1. Find the harmonic content of a six-step line-to-neutral voltage output waveform from a six-step voltage sourced converter as shown in Figure 1. Use a DC line voltage of 300 Volts to determine the voltage levels and a switching frequency of 50 Hertz (for the entire cycle, not merely the individual steps) to determine the appropriate time intervals. In your answer, give the magnitude of the first five non-zero voltage harmonics.

![Figure 1. Six step voltage sourced converter line-to-neutral output waveform](image)

2. An induction motor has an equivalent circuit as shown in Figure 2. It is supplied from a power converter that has a harmonic content as follows: Fundamental frequency voltage amplitude is 1.0 per unit; Fifth harmonic voltage amplitude is 0.20 per unit; Seventh harmonic voltage amplitude is 0.143 per unit; All at zero phase angle. If you need a base for your calculations, use 440V and 50kVA. Answers may be expressed in either per unit or SI units for full credit.

   a. Find the currents and the power losses at each frequency. Remember that all inductances and the slip are functions of the frequency at hand. \( \text{slip} = 0.025 \), \( f = 50 \text{Hz} \)

   b. Why is it a bad idea to use a variable speed drive on a deep bar induction machine?

![Figure 2. Induction motor circuit with voltage sourced converter input](image)

There is more on the other side of this page.
3. In class, we saw a MathCAD simulation of a pulse width modulated waveform using Sine Triangle methods. For a 5.0kHz triangle wave of 1.0 peak-to-peak and an average value of zero with a sine wave of 50 Hertz,

   a. Find the voltage harmonic spectrum.

   b. An interesting way to increase the available fundamental component in the output is to add some third harmonic to the sine wave. Some loads filter out third harmonic, so this is an easy way to get greater performance from a sine-triangle inverter. For the inverter at hand, determine how much more fundamental frequency can be added without overmodulating. There exists a closed form analytical solution to this problem, but it is probably easier and more fun to let computer simulation do the work for you. Two significant figures on the percent increase is plenty.

4. Space vectors provide a way to get specific polyphase voltages from an electronic power converter. For a DC link voltage of 100V and a switching interval of 50μs,

   a. Determine the switching time intervals appropriate to obtain an output voltage vector of 47 Volts at a phase angle of 85 degrees.

   b. Find the maximum possible output voltage that can be obtained at the same 85 degrees phase angle.
\[ \vec{u}_2 = \frac{2}{3} u_d \angle 60^\circ \]

\[ \vec{u}_1, t_1 + \vec{u}_2, t_2 = \vec{u}_x, T \]

\[ T = t_1 + t_2 \]
$$T \begin{bmatrix} U_{xg} \\ U_{xd} \end{bmatrix} = \begin{bmatrix} V_{1g} & V_{2g} \\ V_{1d} & V_{2d} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} V_{1g} & V_{2g} \end{bmatrix}^{-1} \begin{bmatrix} U_{xg} \\ U_{xd} \end{bmatrix}^T$$

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \frac{T}{V_{g1} V_{d2} - V_{d1} V_{g2}} \begin{bmatrix} V_{d2} & -V_{g2} \\ -V_{d1} & V_{g1} \end{bmatrix} \begin{bmatrix} U_{xg} \\ U_{xd} \end{bmatrix}$$

$$T = t_1 + t_2$$
\[
T = \frac{t_1 + t_2 + t_0}{t_1 + t_2 + t_0} = \frac{1}{1}
\]

\[
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]
\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5
\end{bmatrix} =
\begin{bmatrix}
V_{1g} & V_{2g} & 0 & 0 \\
V_{1d} & V_{2d} & 0 & 0 \\
1 & 1 & 1 & 1 \\
\end{bmatrix}
\begin{bmatrix}
V_{1b} \\
V_{2b} \\
V_{xb}
\end{bmatrix}
\]
Space vector example

\[ V_i := 100 \quad f := 80 \text{kHz} \quad T_s := \frac{1}{f} = 12.5 \mu s \]

\[ j := \sqrt{-1} \]

Nearest vectors

\[ V_1 := \frac{2}{3} V_i e^{j0 \cdot \text{deg}} \quad V_2 := \frac{2}{3} V_i e^{j60 \cdot \text{deg}} \]

Define the components of the matrix

\[ V_{1q} := \Re(V_1) = 66.667 \quad V_{1d} := -\Im(V_1) = 0 \]

\[ V_{2q} := \Re(V_2) = 33.333 \quad V_{2d} := -\Im(V_2) = -57.735 \]

Define the desired output

\[ V_x := 45 e^{j34 \cdot \text{deg}} \quad V_{xq} := \Re(V_x) = 37.307 \quad V_{xd} := -\Im(V_x) = -25.164 \]

\[
\begin{pmatrix}
V_{xq} \\
V_{xd}
\end{pmatrix} =
\begin{pmatrix}
V_{1q} & V_{2q} & 0 \\
V_{1d} & V_{2d} & 0
\end{pmatrix}
\begin{pmatrix}
t_1 \\
t_2 \\
t_0
\end{pmatrix}
\]

Solve

\[
\begin{pmatrix}
t_1 \\
t_2 \\
t_0
\end{pmatrix} =
\begin{pmatrix}
V_{1q} & V_{2q} & 0 \\
V_{1d} & V_{2d} & 0
\end{pmatrix}^{-1}
\begin{pmatrix}
V_{xq} \\
V_{xd}
\end{pmatrix} =
\begin{pmatrix}
0.342 \\
0.436 \\
0.222
\end{pmatrix}
\]

Find the times:

\[
\begin{pmatrix}
t_1 \\
t_2 \\
t_0
\end{pmatrix} \cdot T_s =
\begin{pmatrix}
4.271 \\
5.448 \\
2.781
\end{pmatrix} \mu s
\]
Space vector PWM

The basic notion is that we can get a voltage result that is the vector sum of these two vectors weighted by the time each is applied.
We would like the switching frequency to be constant. It makes the switches easier to design and less expensive. Therefore, we add in a third state for the switches.