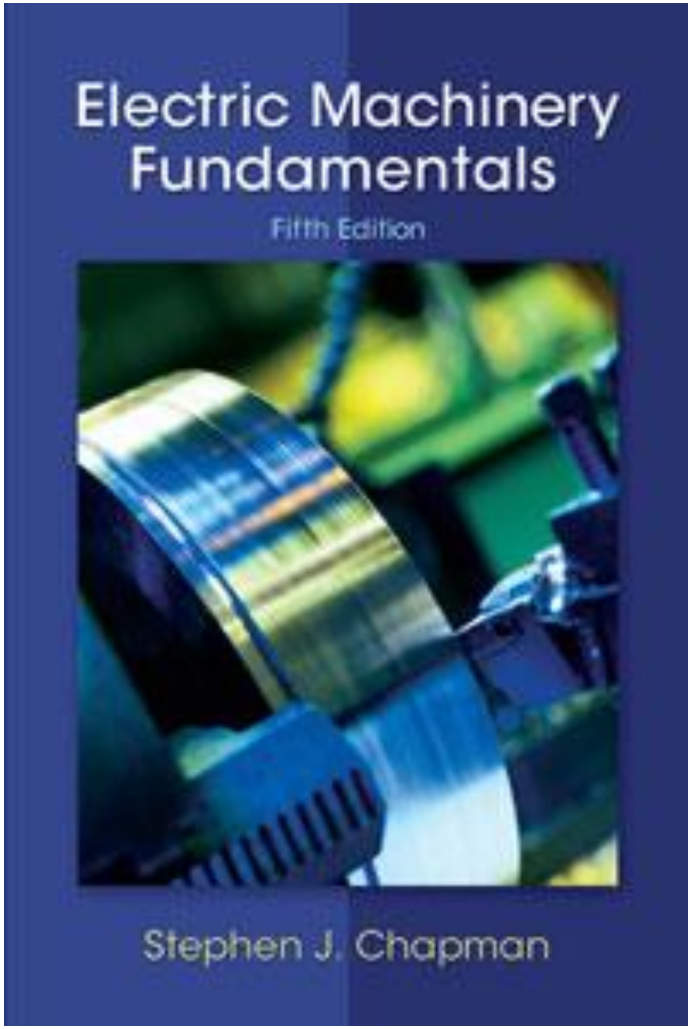

Chapter 6

Induction Motors

The Development of Induced Torque in an Induction Motor

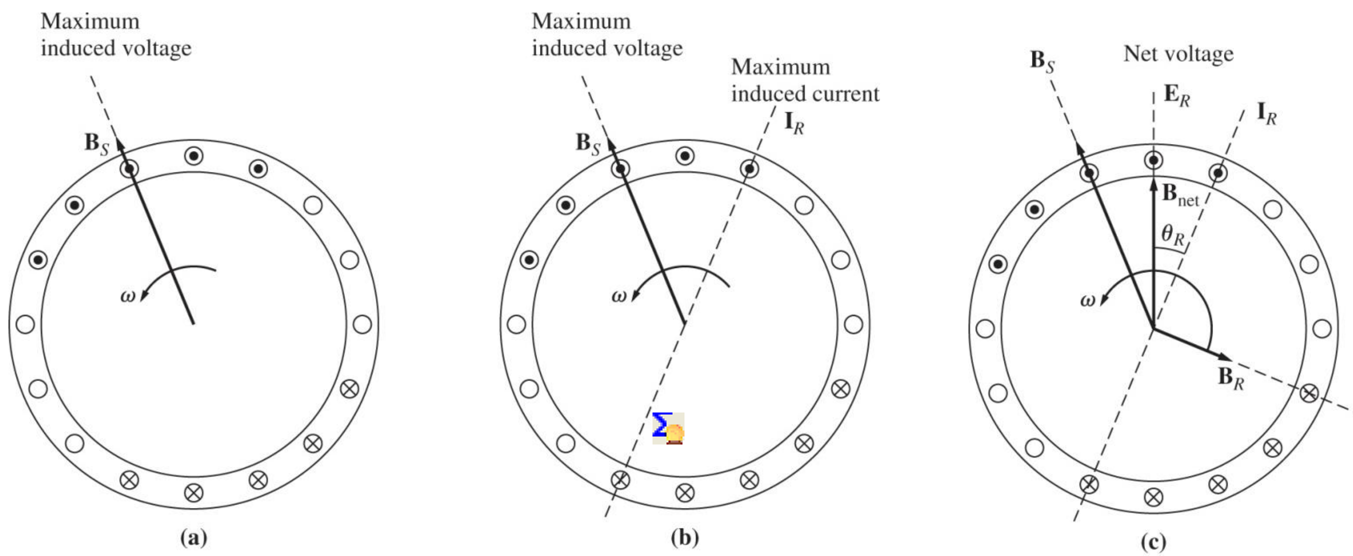

Figure 6-6
The development of induced torque in an induction motor. (a) The rotating stator field B_S induces a voltage in the rotor bars; (b) the rotor voltage produces a rotor current flow, which lags behind the voltage due to rotor inductance; (c) the rotor current produces a magnetic field B_R lagging rotor current by 90^o. Interaction between B_R and B_S produces a torque in the machine.

The Concept of Rotor Slip

- Slip speed is defined as the difference between synchronous speed and rotor speed:

$$n_{slip} = n_{sync} - n_m$$

Where $n_{slip} =$ slip speed of the machine

$n_{sync} =$ speed of the magnetic field

$n_m =$ rotor mechanical speed

$$\text{slip} = s = \frac{n_{slip}}{n_{sync}} = \frac{n_{sync} - n_m}{n_{sync}}$$

The Equivalent Circuit of an Induction Motor

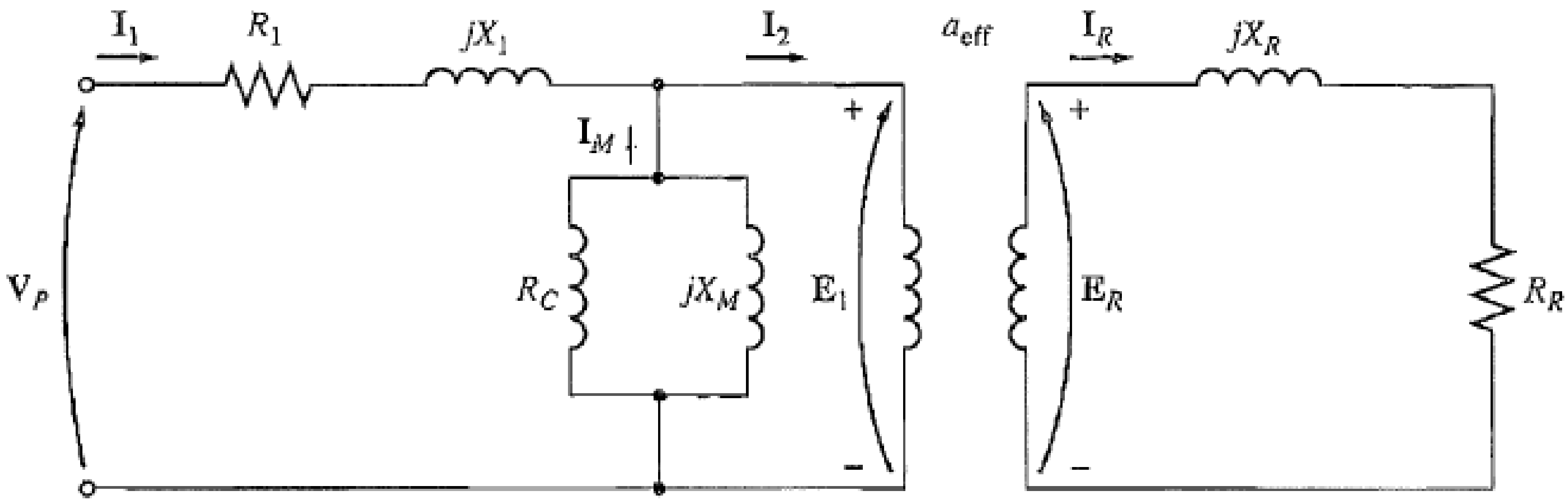

Figure 6-7 Stator Circuit Model Rotor Circuit Model

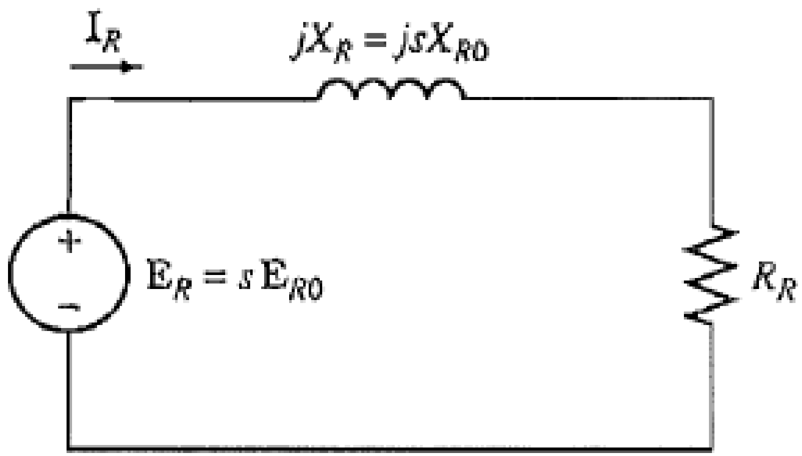

Figure 6-9 Rotor Circuit Model

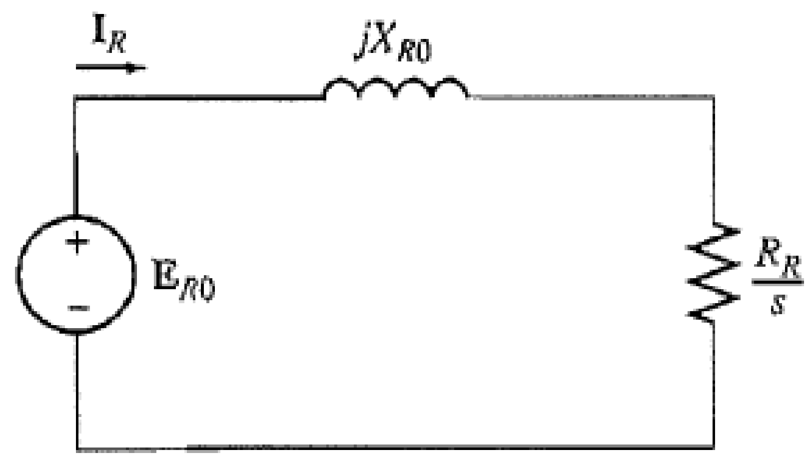

Figure 6-10 Rotor Circuit Model

The Equivalent Circuit of an Induction Motor

R_1 = Stator resistance/phase

X_1 = Stator leakage reactance/phase

R_2= Rotor resistance referred to stator/phase

X_2 = Rotor leakage reactance referred to stator/phase

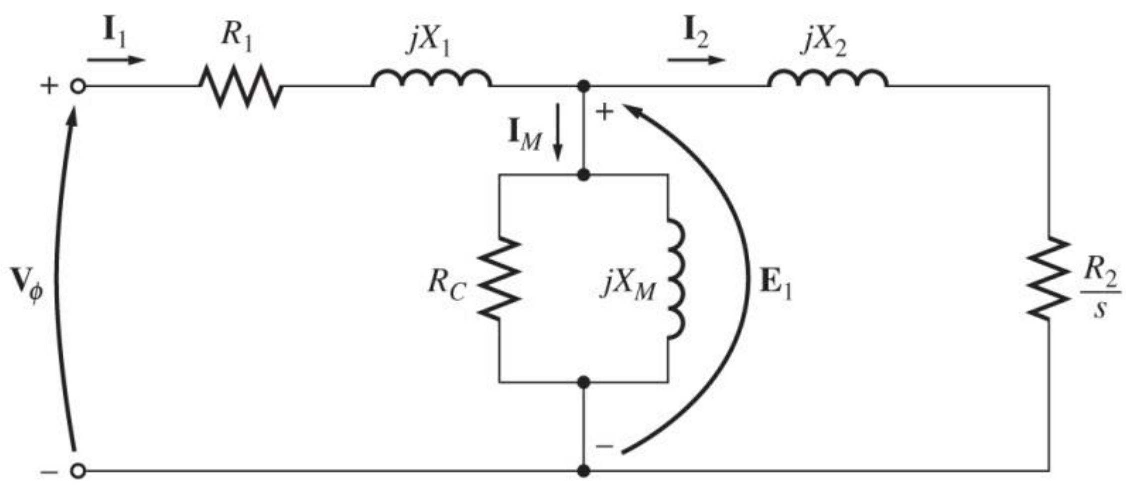

Figure 6-12
The per-phase equivalent circuit of an induction motor.

- Unlike a transformer, in an induction motor, due to the presence of an air gap, the magnetizing current is significant and its effect may not be ignored. However, the core-loss resistance may be removed from the equivalent circuit and its effect accounted for by including core losses in our calculations.

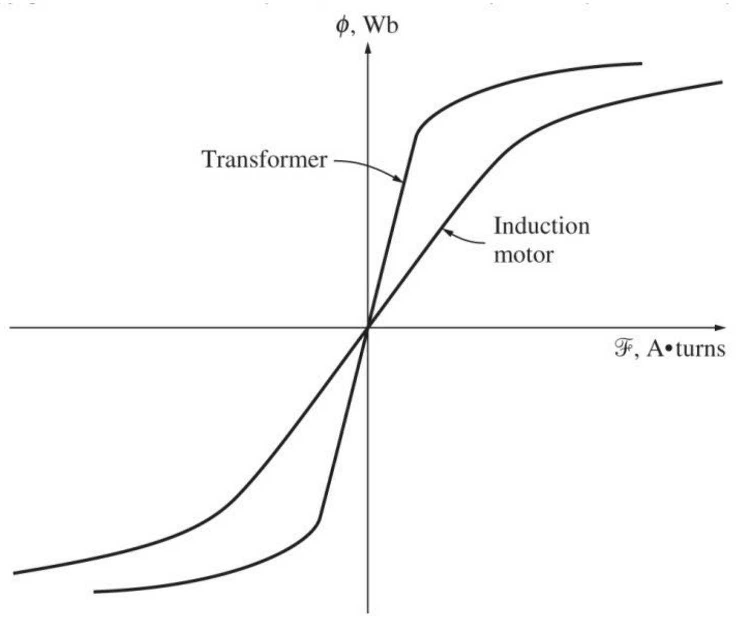

Figure 6-8

The magnetization curve of an induction motor compared to that of a transformer.

Power Flow and Losses of an Induction Motor

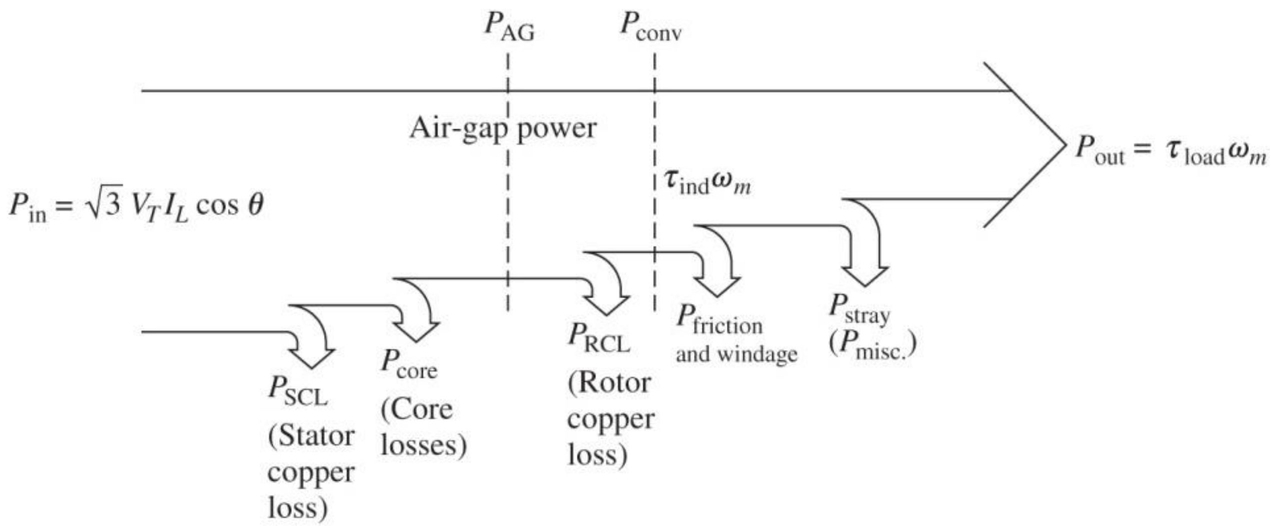

Figure 6-13
The power-flow diagram of an induction motor

Power and Torque in an Induction Motor

- The input impedance of the motor is given by

$$Z_{eq} = (R_1 + jX_1) + (R_c) || (jX_m) || (R_2/s + jX_2)$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{eq}} = I_1 \angle \theta_1$$

$$P_{in} = 3V_\phi I_1 \cos(\theta_1)$$

$$P_{AG} = P_{in} - P_{SCL} - P_{core}$$

$$P_{SCL} = 3 I_1^2 R_1$$

The *only* element in the equivalent circuit where the air-gap power can be consumed is in the resistor R_2/s, therefore,

$$P_{AG} = 3I_2^2 \frac{R_2}{s}$$

- The power converted from electrical to mechanical form, P_{conv}, is given by

$$P_{conv} = P_{AG} - P_{RCL}$$

$$P_{RCL} = 3I_2^2 R_2$$

$$P_{conv} = (1-s)P_{AG}$$

- The output power can be found as

$$P_{out} = P_{conv} - P_{F\&W} - P_{misc}$$

- The induced torque is given by the equation

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{(1-s)P_{AG}}{(1-s)\omega_{sync}} = \frac{P_{AG}}{\omega_{sync}}$$

$$\tau_{ind} = \frac{3}{\omega_{sync}} I_2^2 \left(\frac{R_2}{s} \right)$$

Example 6–3. A 460-V, 25-hp, 60-Hz, four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641\ \Omega \qquad R_2 = 0.332\ \Omega$$
$$X_1 = 1.106\ \Omega \qquad X_2 = 0.464\ \Omega \qquad X_M = 26.3\ \Omega$$

The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency, find the motor's

(a) Speed

(b) Stator current

(c) Power factor

(d) P_{conv} and P_{out}

(e) τ_{ind} and τ_{load}

(f) Efficiency

Solution

The per-phase equivalent circuit of this motor is shown in Figure 6–12, and the power-flow diagram is shown in Figure 6–13. Since the core losses are lumped together with the friction and windage losses and the stray losses, they will be treated like the mechanical losses and be subtracted after P_{conv} in the power-flow diagram.

(a) The synchronous speed is

$$n_{\text{sync}} = \frac{120 f_{se}}{P} = \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

or $$\omega_{\text{sync}} = (1800\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 188.5\text{ rad/s}$$

The rotor's mechanical shaft speed is

$$\begin{aligned} n_m &= (1 - s)n_{\text{sync}} \\ &= (1 - 0.022)(1800\text{ r/min}) = 1760\text{ r/min} \end{aligned}$$

or $$\begin{aligned} \omega_m &= (1 - s)\omega_{\text{sync}} \\ &= (1 - 0.022)(188.5\text{ rad/s}) = 184.4\text{ rad/s} \end{aligned}$$

(*b*) To find the stator current, get the equivalent impedance of the circuit. The first step is to combine the referred rotor impedance in parallel with the magnetization branch, and then to add the stator impedance to that combination in series. The referred rotor impedance is

$$Z_2 = \frac{R_2}{s} + jX_2$$

$$= \frac{0.332}{0.022} + j0.464$$

$$= 15.09 + j0.464\ \Omega = 15.10\angle 1.76^\circ\ \Omega$$

The combined magnetization plus rotor impedance is given by

$$Z_f = \frac{1}{1/jX_M + 1/Z_2}$$

$$= \frac{1}{-j0.038 + 0.0662\angle -1.76^\circ}$$

$$= \frac{1}{0.0773\angle -31.1^\circ} = 12.94\angle 31.1^\circ\ \Omega$$

Therefore, the total impedance is

$$Z_{\text{tot}} = Z_{\text{stat}} + Z_f$$
$$= 0.641 + j1.106 + 12.94\angle 31.1^\circ\ \Omega$$
$$= 11.72 + j7.79 = 14.07\angle 33.6^\circ\ \Omega$$

The resulting stator current is

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{\text{tot}}}$$
$$= \frac{266\angle 0^\circ\ \text{V}}{14.07\angle 33.6^\circ\ \Omega} = 18.88\angle -33.6^\circ\ \text{A}$$

(c) The power motor power factor is

$$\text{PF} = \cos 33.6^\circ = 0.833 \qquad \text{lagging}$$

(d) The input power to this motor is

$$P_{\text{in}} = \sqrt{3} V_T I_L \cos\theta$$
$$= \sqrt{3}(460\ \text{V})(18.88\ \text{A})(0.833) = 12{,}530\ \text{W}$$

The stator copper losses in this machine are

$$P_{SCL} = 3I_1^2 R_1$$

$$= 3(18.88\text{ A})^2(0.641\ \Omega) = 685\text{ W}$$

The air-gap power is given by

$$P_{AG} = P_{in} - P_{SCL} = 12{,}530\text{ W} - 685\text{ W} = 11{,}845\text{ W}$$

Therefore, the power converted is

$$P_{conv} = (1 - s)P_{AG} = (1 - 0.022)(11{,}845\text{ W}) = 11{,}585\text{ W}$$

The power P_{out} is given by

$$P_{out} = P_{conv} - P_{rot} = 11{,}585\text{ W} - 1100\text{ W} = 10{,}485\text{ W}$$

$$= 10{,}485\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = 14.1\text{ hp}$$

(*e*) The induced torque is given by

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}}$$

$$= \frac{11{,}845 \text{ W}}{188.5 \text{ rad/s}} = 62.8 \text{ N} \bullet \text{m}$$

and the output torque is given by

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m}$$

$$= \frac{10{,}485 \text{ W}}{184.4 \text{ rad/s}} = 56.9 \text{ N} \bullet \text{m}$$

(In English units, these torques are 46.3 and 41.9 lb-ft, respectively.)

(*f*) The motor's efficiency at this operating condition is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

$$= \frac{10{,}485 \text{ W}}{12{,}530 \text{ W}} \times 100\% = 83.7\%$$

Induced Torque in an Induction Motor

- The induced torque in an induction motor was to be

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{P_{AG}}{\omega_{sync}} = \frac{3}{\omega_{sync}} I_2^2 \left(\frac{R_2}{s} \right)$$

- To find rotor current I_2, the stator circuit is replaced with its Thevenin equivalent circuit.

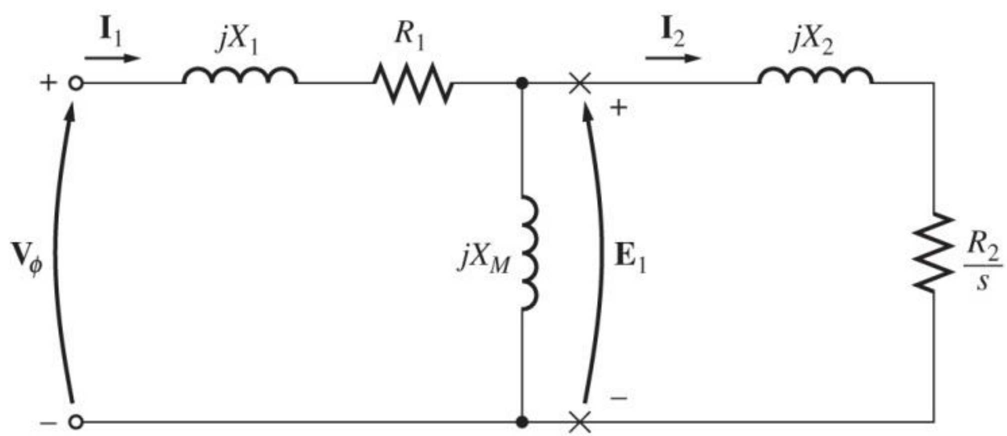

Figure 6-17

Per-phase equivalent circuit of an induction motor.

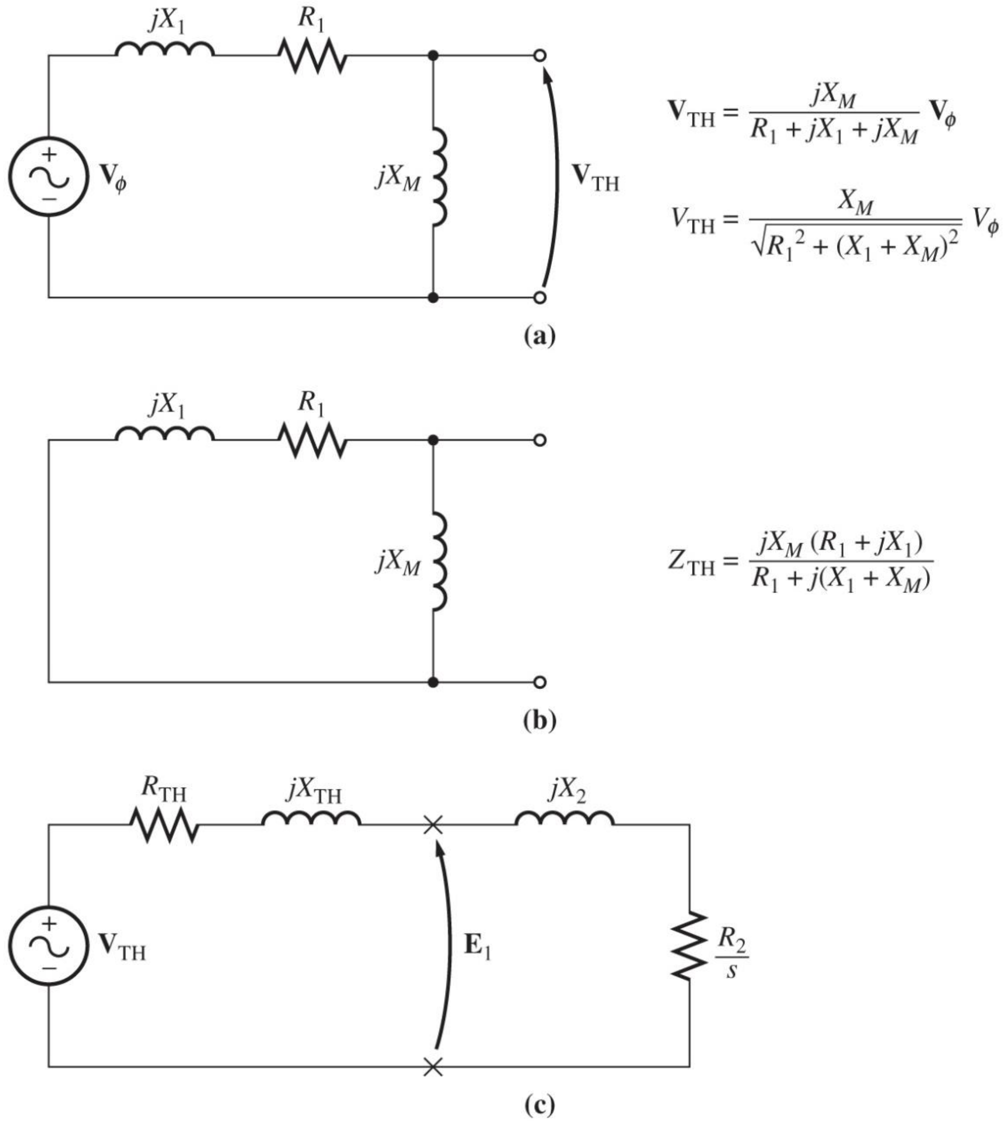

Figure 6-18
(a) The Thevenin equivalent voltage of the stator circuit. (b) The Thevenin impedance. (c) The resulting simplified equivalent circuit of an induction motor

$$\mathbf{V}_{TH} = \mathbf{V}_{\phi} \frac{jX_M}{R_1 + j(X_1 + X_M)}$$

$$Z_{TH} = R_{TH} + jX_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_{TH}}{R_{TH} + R_2/s + j(X_{TH} + X_2)}$$

$$I_2 = \frac{V_{TH}}{\sqrt{\left(R_{TH} + R_2/s\right)^2 + \left(X_{TH} + X_2\right)^2}}$$

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{3V_{TH}^2}{\omega_{sync}} \frac{R_2/s}{\left(R_{TH} + R_2/s\right)^2 + \left(X_{TH} + X_2\right)^2}$$

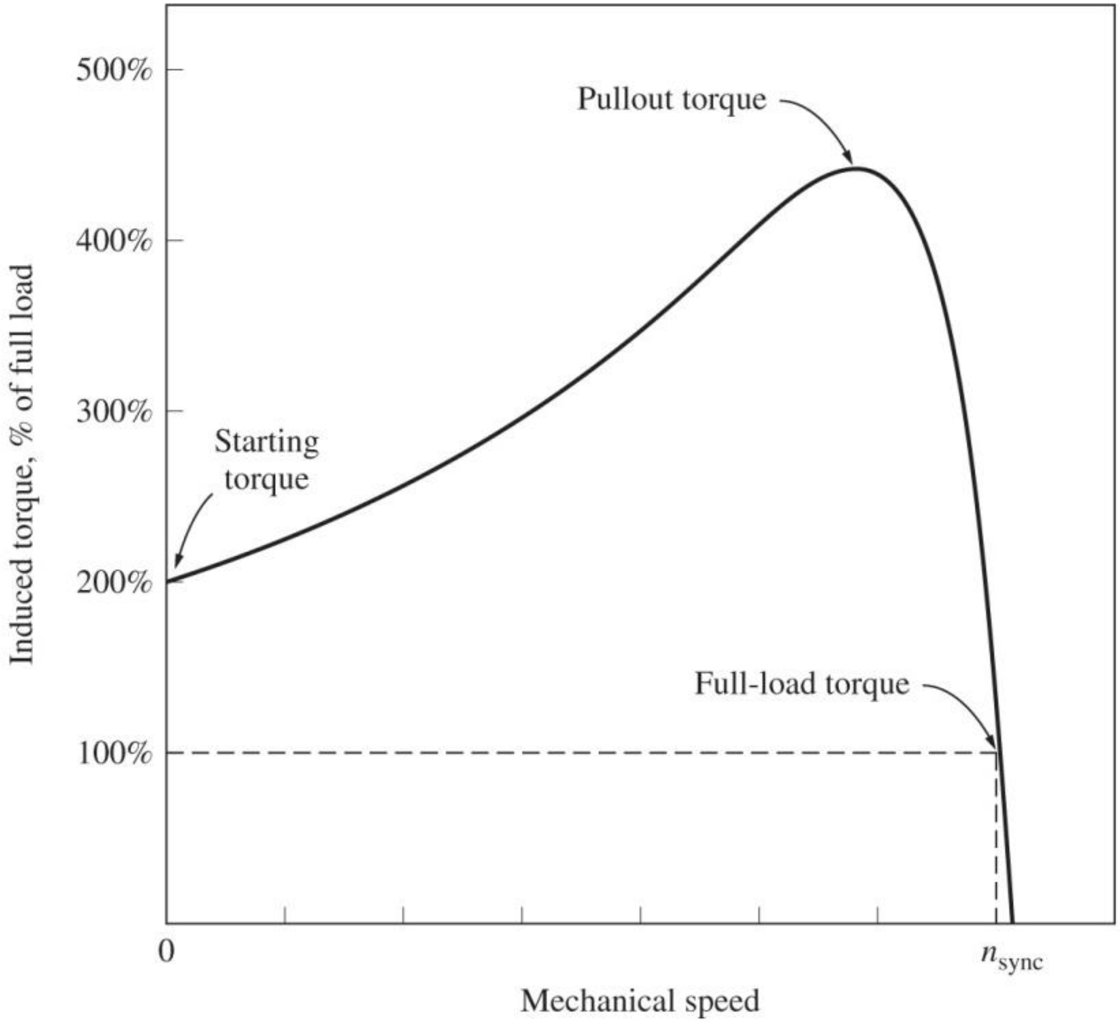

Figure 6-19
A typical induction motor torque-speed characteristic curve

Maximum (Pullout) Torque in an Induction Motor

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{3V_{TH}^2}{\omega_{sync}} \frac{R_2/s}{\left(R_{TH} + R_2/s\right)^2 + \left(X_{TH} + X_2\right)^2}$$

$$\frac{d\,\tau_{ind}}{ds} = 0$$

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}}$$

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}\right]}$$

- Slip at maximum torque can be varied by changing rotor resistance while the corresponding maximum torque is independent of R_2

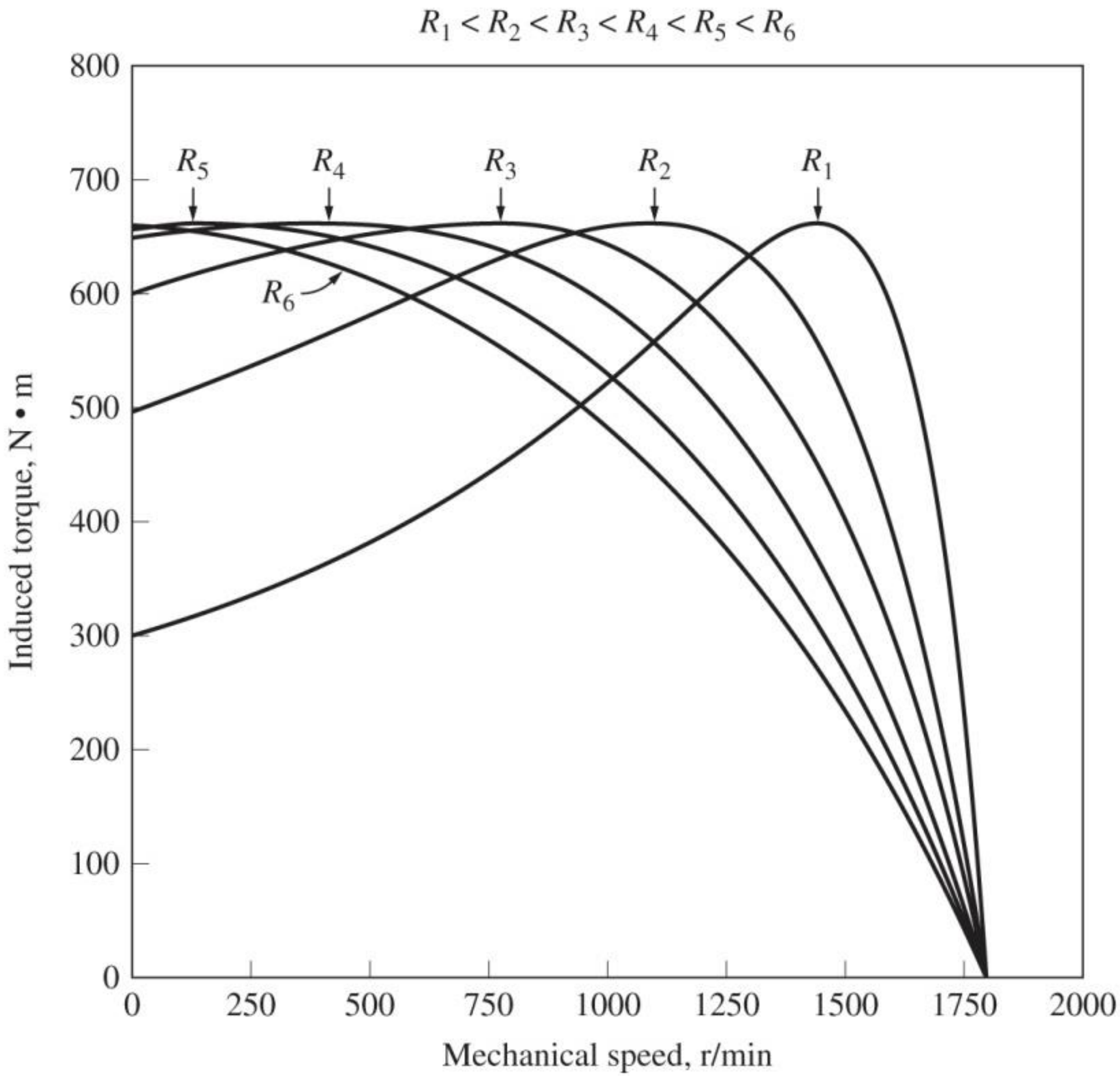

Figure 6-22
The effect of varying rotor resistance on the torque-speed characteristic of a wound-rotor induction motor.

Example 6–4. A two-pole, 50-Hz induction motor supplies 15 kW to a load at a speed of 2950 r/min.

(a) What is the motor's slip?
(b) What is the induced torque in the motor in N • m under these conditions?
(c) What will the operating speed of the motor be if its torque is doubled?
(d) How much power will be supplied by the motor when the torque is doubled?

Solution

(a) The synchronous speed of this motor is

$$n_{\text{sync}} = \frac{120 f_{se}}{P} = \frac{120(50\text{ Hz})}{2\text{ poles}} = 3000\text{ r/min}$$

Therefore, the motor's slip is

$$s = \frac{n_{\text{sync}} - n_{\text{m}}}{n_{\text{sync}}}(\times\ 100\%)$$

$$= \frac{3000\text{ r/min} - 2950\text{ r/min}}{3000\text{ r/min}}(\times\ 100\%)$$

$$= 0.0167\text{ or }1.67\%$$

(*b*) The induced torque in the motor must be assumed equal to the load torque, and P_{conv} must be assumed equal to P_{load}, since no value was given for mechanical losses. The torque is thus

$$\tau_{ind} = \frac{P_{conv}}{\omega_m}$$

$$= \frac{15 \text{ kW}}{(2950 \text{ r/min})(2\pi \text{ rad/r})(1 \text{ min}/60 \text{ s})}$$

$$= 48.6 \text{ N} \bullet \text{m}$$

(*c*) In the low-slip region, the torque–speed curve is linear, and the induced torque is directly proportional to slip. Therefore, if the torque doubles, then the new slip will be 3.33 percent. The operating speed of the motor is thus

$$n_m = (1 - s)n_{sync} = (1 - 0.0333)(3000 \text{ r/min}) = 2900 \text{ r/min}$$

(*d*) The power supplied by the motor is given by

$$P_{conv} = \tau_{ind}\omega_m$$

$$= (97.2 \text{ N} \bullet \text{m})(2900 \text{ r/min})(2\pi \text{ rad/r})(1 \text{ min}/60 \text{ s})$$

$$= 29.5 \text{ kW}$$

Example 6–5. A 460-V, 25-hp, 60-Hz, four-pole, Y-connected wound-rotor induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$R_1 = 0.641\ \Omega \qquad R_2 = 0.332\ \Omega$$
$$X_1 = 1.106\ \Omega \qquad X_2 = 0.464\ \Omega \qquad X_M = 26.3\ \Omega$$

(a) What is the maximum torque of this motor? At what speed and slip does it occur?

(b) What is the starting torque of this motor?

(c) When the rotor resistance is doubled, what is the speed at which the maximum torque now occurs? What is the new starting torque of the motor?

Solution

The Thevenin voltage of this motor is

$$V_{\mathrm{TH}} = V_\phi \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}$$

$$= \frac{(266 \text{ V})(26.3 \ \Omega)}{\sqrt{(0.641 \ \Omega)^2 + (1.106 \ \Omega + 26.3 \ \Omega)^2}} = 255.2 \text{ V}$$

The Thevenin resistance is

$$R_{\mathrm{TH}} \approx R_1 \left(\frac{X_M}{X_1 + X_M} \right)^2$$

$$\approx (0.641 \ \Omega) \left(\frac{26.3 \ \Omega}{1.106 \ \Omega + 26.3 \ \Omega} \right)^2 = 0.590 \ \Omega$$

The Thevenin reactance is

$$X_{\mathrm{TH}} \approx X_1 = 1.106 \ \Omega$$

(a) The slip at which maximum torque occurs is given by Equation

$$= \frac{0.332\ \Omega}{\sqrt{(0.590\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2}} = 0.198$$

This corresponds to a mechanical speed of

$$n_m = (1 - s)n_{\text{sync}} = (1 - 0.198)(1800\ \text{r/min}) = 1444\ \text{r/min}$$

The torque at this speed is

$$\tau_{\max} = \frac{3V_{\text{TH}}^2}{2\omega_{\text{sync}}\left[R_{\text{TH}} + \sqrt{R_{\text{TH}}^2 + (X_{\text{TH}} + X_2)^2}\right]} = 229\ \text{N} \cdot \text{m}$$

(b) $$T_{start} = \frac{3(255.2\ \text{V})^2(0.332\ \Omega)}{(188.5\ \text{rad/s})[(0.590\ \Omega + 0.332\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$$

$$= 104\ \text{N} \bullet \text{m}$$

(c) If the rotor resistance is doubled, then the slip at maximum torque doubles, too. Therefore,

$$s_{\max} = 0.396$$

and the speed at maximum torque is

$$n_m = (1 - s)n_{\text{sync}} = (1 - 0.396)(1800 \text{ r/min}) = 1087 \text{ r/min}$$

The maximum torque is still

$$\tau_{\max} = 229 \text{ N} \bullet \text{m}$$

The starting torque is now

$$\tau_{\text{start}} = \frac{3(255.2 \text{ V})^2(0.664\ \Omega)}{(188.5 \text{ rad/s})[(0.590\ \Omega + 0.664\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$$

$$= 170 \text{ N} \bullet \text{m}$$

Induction Motor Testing

- The No-Load Test: to obtain the rotational losses and information leading to magnetizing reactance. Motor operated at rated voltage and no load.

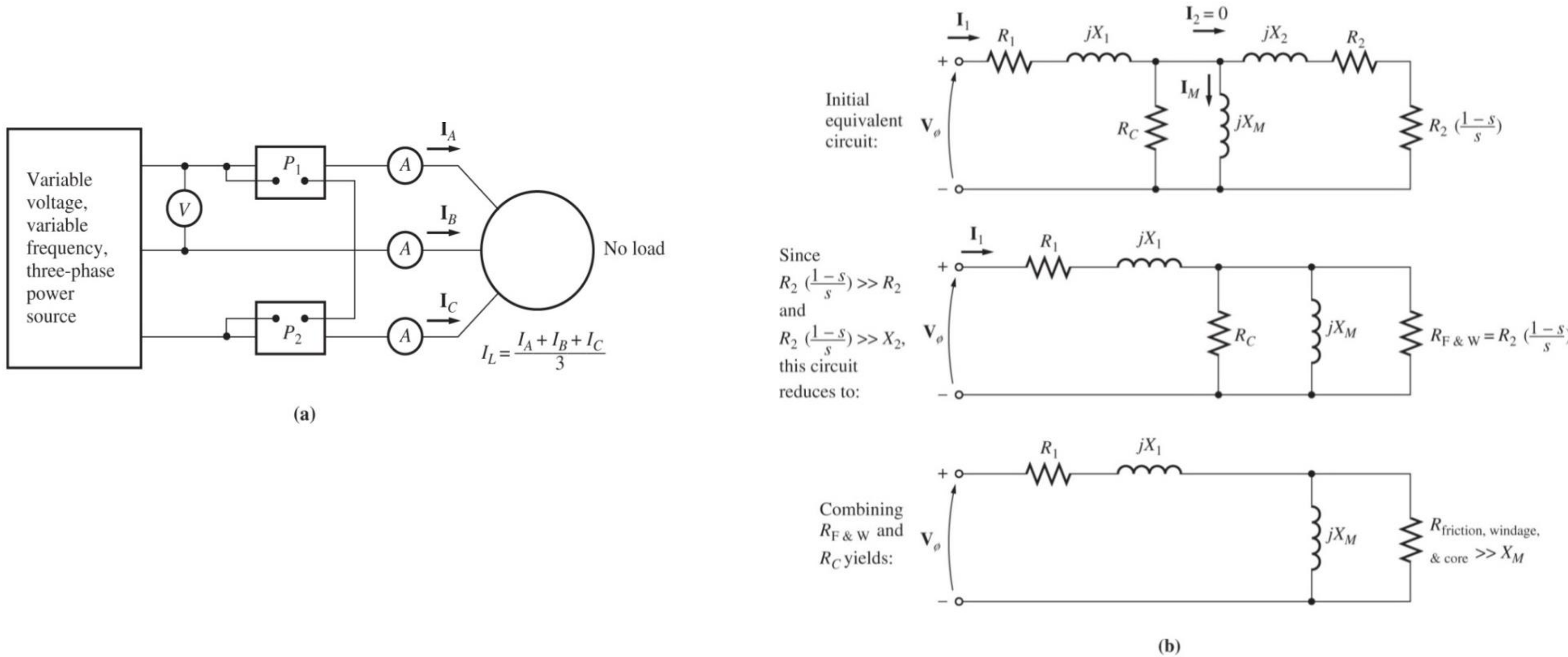

Figure 6-53
The no-load test of an induction motor. (a) test circuit. (b) the resulting equivalent circuit. Note that at no load the motor's impedance is essentially $R_1 + j\,(X_1 + X_M)$.

- The rotational losses of the motor are

$$P_{in} = P_{SCL} + P_{core} + P_{F\&W} + P_{misc} = P_{SCL} + P_{rot}$$

$$P_{SCL} = 3\,I_{1,nl}^{2} R_{1}$$

$$P_{rot} = P_{in} - 3\,I_{1,nl}^{2} R_{1}$$

$$Z_{eq} = \frac{V_{\phi}}{I_{1,nl}} \approx X_{1} + X_{M}$$

  - The stator resistance will be obtained from the dc test.
  - X_1 will be obtained from the locked-rotor test.

- The DC Test: to obtain stator resistance, R_1. An adjusted dc voltage is applied between two terminals of the stator circuit such that rated armature current flows.

$$R_1 = \frac{V_{DC}}{2 I_{DC}}$$

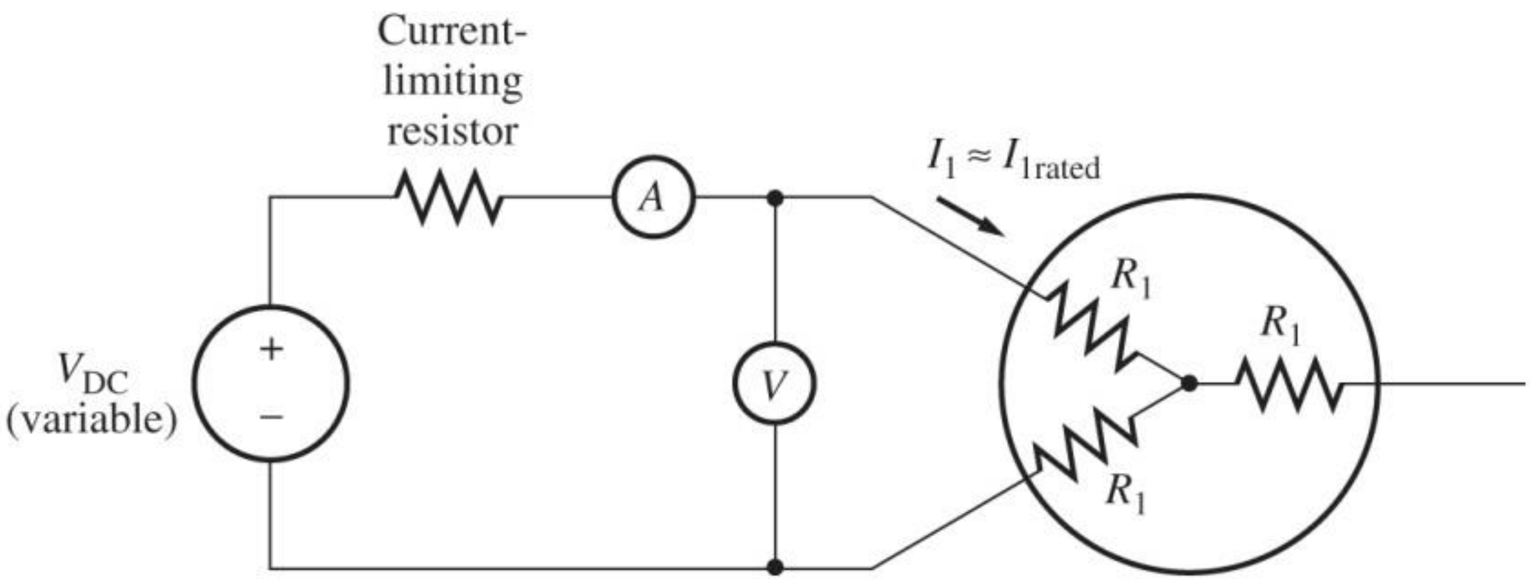

Figure 6-54
The circuit for a dc resistance test.

- The Locked-Rotor (or *Blocked-Rotor*) Test: to obtain R_2, X_1+X_2, and X_M (using the no-load test results).

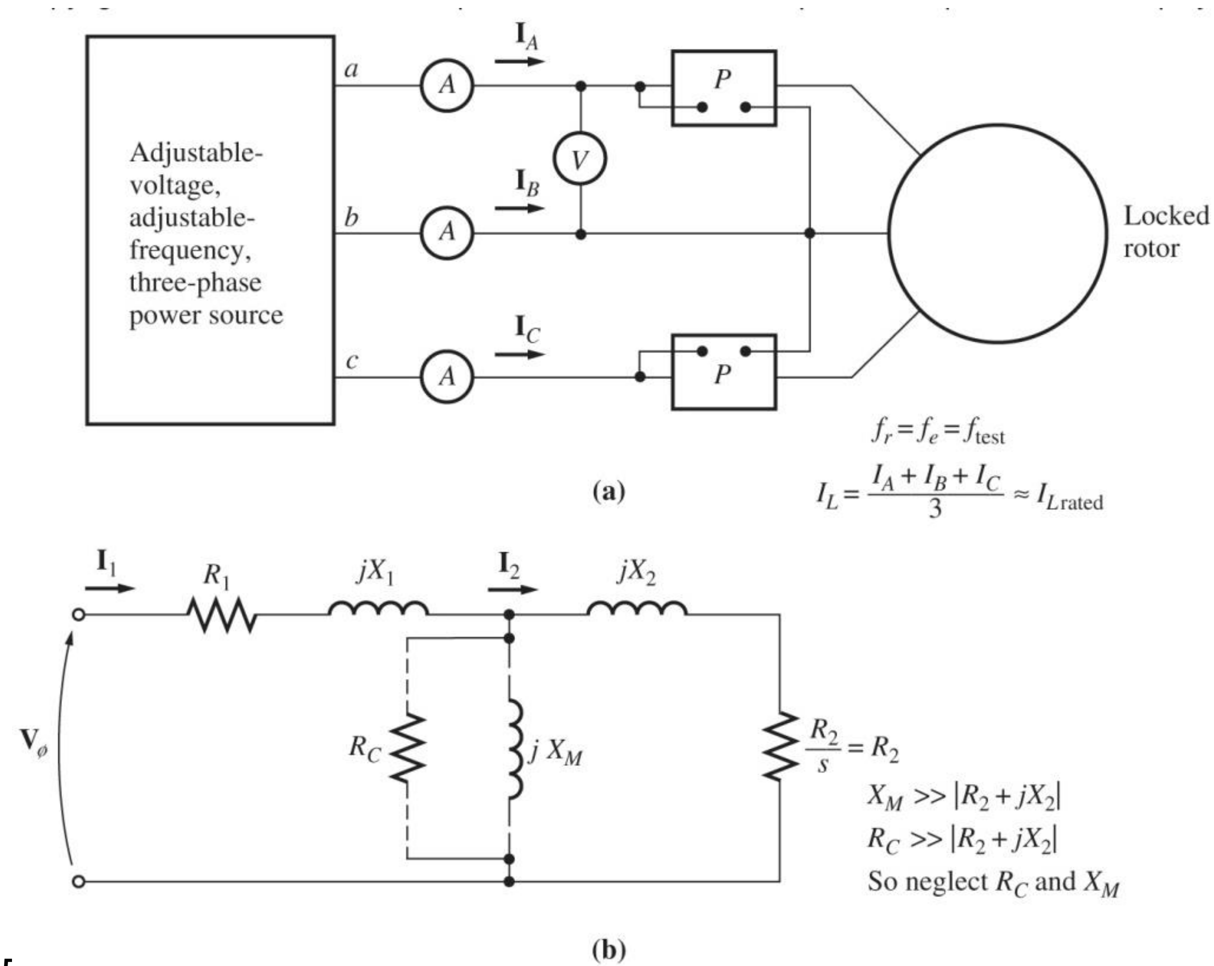

Figure 6-55
The locked-rotor test for an induction motor: (a) test circuit; (b) motor equivalent circuit

- The locked-rotor reactance at test frequency, X'_{LR}, is obtained from

$$|Z_{LR}| = \frac{V_\phi}{I_1} = \frac{V_T}{\sqrt{3}\, I_L}$$

$$Cos\,\theta_{LR} = \frac{P_{in}}{\sqrt{3}\, V_T I_L}$$

$$Z_{LR} = R_{LR} + jX'_{LR} = |Z_{LR}|\angle\theta_{LR}$$

- The locked-rotor reactance at rated frequency, X_{LR}, is

$$X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = X_1 + X_2$$

- X_1 and X_2 are found from rule of thumb based on rotor design.

$$R_{LR} = R_1 + R_2 \rightarrow R_2$$

Example 6–8. The following test data were taken on a 7.5-hp, four-pole, 208-V, 60-Hz, design A, Y-connected induction motor having a rated current of 28 A.

DC test:

$V_{DC} = 13.6 \text{ V}$ $\quad$ $I_{DC} = 28.0 \text{ A}$

No-load test:

$V_T = 208 \text{ V}$ $\quad$ $f = 60 \text{ Hz}$

$I_B = 8.20 \text{ A}$

$I_C = 8.18 \text{ A}$

Locked-rotor test:

$V_T = 25 \text{ V}$ $\quad$ $f = 15 \text{ Hz}$

$I_A = 28.1 \text{ A}$ $\quad$ $P_{in} = 920 \text{ W}$

$I_B = 28.0 \text{ A}$

$I_C = 27.6 \text{ A}$

(a) Sketch the per-phase equivalent circuit for this motor.

(b) Find the slip at the pullout torque, and find the value of the pullout torque itself.

Solution

(a) From the dc test,

$$R_1 = \frac{V_{DC}}{2I_{DC}} = \frac{13.6\text{ V}}{2(28.0\text{ A})} = 0.243\ \Omega$$

From the no-load test,

$$I_{L,av} = \frac{8.12\text{ A} + 8.20\text{ A} + 8.18\text{ A}}{3} = 8.17\text{ A}$$

$$V_{\phi,nl} = \frac{208\text{ V}}{\sqrt{3}} = 120\text{ V}$$

Therefore,

$$|Z_{nl}| = \frac{120\text{ V}}{8.17\text{ A}} = 14.7\ \Omega = X_1 + X_M$$

When X_1 is known, X_M can be found. The stator copper losses are

$$P_{SCL} = 3I_1^2 R_1 = 3(8.17\text{ A})^2(0.243\ \Omega) = 48.7\text{ W}$$

Therefore, the no-load rotational losses are

$$P_{rot} = P_{in,nl} - P_{SCL,nl}$$

$$= 420\text{ W} - 48.7\text{ W} = 371.3\text{ W}$$

From the locked-rotor test,

$$I_{\mathrm{L,av}} = \frac{28.1\ \mathrm{A} + 28.0\ \mathrm{A} + 27.6\ \mathrm{A}}{3} = 27.9\ \mathrm{A}$$

The locked-rotor impedance is

$$\left|Z_{\mathrm{LR}}\right| = \frac{V_\phi}{I_A} = \frac{V_T}{\sqrt{3}I_A} = \frac{25\ \mathrm{V}}{\sqrt{3}(27.9\ \mathrm{A})} = 0.517\ \Omega$$

and the impedance angle θ is

$$\theta = \cos^{-1}\frac{P_{\mathrm{in}}}{\sqrt{3}V_T I_L}$$

$$= \cos^{-1}\frac{920\ \mathrm{W}}{\sqrt{3}(25\ \mathrm{V})(27.9\ \mathrm{A})}$$

$$= \cos^{-1} 0.762 = 40.4^\circ$$

Therefore, $R_{\mathrm{LR}} = 0.517 \cos 40.4^\circ = 0.394\ \Omega = R_1 + R_2$. Since $R_1 = 0.243\ \Omega$, R_2 must be $0.151\ \Omega$. The reactance at 15 Hz is

$$X'_{LR} = 0.517 \sin 40.4^\circ = 0.335\ \Omega$$

The equivalent reactance at 60 Hz is

$$X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = \left(\frac{60\text{ Hz}}{15\text{ Hz}}\right) 0.335\ \Omega = 1.34\ \Omega$$

For design class A induction motors, this reactance is assumed to be divided equally between the rotor and stator, so

$$X_1 = X_2 = 0.67\ \Omega$$

$$X_M = \left|Z_{nl}\right| - X_1 = 14.7\ \Omega - 0.67\ \Omega = 14.03\ \Omega$$

(b) For this equivalent circuit, the Thevenin equivalents are found from Equations (6–41b), (6–44), and (6–45) to be

$$V_{TH} = 114.6\text{ V} \qquad R_{TH} = 0.221\ \Omega \qquad X_{TH} = 0.67\ \Omega$$

Therefore, the slip at the pullout torque is given by

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$= \frac{0.151\ \Omega}{\sqrt{(0.243\ \Omega)^2 + (0.67\ \Omega + 0.67\ \Omega)^2}} = 0.111 = 11.1\%$$

The maximum torque of this motor is given by

$$\tau_{\max} = \frac{3V_{\mathrm{TH}}^2}{2\omega_{\mathrm{sync}}[R_{\mathrm{TH}} + \sqrt{R_{\mathrm{TH}}^2 + (X_{\mathrm{TH}} + X^2)}]}$$

$$= \frac{3(114.6\ \mathrm{V})^2}{2(188.5\ \mathrm{rad/s})[0.221\ \Omega + \sqrt{(0.221\ \Omega)^2 + (0.67\ \Omega + 0.67\ \Omega)^2}]}$$

$$= 66.2\ \mathrm{N \cdot m}$$