Two Axis Transformation

Imitation Measured Currents:

Define array of time and define angular frequency:

$$t := 0\text{sec}, 0.0001\text{sec}.. \frac{6}{60\text{Hz}}$$

$$\omega_0 := 2\cdot\pi\cdot 60\text{Hz} \qquad \omega(t) := \omega_0$$

Voltage as a function of time

$$V_{mag} := 15\text{kV}$$

$$v_a(t) := \sqrt{2}\cdot V_{mag}\cdot\cos(\omega(t)\cdot t)$$

$$v_b(t) := \sqrt{2}\cdot V_{mag}\cdot\cos(\omega(t)\cdot t - 120\text{deg})$$

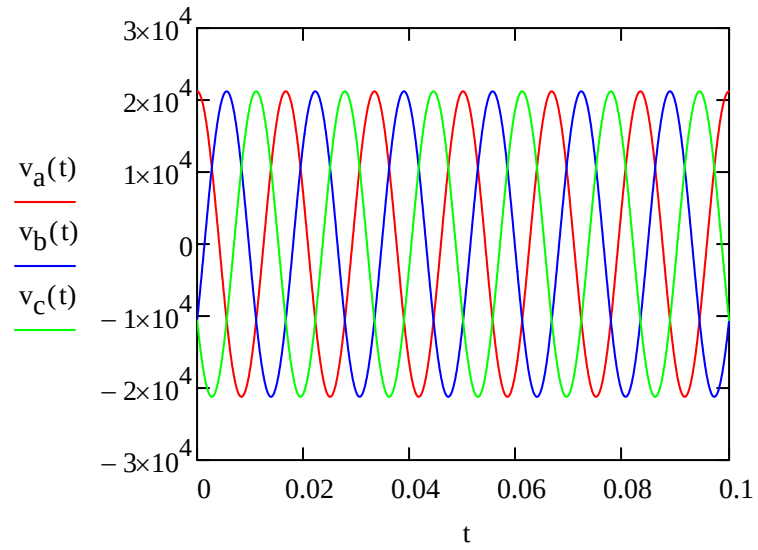
$$v_c(t) := \sqrt{2}\cdot V_{mag}\cdot\cos(\omega(t)\cdot t + 120\text{deg})$$

Transform measured voltage and currents to the stationary dq0 ($\alpha\beta$) reference frame:

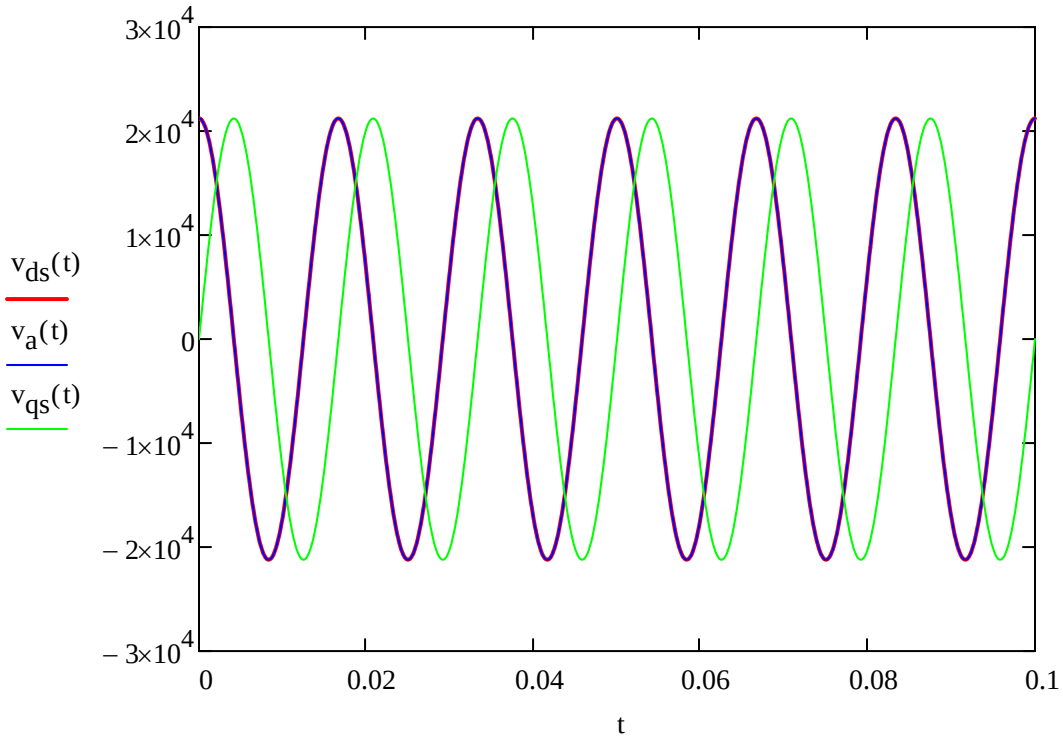
- Use equations from the Clarke Transformation instead of matrix for now

$$v_{ds}(t) := \frac{2}{3}\cdot\left(v_a(t) - 0.5\cdot v_b(t) - 0.5\cdot v_c(t)\right)$$

$$v_{qs}(t) := \frac{\left(v_b(t) - v_c(t)\right)}{\sqrt{3}}$$ Q axis 180 out of phase with some definitions



Transformed voltages (not that $v_{ds}(t)$ in phase with $v_a(t)$



$\theta r(t) := 2 \cdot \pi \cdot 60.0\text{Hz} \cdot t$

- Now apply rotating reference frame transformation in steps

$v_{dr1}(t) := v_{ds}(t) \cdot \cos(\theta r(t))$

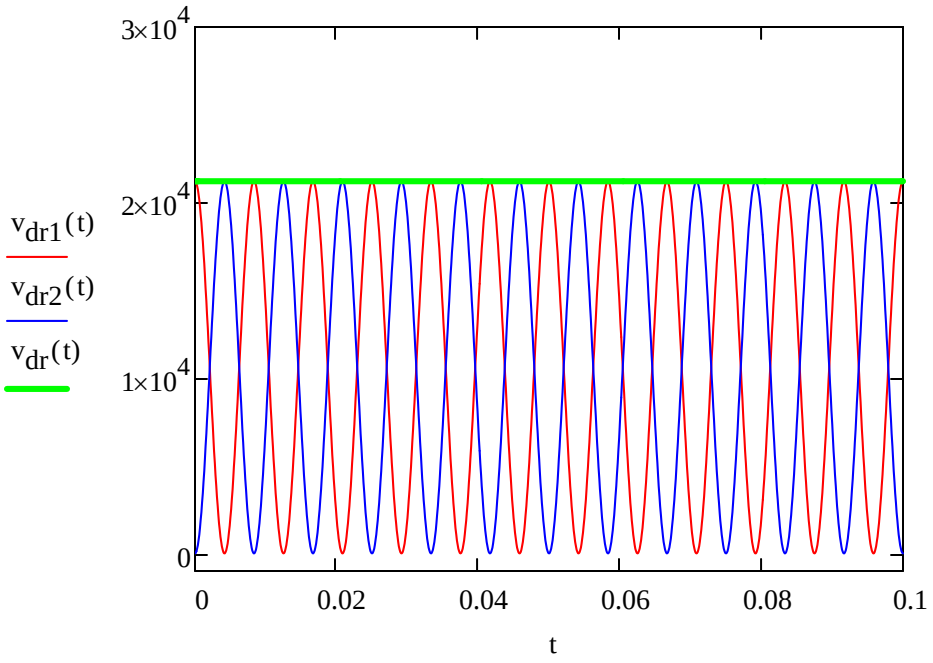
$$v_{dr2}(t) := v_{qs}(t)\cdot\sin(\theta r(t))$$

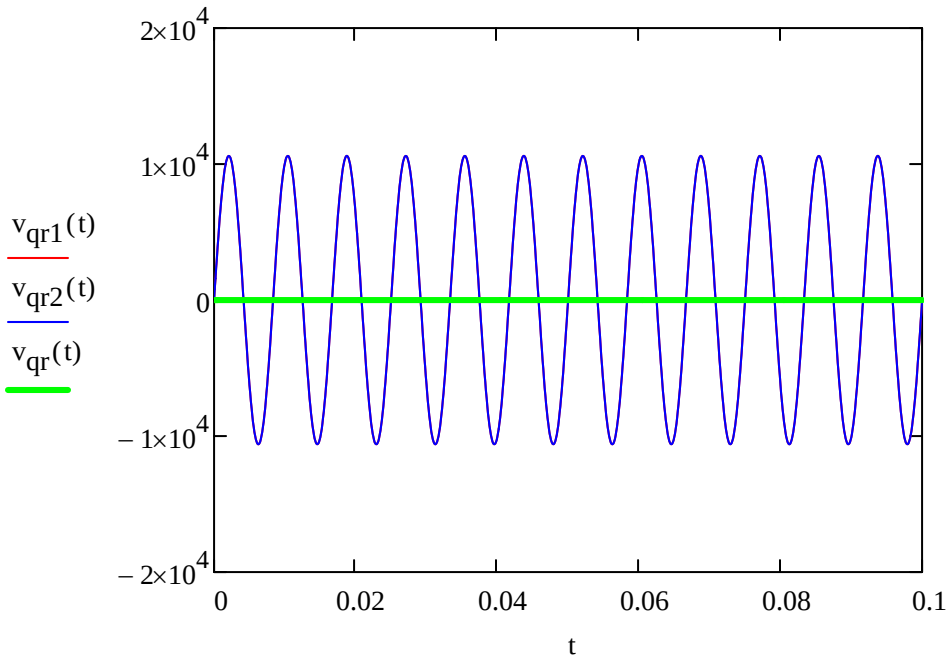
$$v_{dr}(t) := v_{dr1}(t) + v_{dr2}(t)$$

$$v_{qr1}(t) := v_{ds}(t)\cdot\sin(\theta r(t))$$

$$v_{qr2}(t) := v_{qs}(t)\cdot\cos(\theta r(t))$$

$$v_{qr}(t) := -v_{qr1}(t) + v_{qr2}(t)$$





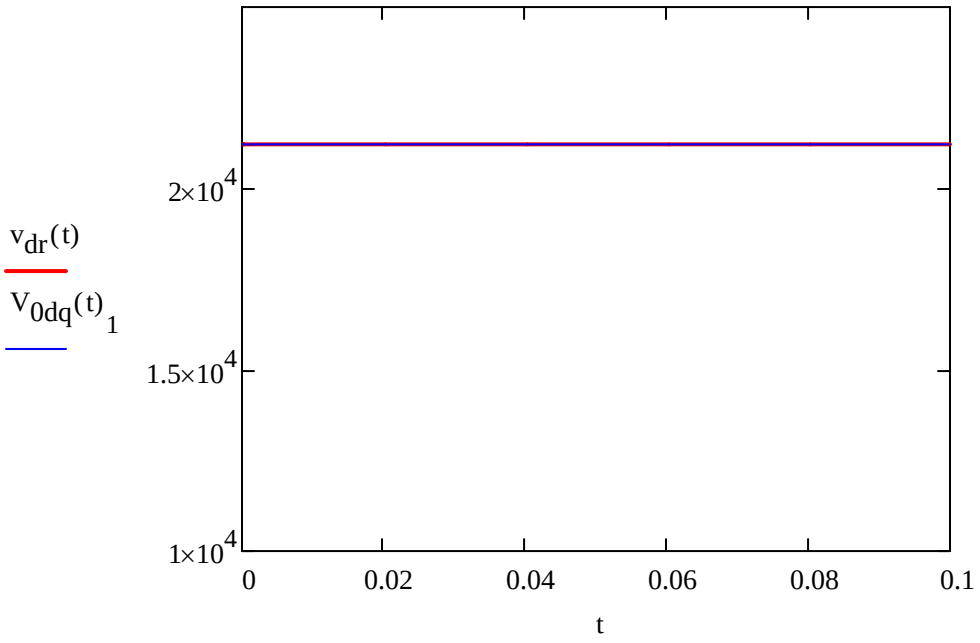
Park's Transformation in Matrix Form

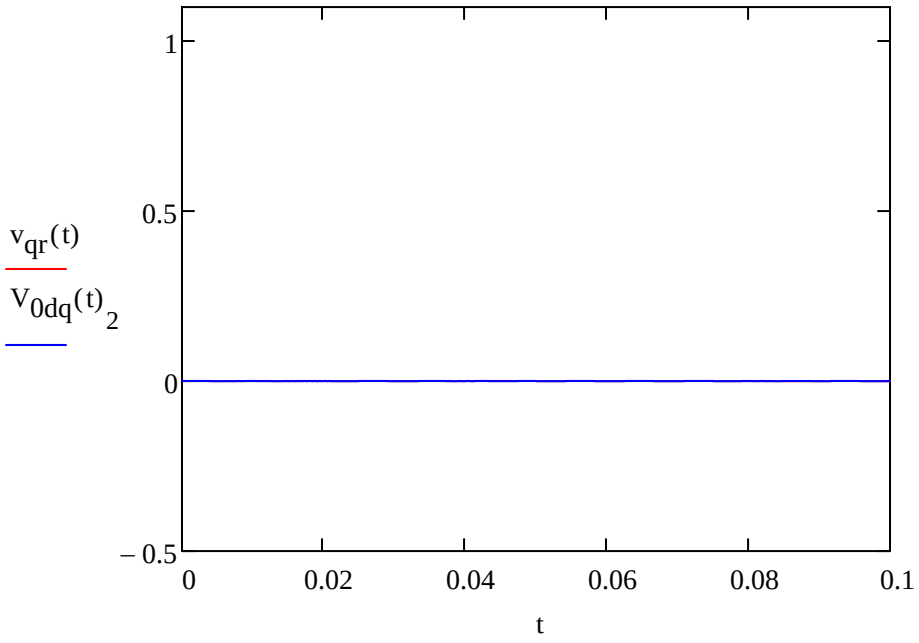
$\theta(t) := \omega_0 \cdot t$ synchronously rotating reference frame, note that this is generally shi by $\pi/2$ for rotating machines.

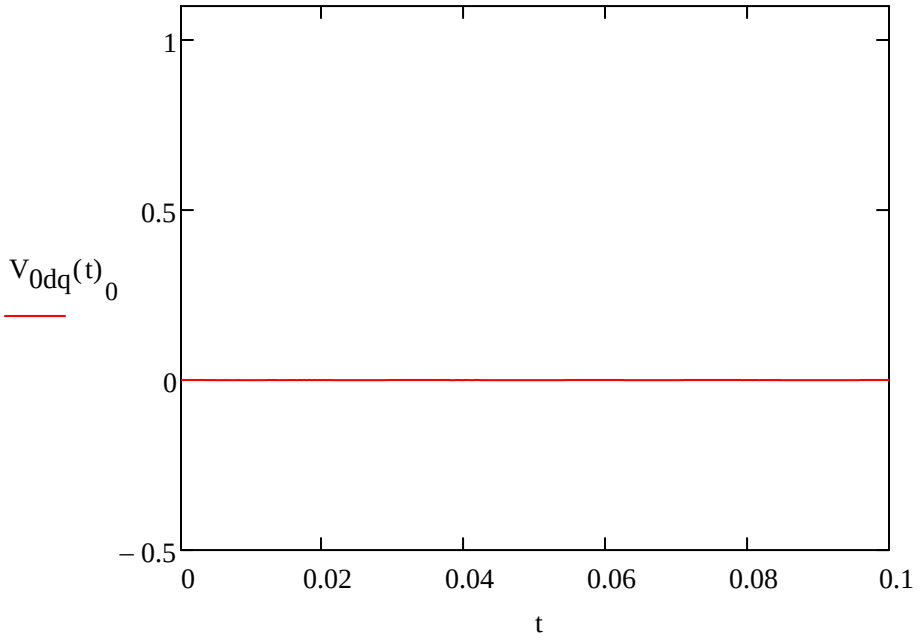
$$P(t) := \frac{2}{3} \cdot \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \cos(\theta(t)) & \cos\left(\theta(t) - \frac{2\cdot\pi}{3}\right) & \cos\left(\theta(t) + \frac{2\cdot\pi}{3}\right) \\ -\sin(\theta(t)) & -\sin\left(\theta(t) - \frac{2\cdot\pi}{3}\right) & -\sin\left(\theta(t) + \frac{2\pi}{3}\right) \end{pmatrix}$$

$$V_{0dq}(t) := P(t)\cdot\begin{pmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{pmatrix}$$

- Compare to earlier results



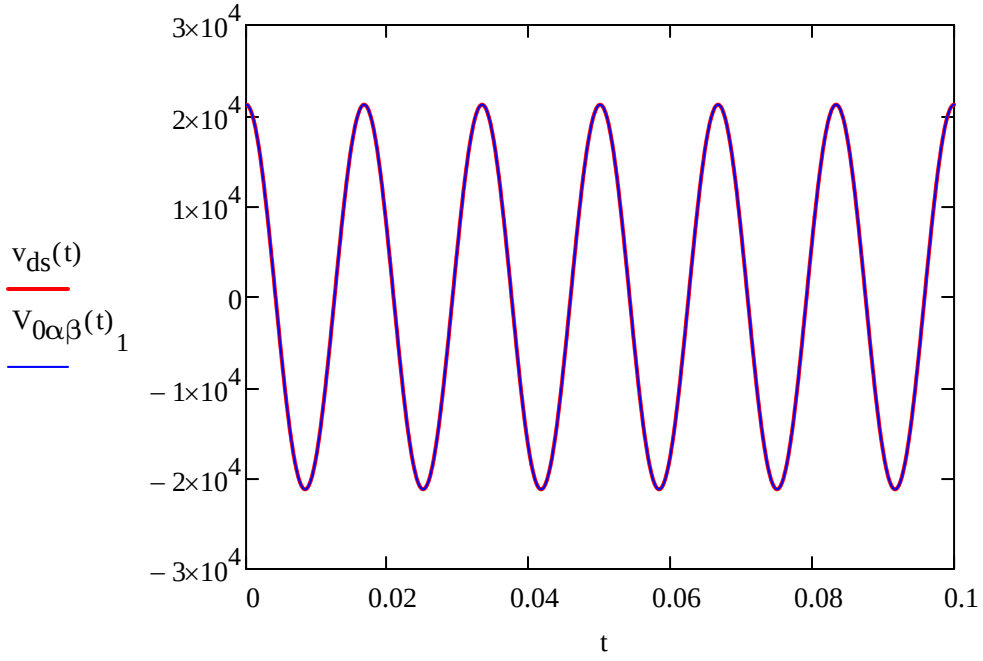


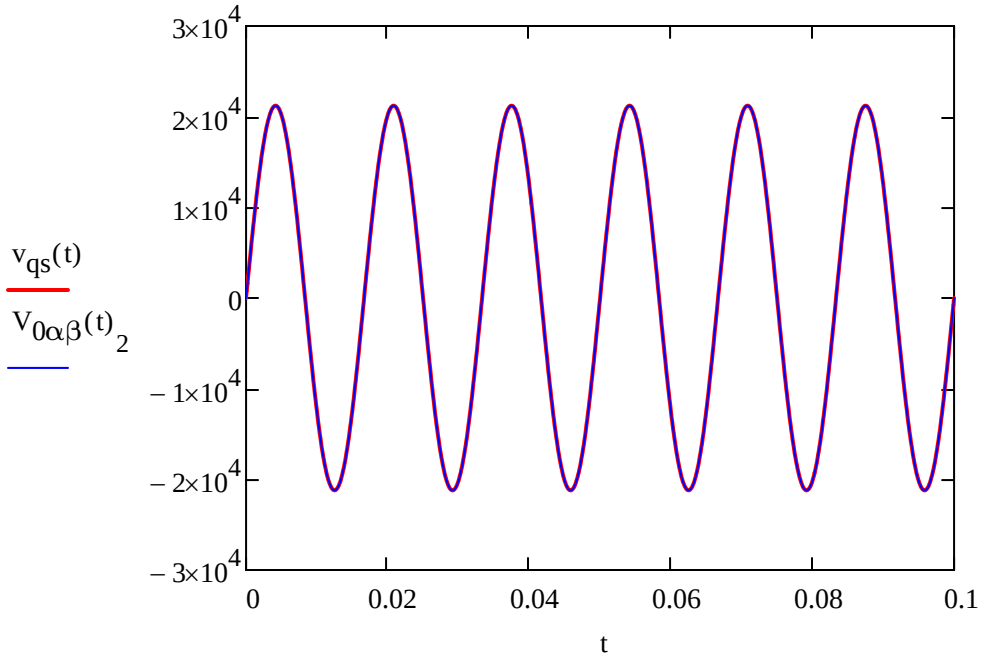


- Balanced three phase source, so zero as expected....

Or if we redefine or reference angle:

$$V_{0\alpha\beta}(t) := P(0)\cdot\begin{pmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{pmatrix}$$





Now add some currents:

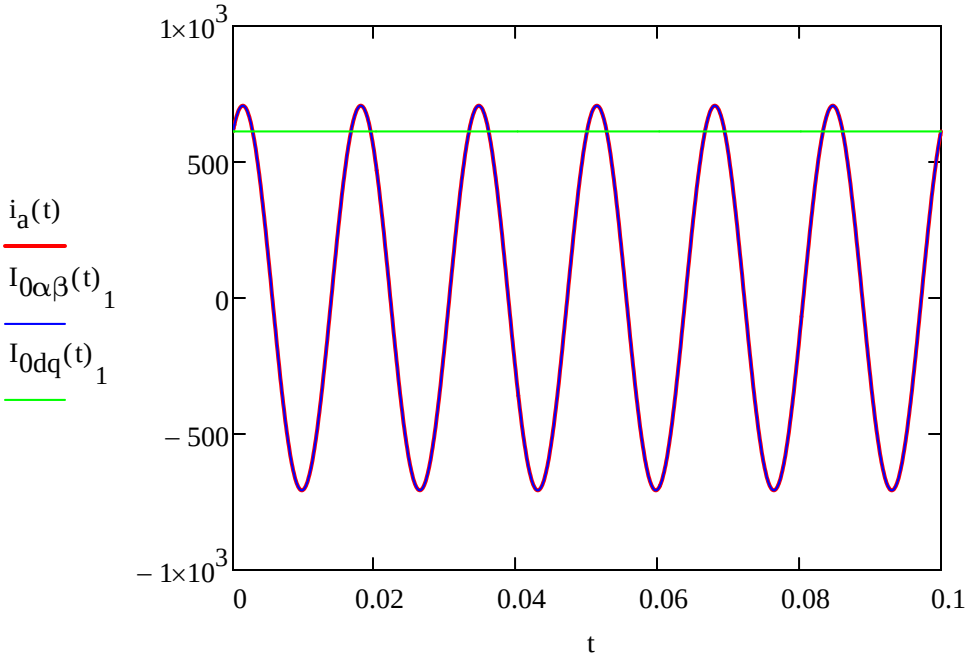
$$I_{mag} := 500A$$

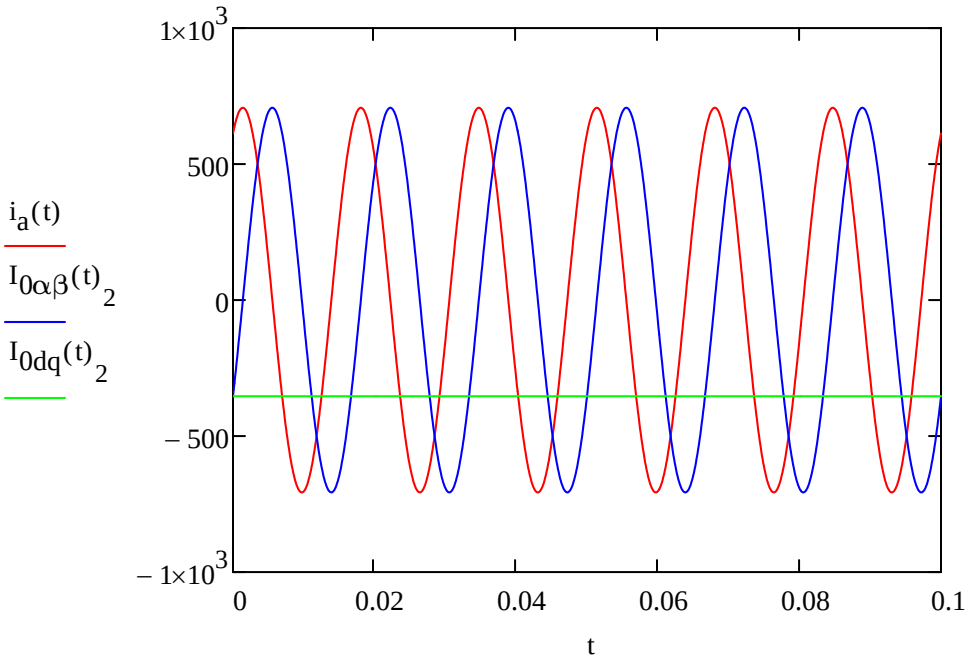
$$i_a(t) := \sqrt{2}\cdot I_{mag}\cdot\cos(\omega(t)\cdot t - 30deg)$$

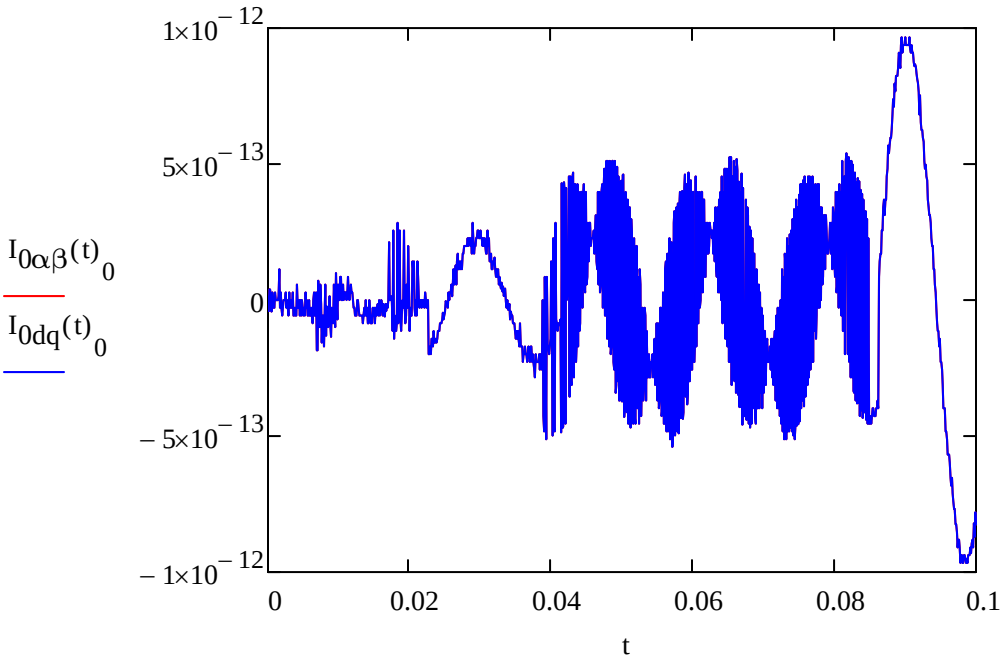
$$i_b(t) := \sqrt{2}\cdot I_{mag}\cdot\cos(\omega(t)\cdot t - 150deg)$$

$$i_c(t) := \sqrt{2}\cdot I_{mag}\cdot\cos(\omega(t)\cdot t + 90deg)$$

$$I_{0dq}(t) := P(t)\cdot\begin{pmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{pmatrix} \qquad I_{0\alpha\beta}(t) := P(0)\cdot\begin{pmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{pmatrix}$$




$i_a(t)$
$I_{0\alpha\beta}(t)_2$
$I_{0dq}(t)_2$
1×10^3
500
0
-500
-1×10^3
0
0.02
0.04
0.06
0.08
0.1
t


$I_{0\alpha\beta}(t)_0$
$I_{0dq}(t)_0$
1×10^{-12}
5×10^{-13}
0
-5×10^{-13}
-1×10^{-12}
0
0.02
0.04
0.06
0.08
0.1
t

Now calculate real and reactive power

$MW := 1000kW \qquad MVA := MW \qquad MVAR := MW$

- Phasor form first:

$$V_a := V_{mag} \cdot e^{j \cdot 0deg}$$

$$I_a := I_{mag} \cdot e^{-j \cdot 30deg}$$

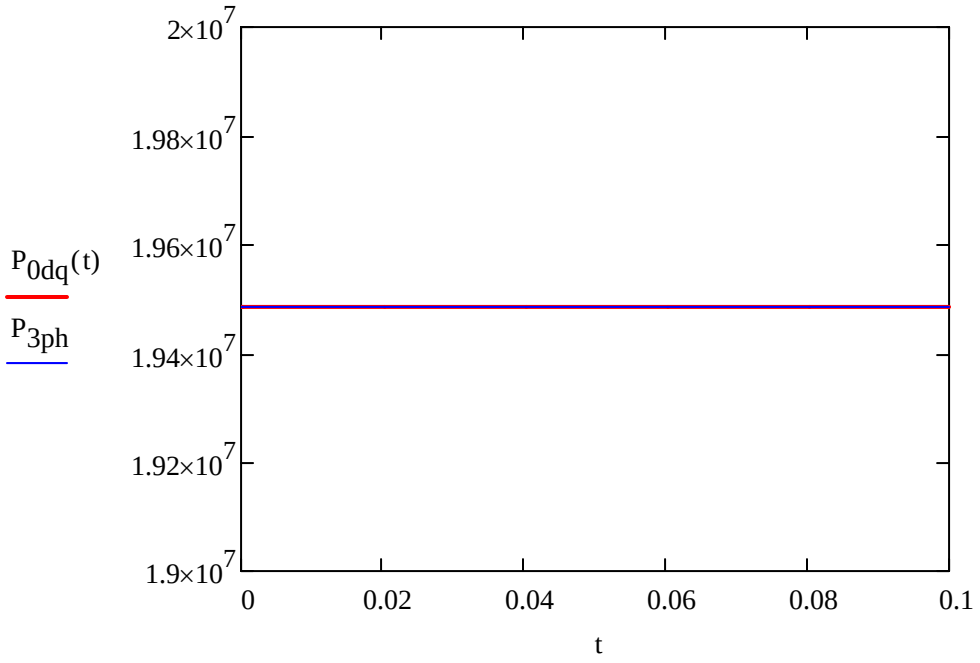
$$P_{3ph} := 3 \cdot Re\left(V_a \cdot \overline{I_a}\right) \qquad P_{3ph} = 19.486 \cdot MW$$

$$Q_{3ph} := 3 \cdot Im\left(V_a \cdot \overline{I_a}\right) \qquad Q_{3ph} = 11.25 \cdot MW$$

- Now in DQ reference frames:

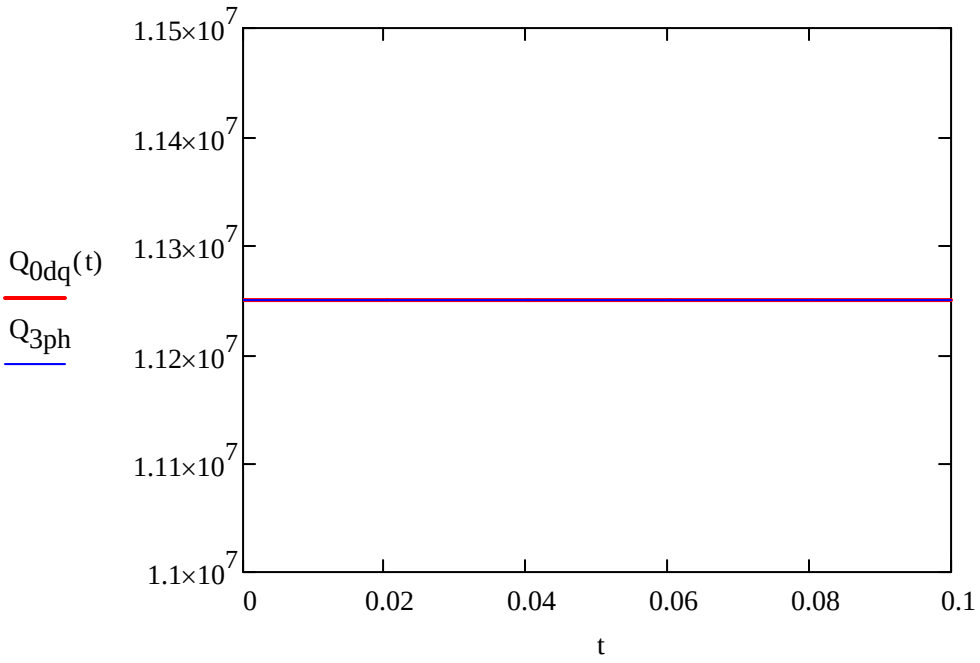
$$P_{0dq}(t) := \frac{3}{2} \cdot \left(V_{0dq}(t)_0 \cdot I_{0dq}(t)_0 + V_{0dq}(t)_1 \cdot I_{0dq}(t)_1 + V_{0dq}(t)_2 \cdot I_{0dq}(t)_2\right)$$

  - Note: we need the 3/2 term because of 2/3 constant in transformation matrix.
  - This will differ if use SQRT(2/3)



$$Q_{0dq}(t) := \frac{3}{2}\cdot\left(V_{0dq}(t)_2\cdot I_{0dq}(t)_1 - V_{0dq}(t)_1\cdot I_{0dq}(t)_2\right)$$

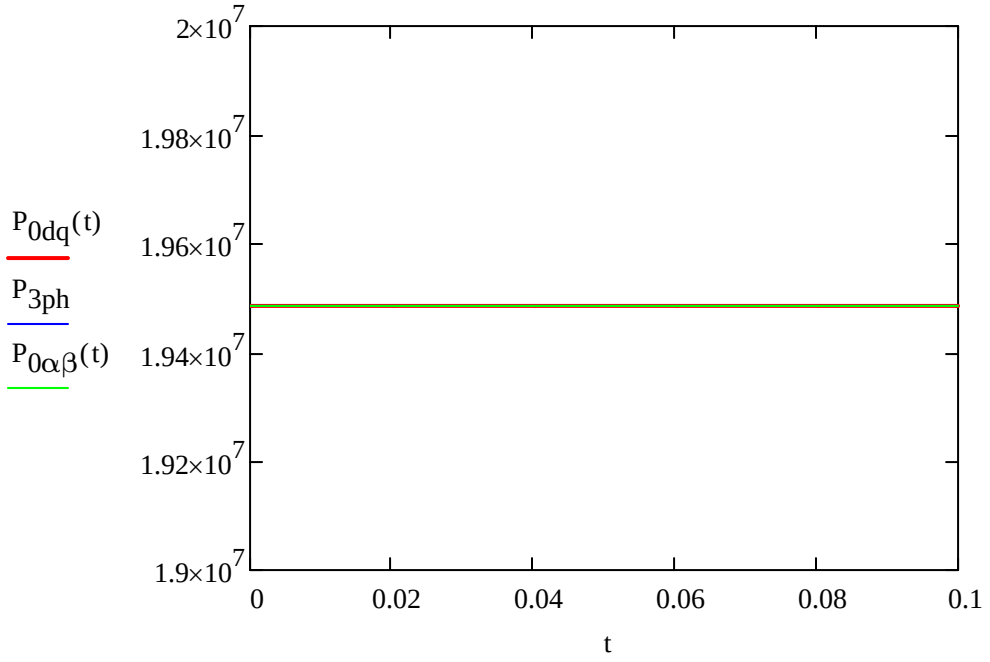
- Note: we need the 3/2 term because of 2/3 constant in transformation matrix.
- This will differ if use SQRT(2/3)

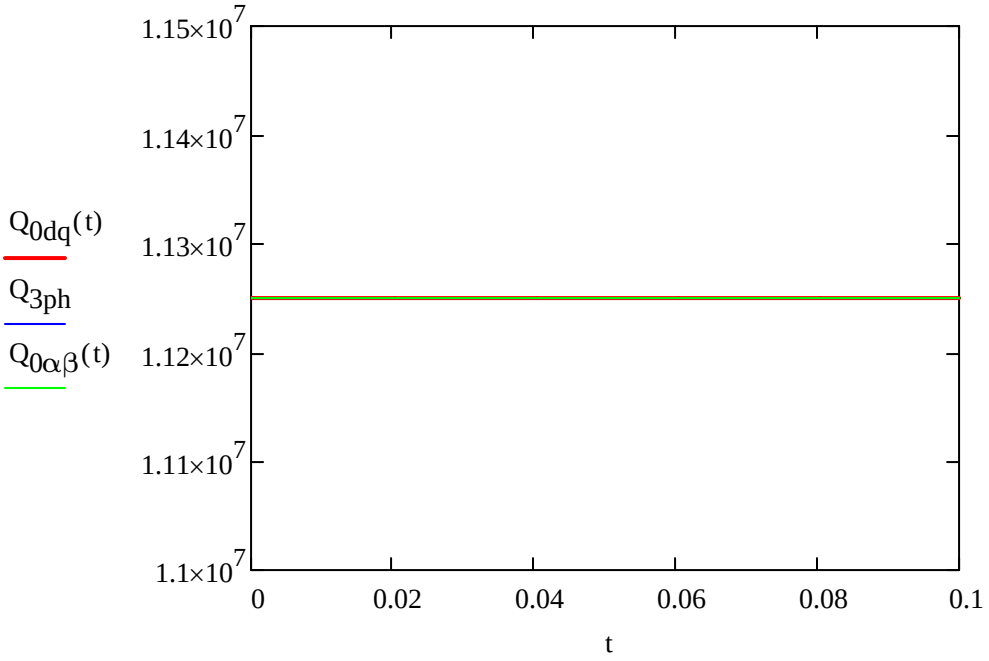


- Finally compare to αβ reference frame

$$P_{0\alpha\beta}(t) := \frac{3}{2}\cdot\left(V_{0\alpha\beta}(t)_0\cdot I_{0\alpha\beta}(t)_0 + V_{0\alpha\beta}(t)_1\cdot I_{0\alpha\beta}(t)_1 + V_{0\alpha\beta}(t)_2\cdot I_{0\alpha\beta}(t)_2\right)$$

$$Q_{0\alpha\beta}(t) := \frac{3}{2}\cdot\left(V_{0\alpha\beta}(t)_2\cdot I_{0\alpha\beta}(t)_1 - V_{0\alpha\beta}(t)_1\cdot I_{0\alpha\beta}(t)_2\right)$$





ATPDraw Example

$\theta_S := 0deg \qquad \theta_R := -20.14deg$

$$V_{S_LN} := \frac{(138kV)}{\sqrt{3}} \cdot e^{j \cdot \theta_S}$$

$$V_{R_LN} := \frac{(132kV)}{\sqrt{3}} \cdot e^{j \cdot \theta_R}$$

$X_{line} := 20ohm$

$$P_{SR} := \left(\frac{3 \cdot \left| V_{S_LN} \right| \cdot \left| V_{R_LN} \right|}{X_{line}} \right) \cdot \sin\left(\theta_S - \theta_R \right)$$

$$P_{SR} = 313.602 \cdot MW$$

$$Q_{SR} := \frac{3 \cdot \left(\left| V_{S_LN} \right| \right)^2}{X_{line}} - \left(\frac{3 \cdot \left| V_{S_LN} \right| \cdot \left| V_{R_LN} \right|}{X_{line}} \right) \cdot \cos\left(\theta_S - \theta_R \right)$$

$$Q_{SR} = 97.092 \cdot MW$$