ECE 320
Homework
Ideal DC / DC Converters

From the text handout by Daniel Hart, do the following problems

6.1 What is the relationship between Vo/Vs and the efficiency of the linear converter in section 6.1?

Input power is

$$P_{in} = V_s \cdot I_L$$

Output power is

$$P_{out} = V_o \cdot I_L$$

Calculate efficiency

$$\eta = \frac{P_{out}}{P_{in}}$$

Simplify the expression by dividing out the current I_L.

$$\eta = \frac{V_o \cdot I_L}{V_s \cdot I_L}$$

$$\eta = \frac{V_o}{V_s}$$

The energy efficiency is, ideally, equal to the voltage ratio. The energy efficiency can be no greater than the voltage ratio in practical linear regulator circuits.

Problem 6.2 A DC power supply must step down a 100V source to 30V. The output power is 100W.

a. Determine the efficiency of the converter of Fig 6.1 when it is used for this application.

b. How much energy is lost in the transistor in one year?

c. using the electric rate in your area, what is the cost of the energy loss for one year?

$V_{in} := 100 \cdot V$ $\quad V_{out} := 30 \cdot V$ $\quad P_{out} := 100 \cdot W$ $\quad cents := 1$ $\quad dollars := 100 \cdot cents$

a. Find the efficiency. First find the current from the power and voltage.

$$I_{out} := \frac{P_{out}}{V_{out}} \qquad I_{out} = 3.333\,A$$

From the figure we see that the circuit is just a series circuit. The output current is the same as the input current.

$$I_{in} := I_{out} \qquad P_{in} := V_{in} \cdot I_{in}$$

Efficiency is power out / power in:

$$\eta := \frac{P_{out}}{P_{in}} \qquad \eta = 0.3$$

b. Energy loss in a year is found:

$T := 1 \cdot yr$ $\quad T = 8766 \cdot hr$ $\quad P_{in} = 333.333\,W$ $\quad P_{out} = 100\,W$

$W_{loss} := (P_{in} - P_{out}) \cdot T$ $\quad W_{loss} = 7.363 \times 10^{9}\,J$ $\quad W_{loss} = 2045 \cdot kW \cdot hr$

c. Cost here is 7.2 cents per kwhr.

$$C_{cost} := W_{loss} \cdot \left(7.2 \cdot \frac{cents}{kW \cdot hr}\right) \qquad C_{cost} = 1.473 \times 10^{4} \cdot cents$$

$$C_{cost} = 147.27 \cdot dollars$$

Problem 6.7 A buck converter has an input of 60V and an output fo 25V. The load resistor is 9Ω, the switching frequency is 20-kHz, L=1mH, and C=200μF.

a. Determine the duty ratio.
b. Determine the average, peak, and rms inductor current.
c. Determine the average source current.
d. Determine the peak and average diode current.

$\mu s := 10^{-6} \cdot sec$

$V_s := 60 \cdot V \quad V_0 := 25 \cdot V \quad R_0 := 9 \cdot \Omega \quad f_s := 20 \cdot kHz \quad L := 1 \cdot mH \quad C := 200 \cdot \mu F$

a. Determine the duty ratio.

$$T_s := \frac{1}{f_s} = 50 \cdot \mu s$$

$$D := \frac{V_0}{V_s} = 0.417$$

b. Determine the average, peak, and rms inductor current.

$$I_0 := \frac{V_0}{R_0} = 2.778\ A$$

Because the capacitor has no average current, the inductor average current is the same as the output average current.

$$I_{Lave} := I_0 = 2.778\ A$$

The change in inductor current is found from the inductor's integral relationship:

$$\Delta I_L := \frac{V_s - V_0}{L} \cdot D \cdot T_s = 0.729\ A$$

Calculating the max and min values,

$$I_{Lmax} := I_{Lave} + \frac{\Delta I_L}{2} = 3.142\ A \qquad I_{Lmin} := I_{Lave} - \frac{\Delta I_L}{2} = 2.413\ A$$

c. Determine the average source current.

$$I_{Save} := I_0 \cdot D = 1.157\ A$$

It is also possible to do this from energy conservation. Ideally, there are no losses in the converter.

$$P_0 := \frac{V_0^{\ 2}}{R_0} = 69.444\ W$$

$$P_s := P_0 = 69.444\ W$$

$$I_{Save} := \frac{P_s}{V_s} = 1.157\ A$$

d. Determine the peak and average diode current.

$$I_{Dpk} := I_{Lmax} = 3.142\ A$$

$$I_{Dave} := I_{Lave} \cdot (1 - D) = 1.62\ A$$

Problem 6.13 Design a buck converter which has an output of 12V from an input of 18V. The output power is 10W. The output voltage ripple must be no more than 100mV peak to peak. Specify the duty ratio, switching frequency, and inductor and capacitor values. Design for continuous inductor current. Assume ideal conditions.

$V_0 := 12 \cdot V \quad V_s := 18 \cdot V \quad P_0 := 10 \cdot W \quad \Delta V_0 := 100 \cdot mV$

The duty ratio is the ratio of the voltages:

$$D := \frac{V_0}{V_s} = 0.667$$

Calculate the output resistance from the given information.

$$R_0 := \frac{V_0^{\,2}}{P_0} = 14.4\,\Omega$$

Specify a switching frequency. This is normally set by the expected power losses in the switch. Here, we will choose a common value,

$$f_s := 500 \cdot kHz$$

Using equation 6-26 in the text handout, we find the minimum inductance for continuous inductor current is ,

$$L_{min} := \frac{D \cdot (1 - D)^2 \cdot R_0}{2 f_s} = 1.067 \cdot \mu H$$

We see the output ripple is defined as

$$\frac{\Delta V_0}{V_0} = \frac{1 - D}{8 \cdot L \cdot C_0 \cdot f_s^{\,2}}$$

Rearrange this to allow us to solve for the remaining unknown, the capacitance,

$$C_0 := \frac{(1 - D) \cdot V_0}{8 \cdot L_{min} \cdot f_s^{\,2} \cdot \Delta V_0} = 18.75 \cdot \mu F$$

Problem 6.15 The boost converter of Fig 6-6 has the following parameters: Vs=20V, D=0.6, R=12.5Ω, L=65μH, C=200μF, and switching frequency = 40kHz.
a. Determine the output voltage.
b. Determine the average, maximum, and minimum inductor current.
c. Determine the output voltage ripple.
d. Dermine the average current in the diode.

$V_s := 20 \cdot V \quad D := 0.60 \quad R_0 := 12.5 \cdot \Omega \quad L := 65 \cdot \mu H \quad C_0 := 200 \cdot \mu F \quad f_s := 40 \cdot kHz$

a. Determine the output voltage.

$$T_s := \frac{1}{f_s} = 25 \cdot \mu s$$

$$V_0 := V_s \cdot \frac{1}{1 - D} = 50\,V$$

b. Determine the average, maximum, and minimum inductor current.

$$P_0 := \frac{{V_0}^2}{R_0} = 200\,W$$

$$P_s := P_0 = 200\,W$$

$$I_s := \frac{P_s}{V_s} = 10\,A \qquad I_L := I_s = 10\,A$$

$$\Delta I_L := \frac{V_s}{L} \cdot D \cdot T_s = 4.615\,A$$

$$I_{Lmax} := I_L + \frac{\Delta I_L}{2} = 12.308\,A \qquad I_{Lmin} := I_L - \frac{\Delta I_L}{2} = 7.692\,A$$

c. Determine the output voltage ripple.

We see the output ripple is defined as

$$\Delta V_0 := \frac{D \cdot V_0}{R_0 \cdot C_0 \cdot f_s} = 300 \cdot mV$$

We can also express this a per unit ripple.

$$\frac{\Delta V_0}{V_0} = 0.006$$

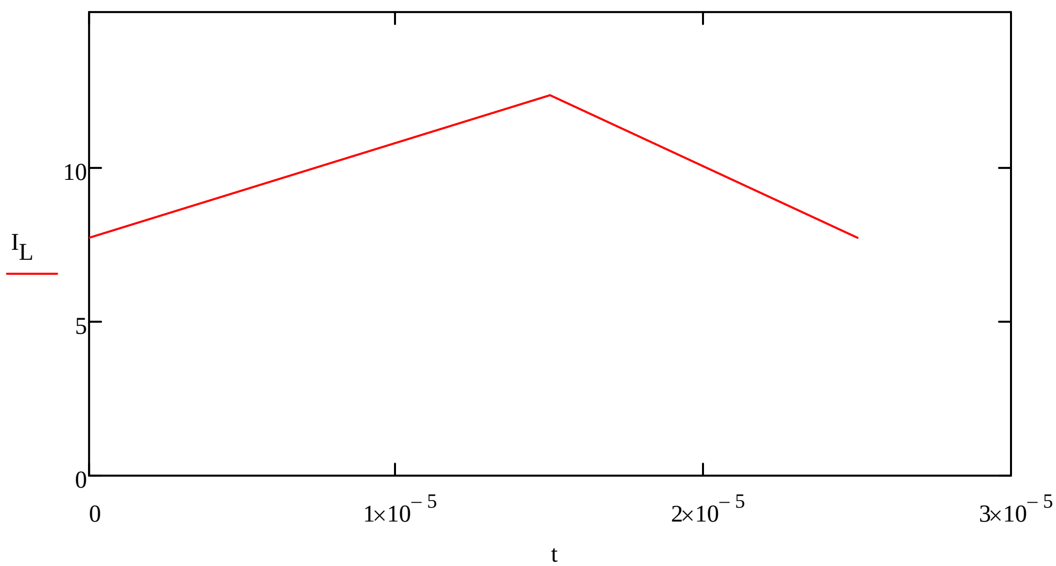
d. Dermine the average current in the diode.

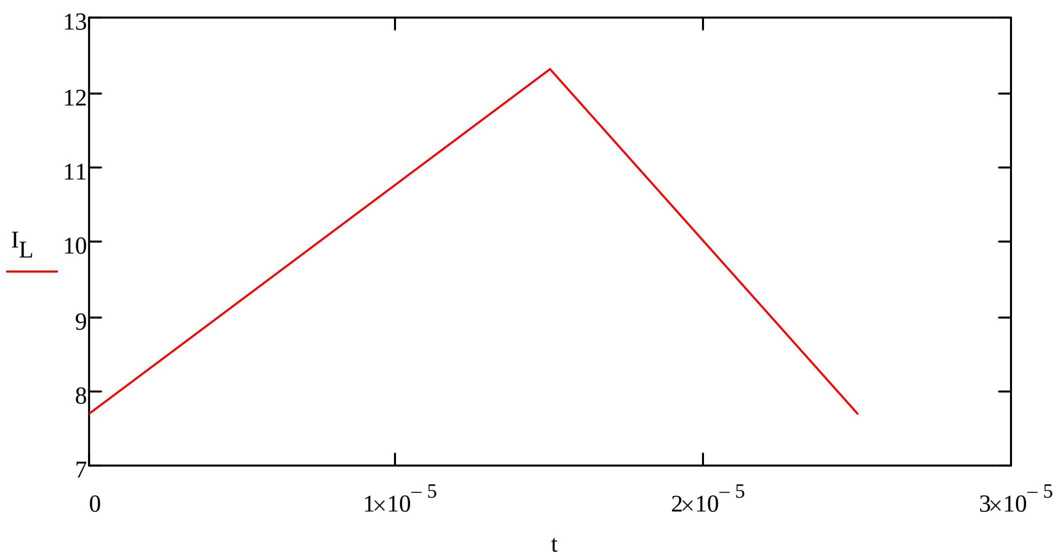
$$I_{Dave} := I_L \cdot (1 - D) = 4\,A$$

Problem 6.17 For the boost converer of Problem 6-15, sketch the inductor and capacitor currents. Determine the rms values of these currents.

$$I_L := \begin{pmatrix} I_{Lmin} \\ I_{Lmax} \\ I_{Lmin} \end{pmatrix} \qquad t := \begin{pmatrix} 0 \cdot s \\ D \cdot T_s \\ T_s \end{pmatrix}$$

Inductor current rises from min to max value in a time equal to DTs and then falls back to its min value to complete the cycle by time Ts





Problem 6.20 The buck-boost converter of Fig 6-8 has the following parameters: Vs=12V, D=0.6, R=10Ω, L=50μH, C=200μF, and switching frequency = 40kHz.
a. Determine the output voltage.
b. Determine the average, maximum, and mimum inductor currents.
c. Determine the output voltage ripple.

$V_s := 12 \cdot V$ $D := 0.60$ $R_0 := 10 \cdot \Omega$ $L := 50 \cdot \mu H$ $C_0 := 200 \cdot \mu F$ $f_s := 40 \cdot kHz$

a. Determine the output voltage.

$$T_s := \frac{1}{f_s} = 25 \cdot \mu s$$

$$V_0 := \frac{D}{1-D} \cdot V_s = 18\,V$$

b. Determine the average, maximum, and mimum inductor currents.

$$I_{Lave} := \frac{V_s \cdot D}{R_0 \cdot (1-D)^2} = 4.5\,A$$

Another way to do this is to calculate the output and input power, then calculate the input current and, from there, calculate the inductor current.

$P_0 := \frac{V_0^{\,2}}{R_0} = 32.4\,W$ $P_s := P_0 = 32.4\,W$ $I_s := \frac{P_s}{V_s} = 2.7\,A$ $I_{Lave} := \frac{I_s}{D} = 4.5\,A$

The change in inductor current is

$$\Delta I_L := \frac{V_s}{L} \cdot D \cdot T_s = 3.6\,A$$

Max and min inductor currents are

$I_{Lmax} := I_{Lave} + \frac{\Delta I_L}{2} = 6.3\,A$ $I_{Lmin} := I_{Lave} - \frac{\Delta I_L}{2} = 2.7\,A$

c. Determine the output voltage ripple.

$$\Delta V_0 := V_0 \cdot \left(\frac{D}{R_0 \cdot C_0 \cdot f_s} \right) = 0.135\,V$$

Per unit ripple is

$$\frac{\Delta V_0}{V_0} = 0.0075$$