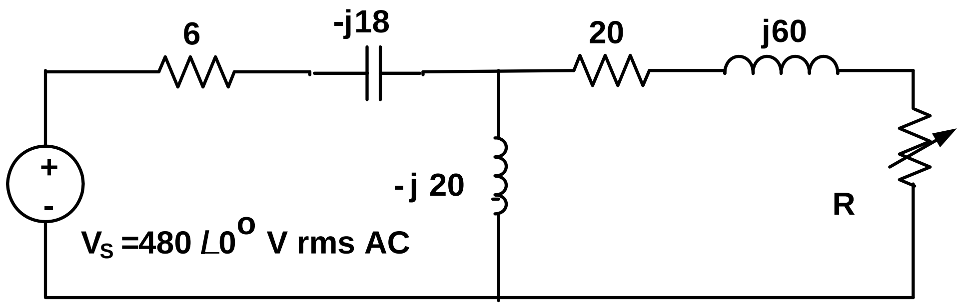
ECE 320

10.34 The variable resistor shown in Figure 10.34 is adjusted until the average power it absorbs is maximum.

A. Find R.
B. Find the maximum average power.



Units for impedances are Ohms

Figure 10.34

Solve this one by first finding the Thevenin equivalent impedance for the network without load R. Then match the impedance magnitudes.

$$j := \sqrt{-1}$$

$$Z_{eq} := (20\cdot\Omega + j\cdot 60\cdot\Omega) + \frac{(8\cdot\Omega - j\cdot 18\cdot\Omega)\cdot(-j\cdot 20\cdot\Omega)}{(8\cdot\Omega - j\cdot 18\cdot\Omega) + (-j\cdot 20\cdot\Omega)} \qquad Z_{eq} = (22.122 + 50.08i)\,\Omega$$

Then match the impedance magnitudes.

$$R_{max} := \left|Z_{eq}\right| \qquad R_{max} = 54.748\,\Omega$$

Find the voltage of the Thevenin circuit and the voltage of the load.

$$V_{th} := (480\cdot V)\cdot\left(\frac{j\cdot 20\cdot\Omega}{8\cdot\Omega - j\cdot 18\cdot\Omega + j\cdot 20\cdot\Omega}\right) \qquad V_{th} = \left(282.353 + 1.129i \times 10^{3}\right)V$$

$$V_{Rmax} := V_{th}\cdot\left(\frac{R_{max}}{R_{max} + Z_{eq}}\right) \qquad V_{Rmax} = (509.073 + 472.732i)\,V$$

Calculate the load power.

$$P_{max} := \frac{\left(\left|V_{Rmax}\right|\right)^2}{R_{max}} \qquad P_{max} = 8.815\cdot kW$$

2. Three loads are connected in parallel across a 480V rms line. Load 1 absorbs 12kW and 6.7 kVAr. Load 2 absorbs 4kVA at 0.96 power factor leading. Load 3 absorbs 15kW at 1.00 power factor.

a. Find the current in Load 1.
b. Load 2 is a combination of a resistance and a capacitance. If these two elements are in parallel with each other, find the resistance and reactance values of each of them.
c. Find the real power, reactive power, and apparent power absorbed by the combined three loads.

Restate the given.

$j := \sqrt{-1}$ $\quad kVAr := kV \cdot A$ $\quad leading := 1$

$P_1 := 12 \cdot kW$ $\quad Q_1 := 6.7 \cdot kVAr$ $\quad V_L := 480 \cdot V$ $\quad lagging := 1$

$S_2 := 4.0 \cdot kV \cdot A$ $\quad pf_2 := 0.96 \cdot leading$

$P_3 := 15 \cdot kW$ $\quad pf_3 := 1.00$

Calculate the current in load 1 from the complex power and the terminal voltage.

$$I_1 := \overline{\left(\frac{P_1 + j \cdot Q_1}{V_L}\right)} = (25 - 13.958i)\,A \qquad |I_1| = 28.633\,A \qquad \arg(I_1) = -29.176 \cdot deg$$

This is the answer for problem 2a.

Find the real and reactive power in load 2.

$$P_2 := S_2 \cdot pf_2 = 3.84 \cdot kW \qquad Q_2 := -\sqrt{S_2 \cdot S_2 - P_2 \cdot P_2} = -1.12 \cdot kVAr$$

For parallel connection, each element has the same terminal voltage. Finding the impedance elements:

$$R_2 := \frac{(|V_L|)^2}{P_2} = 60\,\Omega \qquad X_2 := \frac{(|V_L|)^2}{Q_2} = -205.714\,\Omega$$

This is the answer for problem 2b.

Checking, find the current I_2.

$$\theta_2 := -\operatorname{acos}(0.96) = -0.284$$

$$I_2 := \overline{\left(\frac{S_2}{V_L} \cdot e^{j \cdot \theta_2}\right)} = (8 + 2.333i)\,A \qquad |I_2| = 8.333\,A \qquad \arg(I_2) = 16.26 \cdot deg$$

Using Ohm's Law, find the parallel resistance and reactance. If the voltage has a zero phase angle, which we have already set, then the real part of the current will appear in the resistance and the imaginary part of the current will appear in the reactance.

$$R_2 := \frac{V_L}{\operatorname{Re}(I_2)} = 60\,\Omega \qquad X_2 := \frac{V_L}{\operatorname{Im}(I_2)} = 205.714\,\Omega$$

Collect all the orthogonal components of real and reactive power so we can make a vector addition.

$P_1 = 12 \cdot kW$ $\quad\quad$ $Q_1 = 6.7 \cdot kVAr$

$P_2 = 3.84 \cdot kW$ $\quad\quad$ $Q_2 = -1.12 \cdot kVAr$

$P_3 = 15 \cdot kW$ $\quad\quad$ $Q_3 := 0 \cdot kVAr$ for a 1.00 power factor

Add the components together as orthogonal components of a vector.

$P_T := P_1 + P_2 + P_3 = 30.84 \cdot kW$ $\quad\quad$ $Q_T := Q_1 + Q_2 + Q_3 = 5.58 \cdot kVAr$

$S_T := P_T + j \cdot Q_T = (30.84 + 5.58i) \cdot kV \cdot A$

$\left|S_T\right| = 31.341 \cdot kV \cdot A$

This is the answer for problem 2c.

Checking, find the currents and add them together. We already have I_1 and I_2.

$$I_3 := \frac{P_3}{V_L \cdot pf_3} \cdot e^{j \cdot 0 \cdot deg} = 31.25\ A$$

$I_T := I_1 + I_2 + I_3 = (64.25 - 11.625i)\ A$

Finally, calculate the total kVA.

$S_T := V_L \cdot \overline{I_T} = (30.84 + 5.58i) \cdot kV \cdot A$

3. A certain load has a voltage of 120V AC at 60 Hertz. It draws 3.0 Amps.

a. Let the voltage and current be in phase. Determine and sketch the power that this load draws as a function of time. Draw the voltage on the same set of axes for reference.

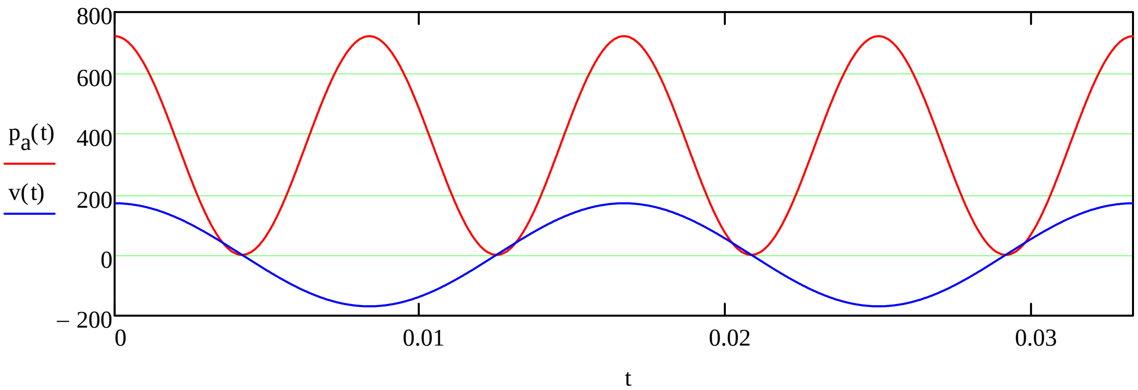
b. Let the power factor be 0.90 lagging; determine and sketch the power that this load draws as a function of time. Draw the voltage on the same set of axes for reference.

The given voltage is rms (not otherwise stated); the current is also rms for the same reason.

$$\omega_s := 2\cdot\pi\cdot 60\cdot\frac{rad}{sec} = 376.991\cdot\frac{rad}{sec}$$

$$v(t) := 120\cdot\sqrt{2}\cdot\cos(\omega_s\cdot t)\cdot V \qquad i_a(t) := 3\cdot\sqrt{2}\cdot\cos(\omega_s\cdot t)\cdot A$$

$$p_a(t) := v(t)\cdot i_a(t) \qquad P_a := \frac{\omega_s}{2\cdot\pi}\cdot\int_0^{\frac{2\cdot\pi}{\omega_s}} p_a(t)\,dt = 360\,W$$

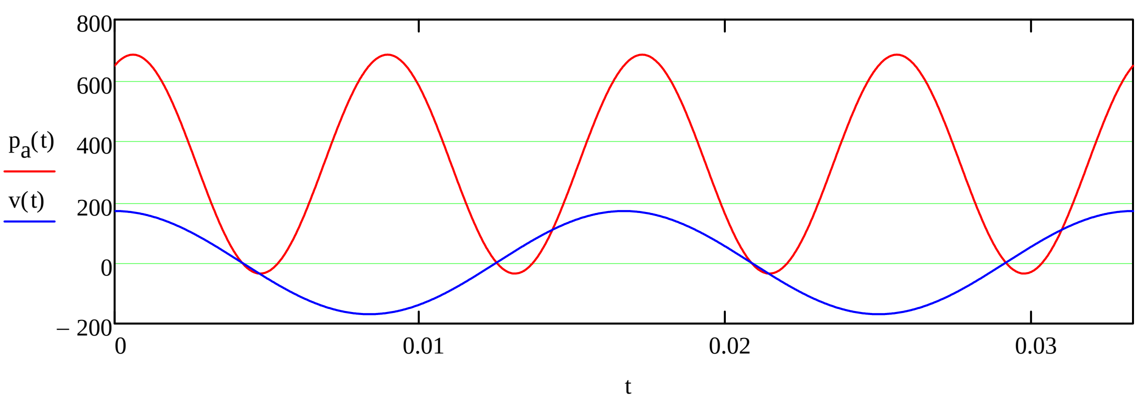


$$\theta_{3b} := acos(0.90) = 25.842\cdot deg$$

$$v(t) := 120\cdot\sqrt{2}\cdot\cos(\omega_s\cdot t)\cdot V \qquad i_a(t) := 3\cdot\sqrt{2}\cdot\cos(\omega_s\cdot t - \theta_{3b})\cdot A$$

$$p_a(t) := v(t)\cdot i_a(t) \qquad P_a := \frac{\omega_s}{2\cdot\pi}\cdot\int_0^{\frac{2\cdot\pi}{\omega_s}} p_a(t)\,dt = 324\,W$$

See the phase shift and the reduced average value of the power waveform.



Problem 4. (10.37) A factory has an electrical load of 1600 kW at a lagging power factor of 0.80. An additional variable power factor load is to be added to the factory. The new load will add 320 kW to the real power load of the factory. The power factor of the added load is to be adjusted so that the overall power factor of the factory is 0.96 leading.

A. Specify the reactive power associated with the added load.
B. Does the added load absorb or deliver magnetizing VArs?
C. What is the power factor of the additional load?
D. Assume that the rms voltage at the factory is 2400V. What is the rms magnitude of the current into the factory before the variable power factor load is added?
E. What is the rms magnitude of the current into the factory after the variable power factor load has been added?

$kVAr := kV \cdot A$

$P_F := 1600 \cdot kW$ $\quad pf_F := 0.80$ lagging $\quad P_N := 320 \cdot kW$ $\quad pf_{all} := 0.96$ leading

$MVAr := 10^6 \cdot V \cdot A$

$MVA := 10^6 \cdot V \cdot A$

Calculate the real and reactive power for the original factory load.

$S_F := \frac{P_F}{pf_F}$ $\quad S_F = 2 \times 10^3 \cdot kV \cdot A$ $\quad Q_F := \sqrt{S_F \cdot S_F - P_F \cdot P_F}$ $\quad Q_F = 1.2 \times 10^3 \cdot kVAr$

Calculate the real and reactive power for the combined load.

$P_{all} := P_F + P_N$ $\quad P_{all} = 1.92 \cdot MW$

$S_{all} := \frac{P_{all}}{pf_{all}}$ $\quad S_{all} = 2 \cdot MVA$ $\quad Q_{all} := -\sqrt{S_{all} \cdot S_{all} - P_{all} \cdot P_{all}}$ $\quad Q_{all} = -0.56 \cdot MVAr$

The added reactive power is the difference between the original and combined values thereof.

$Q_N := Q_{all} - Q_F$ $\quad Q_N = -1.76 \cdot MVAr$ **This is the answer to part a.**

The added load delivers magnetizing VArs. The added load is capacitive in nature.

This is the answer to part b.

Use the definition of power factor to get the new load's power factor.

$S_N := \sqrt{P_N \cdot P_N + Q_N \cdot Q_N}$ $\quad S_N = 1.789 \cdot MVA$

$pf_N := \frac{P_N}{S_N}$ $\quad pf_N = 0.179$ leading **This is the answer to part c.**

Use the equation for real power to find the current, before and after adding the load. $V_F := 2400 \cdot V$

$P_F = V_F \cdot I_F \cdot pf_F$ $\quad I_F := \frac{P_F}{V_F \cdot pf_F}$ $\quad I_F = 833.333\ A$ **This is the answer to part d.**

$I_{all} := \frac{P_{all}}{V_F \cdot pf_{all}}$ $\quad I_{all} = 833.333\ A$ **This is the answer to part e.**

10.44 If a third resistor is added to a hari dryer circuit, it is possible to design three independent power specifications. If the resistor R3 is added in series with the thermal fuse, then the corresponding LOW, MEDIUM, and HIGH power circuit diagrams are as shown in Figure 10.44. If the three power settings are 600W, 900W, and 1200W, respectively, when connected to a 120V rms supply, what resistor values should be used?

Using a power method, we define the three cases. Guess $R_1 := 10 \cdot \Omega \quad R_2 := 10 \cdot \Omega \quad R_3 := 10 \cdot \Omega$

Given

$$600 \cdot W = \frac{(120 \cdot V)^2}{R_1 + R_2 + R_3} \qquad 900 \cdot W = \frac{(120 \cdot V)^2}{R_2 + R_3} \qquad 1200 \cdot W = \frac{(120 \cdot V)^2}{R_3 + \dfrac{R_1 \cdot R_2}{R_1 + R_2}}$$

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} := \text{Find}(R_1, R_2, R_3) = \begin{pmatrix} 8 \\ 8 \\ 8 \end{pmatrix} \Omega$$

For those who prefer to see more algebra, we see that rearranging the first two cases leaves a way to find R_1.

$$R_1 + R_2 + R_3 = \frac{(120 \cdot V)^2}{600 \cdot W} \qquad R_2 + R_3 = \frac{(120 \cdot V)^2}{900 \cdot W}$$

$$R_1 := \frac{(120 \cdot V)^2}{600 \cdot W} - \frac{(120 \cdot V)^2}{900 \cdot W} \qquad R_1 = 8\,\Omega$$

The other two expressions give two equations and two unknowns:

$$900 \cdot W = \frac{(120 \cdot V)^2}{R_2 + R_3}$$

$$R_2 + R_3 = 16$$

$$R_2 + R_3 = 16$$

$$1200 \cdot W = \frac{(120 \cdot V)^2}{R_3 + \dfrac{R_1 \cdot R_2}{R_1 + R_2}}$$

$$R_3 \cdot (R_1 + R_2) + R_1 \cdot R_2 = 12 \cdot (R_1 + R_2)$$

$$R_3 \cdot R_1 + R_3 \cdot R_2 + R_1 \cdot R_2 = 12 \cdot R_1 + 12 \cdot R_2$$

Subsitute for R1.

$$8 \cdot R_3 + R_2 \cdot R_3 + 8 \cdot R_2 - 12 \cdot 8 - 12 \cdot R_2 = 0$$

Simplify again.

$$8 \cdot R_3 + R_2 \cdot R_3 - 4 \cdot R_2 - 96 = 0$$

The two equations simplify to

$$R_2 + R_3 = 16$$

$$8 \cdot R_3 + R_2 \cdot R_3 - 4 \cdot R_2 - 96 = 0$$

Substitute for R2.

$$8 \cdot R_3 + (16 - R_3) \cdot R_3 - 4(16 - R_3) - 96 + 0$$

Simplify into a quadratic equation.

$$8 \cdot R_3 + 16 \cdot R_3 + 4 \cdot R_3 - 96 - R_3^{\ 2} - 64 = 0$$

$$R_3^{\ 2} - 28 \cdot R_3 + 160 = 0$$

Factor this

$$(R_3 - 20) \cdot (R_3 - 8) = 0$$

Solve for R_2 to find the bogus root.

$R_3 := 20$ $\qquad$ $R_2 := 16 - R_3 = -4$

$R_3 := 8$ $\qquad$ $R_2 := 16 - R_3 = 8$

The answer is $\quad R_1 = 8\,\Omega \qquad R_2 = 8 \qquad R_3 = 8$ $\quad$ All resistances are positive in this case, making it the realistic solution.

If we fail to see the factors, use the quadratic formula.

$$R_3 := \frac{28 + \sqrt{28^2 - 4 \cdot 1 \cdot 160}}{2} \qquad R_3 = 20$$

$$R_3 := \frac{28 - \sqrt{28^2 - 4 \cdot 1 \cdot 160}}{2} \qquad R_3 = 8$$

All resistances are positive in this case, making it the realistic solution.

$R_1 = 8\,\Omega \qquad R_2 := 8 \cdot \Omega \qquad R_3 := 8 \cdot \Omega$