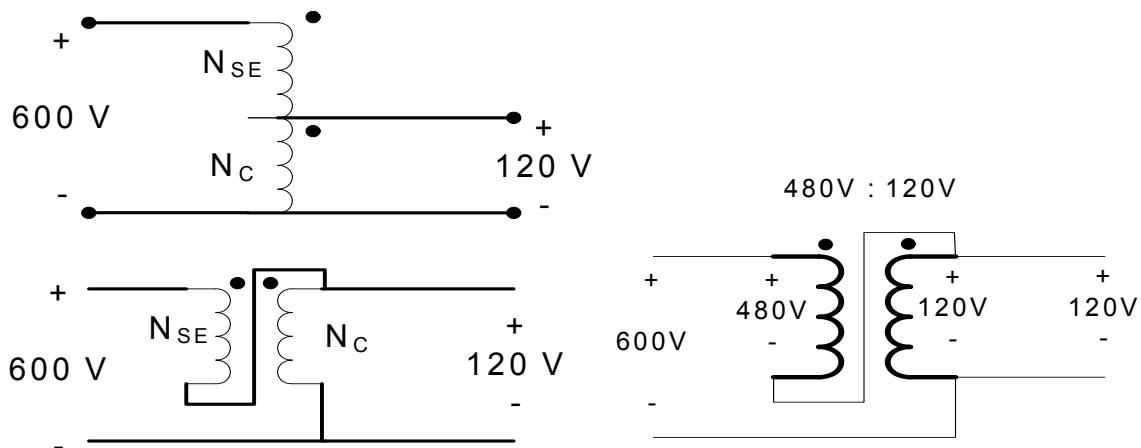
Problem 2.15 A 10,000 VA, 480V / 120V conventional transformer is to be used to supply power from a 600V source to 120V load. Consider the transformer to be ideal and assume that all insulation can handle 600V.

- Sketch the transformer connection that will do the required job.
- Find the kVA rating of the transformer in the configuration.
- Find the maximum primary and secondary currents under these conditions.

$$S_T := 10000 \cdot \text{V} \cdot \text{A} \quad V_P := 480 \cdot \text{V} \quad V_S := 120 \cdot \text{V}$$

a. Sketch the transformer connection that will do the required job.

Here are three ways to connect the transformer. They are actually the same, but the diagram for the transformer itself is different in each case.



b. Find the kVA rating of the transformer in the configuration.

$$V_{\text{in}} := 600 \cdot \text{V} \quad V_{\text{out}} := 120 \cdot \text{V}$$

$$I_P := \frac{S_T}{V_P} \quad I_P = 20.833 \text{ A} \quad I_S := \frac{S_T}{V_S} \quad I_S = 83.333 \text{ A}$$

The input has the same current as the original transformer's primary

$$I_{in} := I_P \quad I_{in} = 20.833 \text{ A}$$

the output current is the sum of the two winding currents of the original transformer.

$$I_{out} := I_P + I_S \quad I_{out} = 104.167 \text{ A}$$

The kVA rating can be found from either side, if the transformer is considered to be ideal.

$$KVA_{in} := V_{in} \cdot I_{in} \quad KVA_{in} = 12.5 \cdot \text{kV} \cdot \text{A}$$

$$KVA_{out} := V_{out} \cdot I_{out} \quad KVA_{out} = 12.5 \cdot \text{kV} \cdot \text{A}$$

c. Find the maximum primary and secondary currents under these conditions.

$$I_P := \frac{S_T}{V_P} \quad I_P = 20.833 \text{ A} \quad I_S := \frac{S_T}{V_S} \quad I_S = 83.333 \text{ A}$$

The input current is the same as the primary current; the output current is, as found above, larger than the secondary current.

$$I_{out} = 104.167 \text{ A}$$