Chapter 2

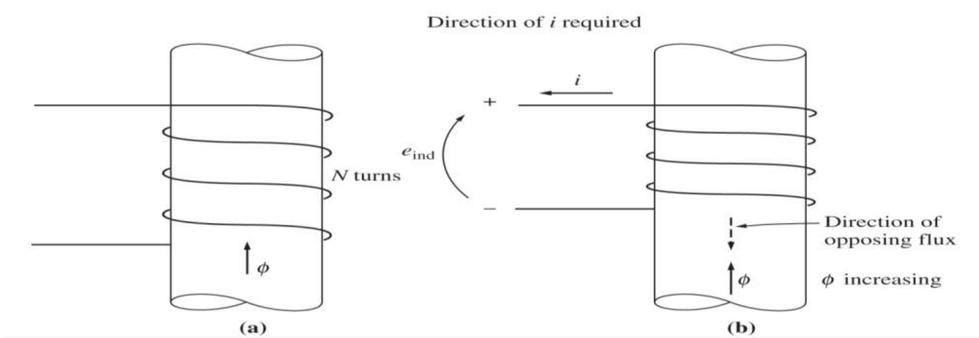
Transformers

Faraday's Law

If a flux φ passes through N turn of a coil, the induced in the coil is given by

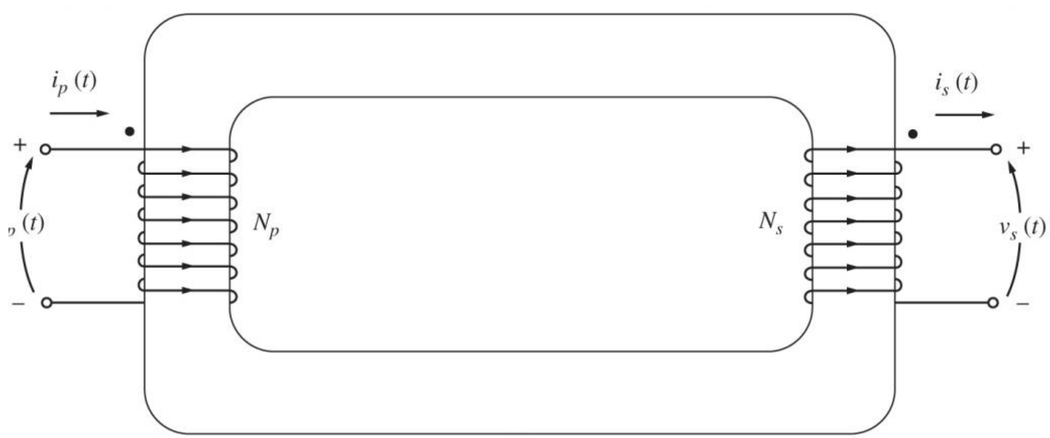
$$e_{ind} = -N\frac{d\varphi}{dt}$$

The negative sign is the statement of the Lenz's law stating that the polarity of the induced voltage should be such that a current produced by it produces a flux in the opposite of the original flux. This is illustrated below



Ideal Transformer

- The relative permeability of the core is very high
- No Leakage Flux, hence Leakage Flux of the two windings are neglected.
- Ideal windings has no resistances.
- Ideal core has no reluctance.
- Core losses are neglected.



- From Farady’s Law

$$\begin{cases} v_P(t) = -N_P \, d\varphi/dt \\ v_S(t) = -N_S \, d\varphi/dt \end{cases} \rightarrow \frac{v_P(t)}{v_S(t)} = \frac{N_P}{N_S} = a$$

- Since there is no magnetic potential drop in the ideal core,

$$N_s I_p(t) = N_s I_s(t)$$

$$\frac{I_p(t)}{I_s(t)} = \frac{N_s}{N_p} = \frac{1}{a}$$

Dot Convention

- The “dots” help to determine the polarity of the voltage and direction of the current in the secondary winding.
- If the primary *voltage* is positive at the dotted end of the winding with respect to the undotted end, then the secondary voltage will be positive at the dotted end also.
- If the primary *current* of the transformer flows into the dotted end of the primary winding, the secondary current will flow out of the dotted end of the secondary winding.

Impedance Transformation Through an Ideal Transformer

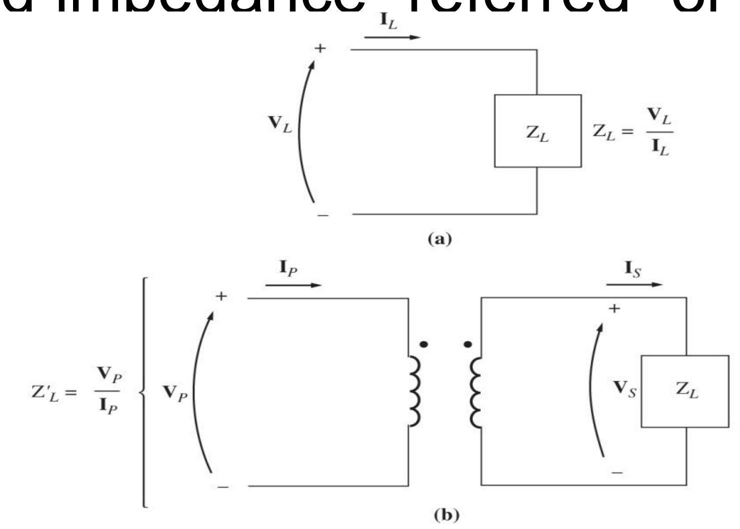
- The load impedance Z_L is given by

$$Z_L = \frac{V_s}{I_s}$$

- The apparent impedance of the primary circuit is

$$Z'_L = \frac{V_p}{I_p} = \frac{aV_s}{I_s / a} = a^2 \frac{V_s}{I_s} = a^2 Z_L$$

- Z'_L is the load impedance "referred" or "reflected" to the primary side



- The primary voltage can be expressed as

$$V_p = aV_s$$

- This is the secondary voltage as "seen" from the primary side of the ideal transformer. This referred voltage may be defined as

$$V_s^{'} \triangleq aV_s$$

- Similarly, the secondary current "referred to" or "as seen from" primary side of the ideal transformer may be defined as

$$I_s^{'} @ \frac{I_s}{a}$$

Theory of Operation of Non-Ideal Single-Phase Transformers

In a real transformer, the following non-ideal facts must be considered:

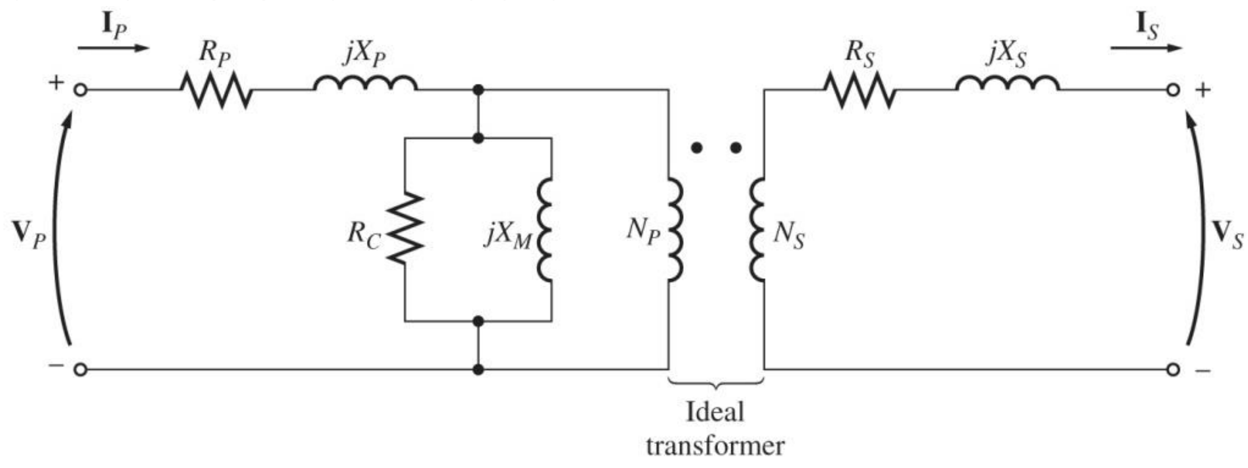
•Copper Losses in primary and secondary windings: these are modeled as series resistors R_p and R_s for the primary and secondary windings, respectively.

•Small portion of the core flux leaks outside the core and passes through one winding only. This flux will be presented by a leakage inductance. Both primary and secondary coils generate leakage flux which are modeled as series inductances X_p and X_s in the primary and secondary windings, respectively.

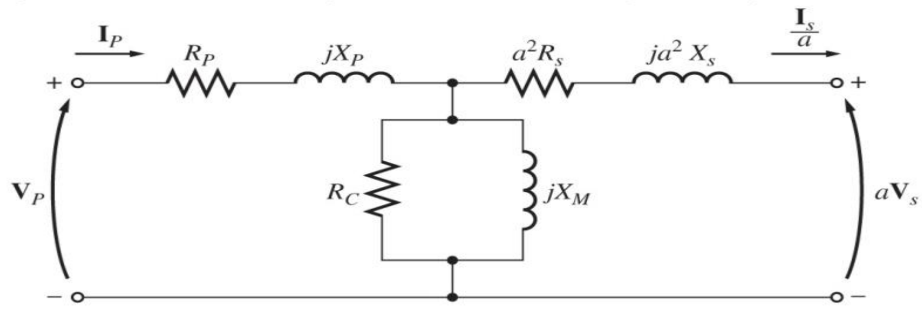
- Core Losses which are due to heating losses produced by Eddy Current and Hysteresis Losses in the core. These losses are presented by a shunt resistor R_C in the primary winding.

- Magnetizing Current which flows in the primary current to establish the flux in the core. This is modeled as a shunt inductance X_m in the primary winding.

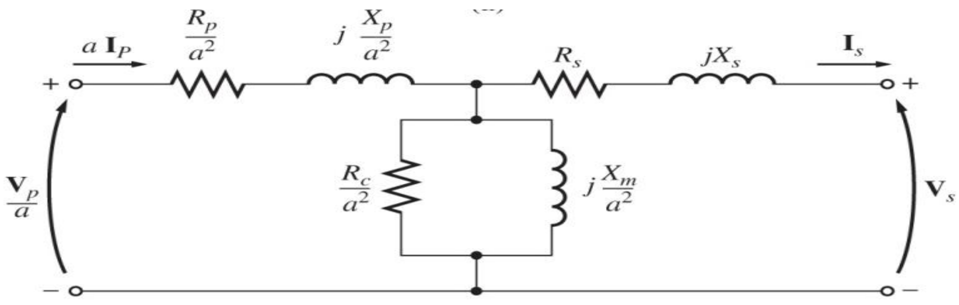
The equivalent circuit for a single-phase non-ideal transformer is shown below:



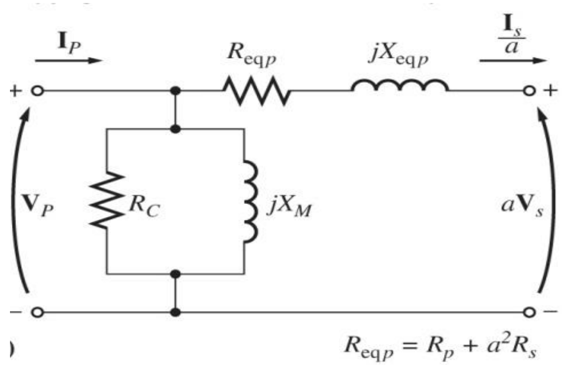
- The equivalent circuit may be simplified by *reflecting* impedances, voltages, and currents from the secondary to the primary as shown below:



- Below is the transformer model referred to *Secondary Side*.



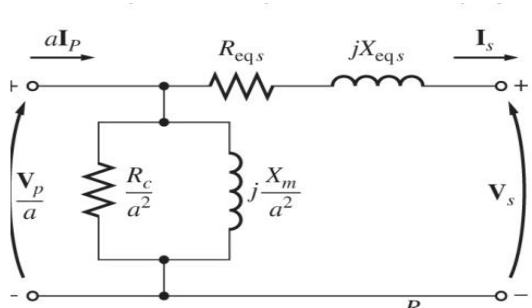
- *Simplified* equivalent circuit referred to *primary side:*



$$R_{\text{eq}P} = R_p + a^2 R_s$$

$$X_{\text{eq}P} = X_p + a^2 X_s$$

- *Simplified* equivalent circuit referred to *secondary side:*



$$R_{\text{eq}s} = \frac{R_p}{a^2} + R_s$$

$$X_{\text{eq}s} = \frac{X_p}{a^2} + X_s$$

Determining the Values of Components in the Transformer Model

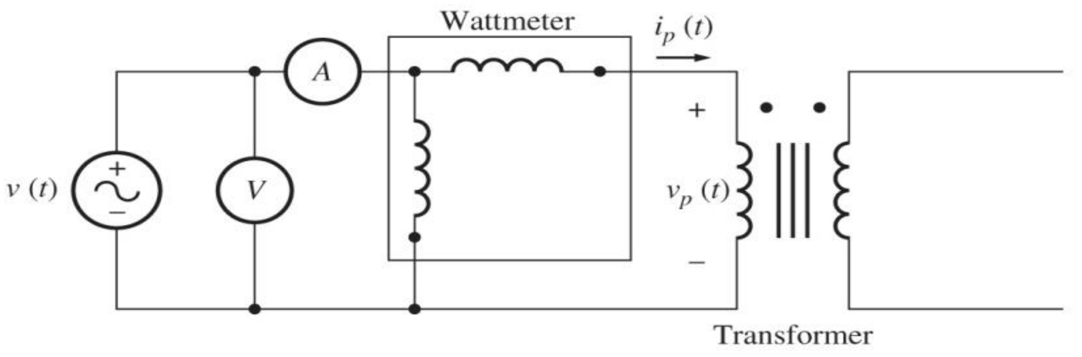
Transformer impedances may be obtained from two tests:

- Open-circuit test: to determine core losses and magnetizing reactance (R_c and X_m)

- Short-circuit test: to determine equivalent Series Impedance R_{eq}. and equivalent leakage reactances, X_{eq})

Open-Circuit Test to Determine R_c and X_m

With the high voltage side open, V_{oc}, I_{oc}, and P_{oc} are measured on the low voltage side.

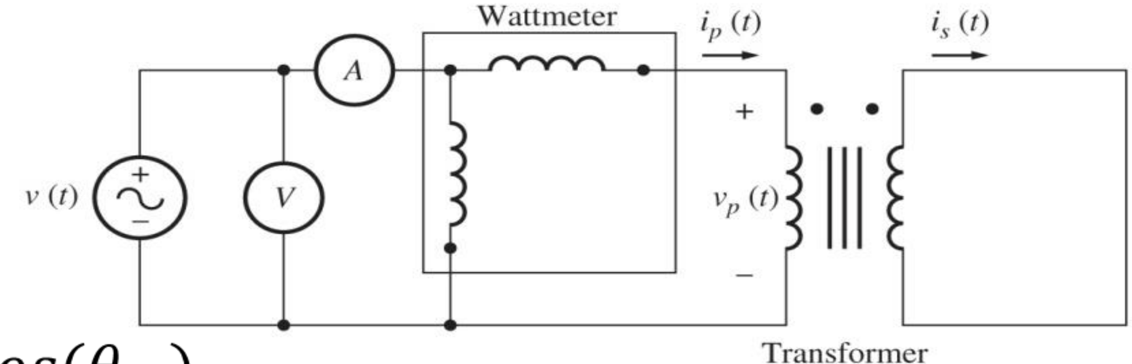


$$PF = cos\theta = \frac{P_{oc}}{|V_{oc}||I_{oc}|}$$

$$Y_{oc} = \frac{|I_{oc}|}{|V_{oc}|}\angle - \theta = \frac{|I_{oc}|}{|V_{oc}|}\angle(-cos^{-1}PF) = \frac{1}{R_c} - j\,\frac{1}{X_M}$$

Short-Circuit Test to Determine R_{eq} and X_{eq}

With low voltage side shorted, a **reduced voltage** is applied to the high voltage side such that **rated current** flows in the high voltage side. V_{SC}, I_{SC}, and P_{SC} are measured on the high voltage side.



$$PF = \frac{P_{sc}}{|I_{sc}||V_{sc}|} = cos(\theta_{sc})$$

$$Z_{eq} = \frac{|V_{sc}|}{|I_{sc}|}\angle(cos^{-1}PF) = R_{eq} + jX_{eq}$$

Transformer Voltage Regulation

- Due to the series impedances, the output voltage (secondary) will change as the load current changes.

- Voltage Regulation, VR, compares the output voltage of the transformer at no load with the output voltage at full load while the input voltage is kept constant at a value corresponding to the full load condition.

$$VR = \frac{|V_{s,nl}| - |V_{s,fl}|}{|V_{s,fl}|} \cdot 100\% = \frac{|V_{p/a}| - |V_{s,fl}|}{|V_{s,fl}|} \cdot 100\%$$

Per-Unit System

- When power system quantities such as currents, voltages, powers, and impedances are expressed as a fraction of their rated or “base” values, system’s *per unit* values are obtained.

- The first step is to define *base* or “normal” values for V, I, Z, and Power quantities.

- Since S =V.I and Z=V/I, only 2 of the 4 quantities needed as base value (Typically S and V).

- For machines and transformers, their rated voltages and VA are selected as the base values.

- Given S_b and V_b,

$$I_b = \frac{S_b}{V_b}$$

$$Z_b,\ R_b,\ \text{or}\ X_b = \frac{V_b}{I_b} == \frac{\left(V_b\right)^2}{S_b}$$

Note: For regions of the system separated by transformers

- S_b is the same in all regions.
- V_b change from region to region following the transformer's turns ratio.

Transformer Efficiency

- $P_{out} = P_s = |V_s||I_s|\text{Cos}(\theta_s)$

- $P_{in} = P_s + P_{Losses} = |V_s||I_s|\ Cos(\theta_s) + P_{core} + P_{cu}$

$$\eta = \frac{P_{out}}{P_{in}}$$

Three-Phase Transformers

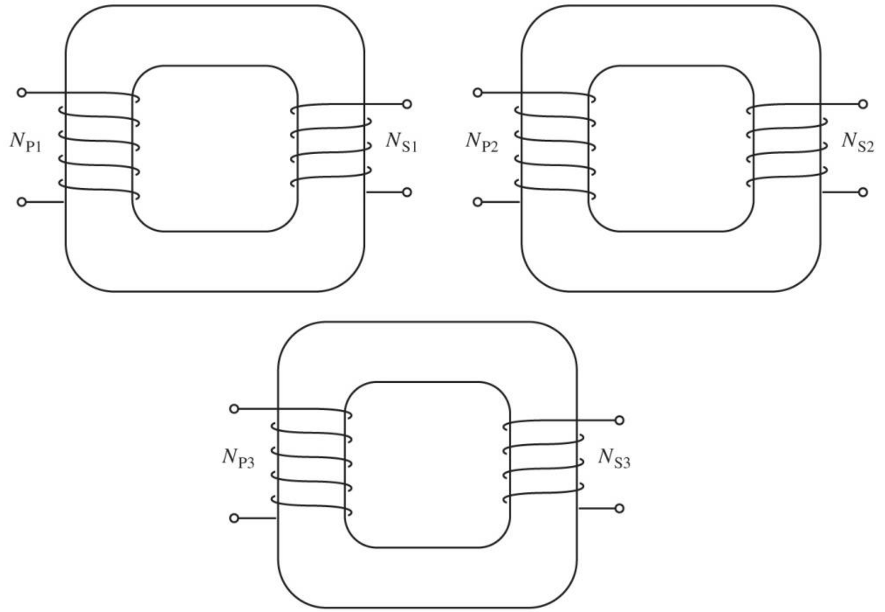


Figure 2-35

A three-phase transformer bank composed of independent transformers

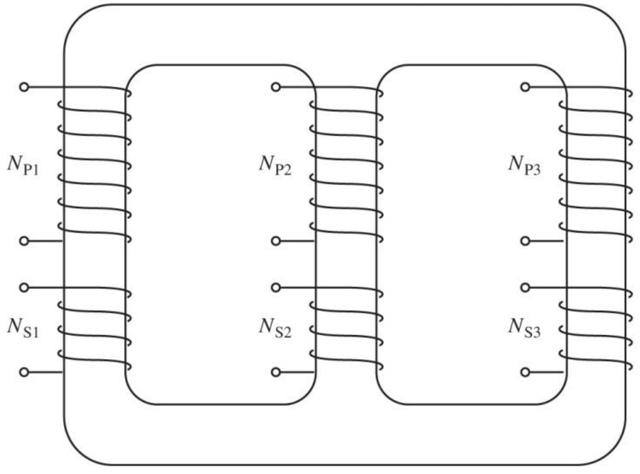


Figure 2-36

A three-phase transformer wound on a single three-legged core.

Transformer Connections

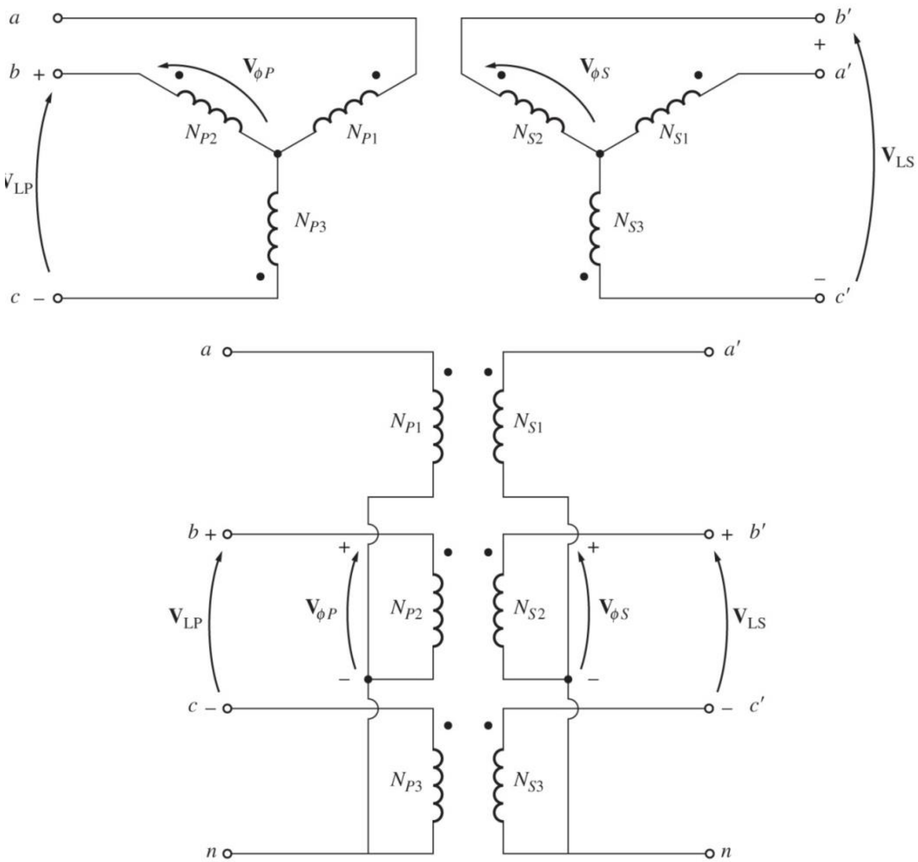


Figure 2-37(a)

Three-phase transformer Y-Y connections and wiring diagram.

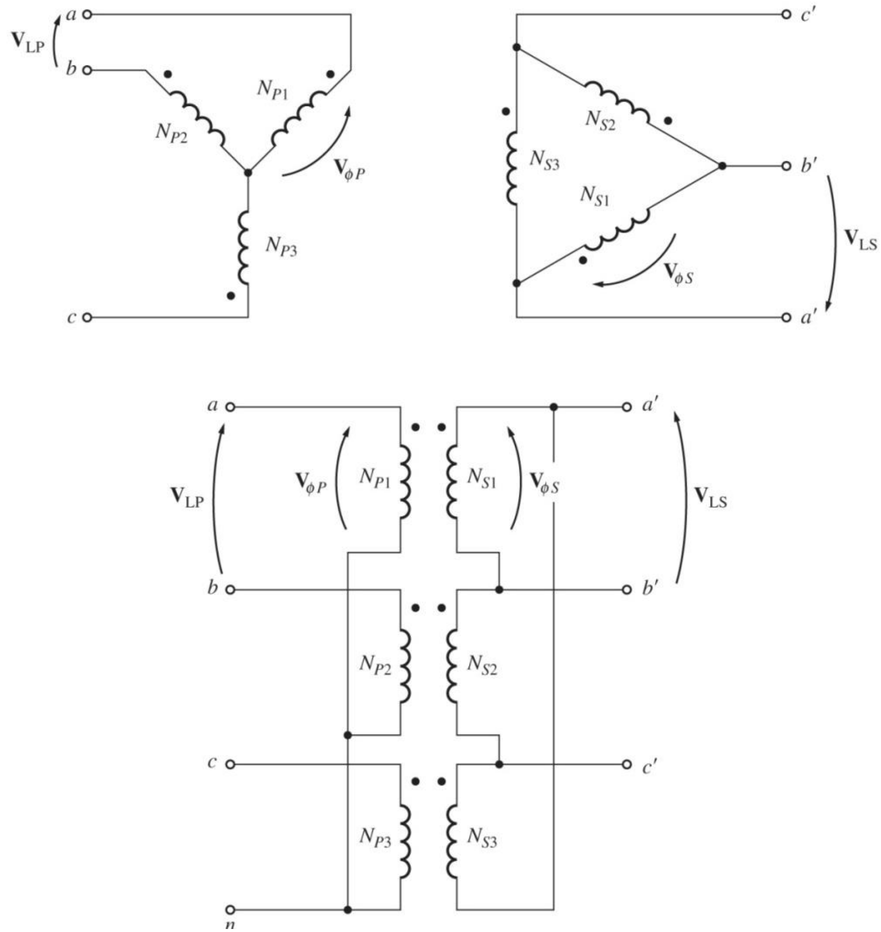


Figure 2-37(b)

Three-phase transformer Y-Δ connections and wiring diagram.

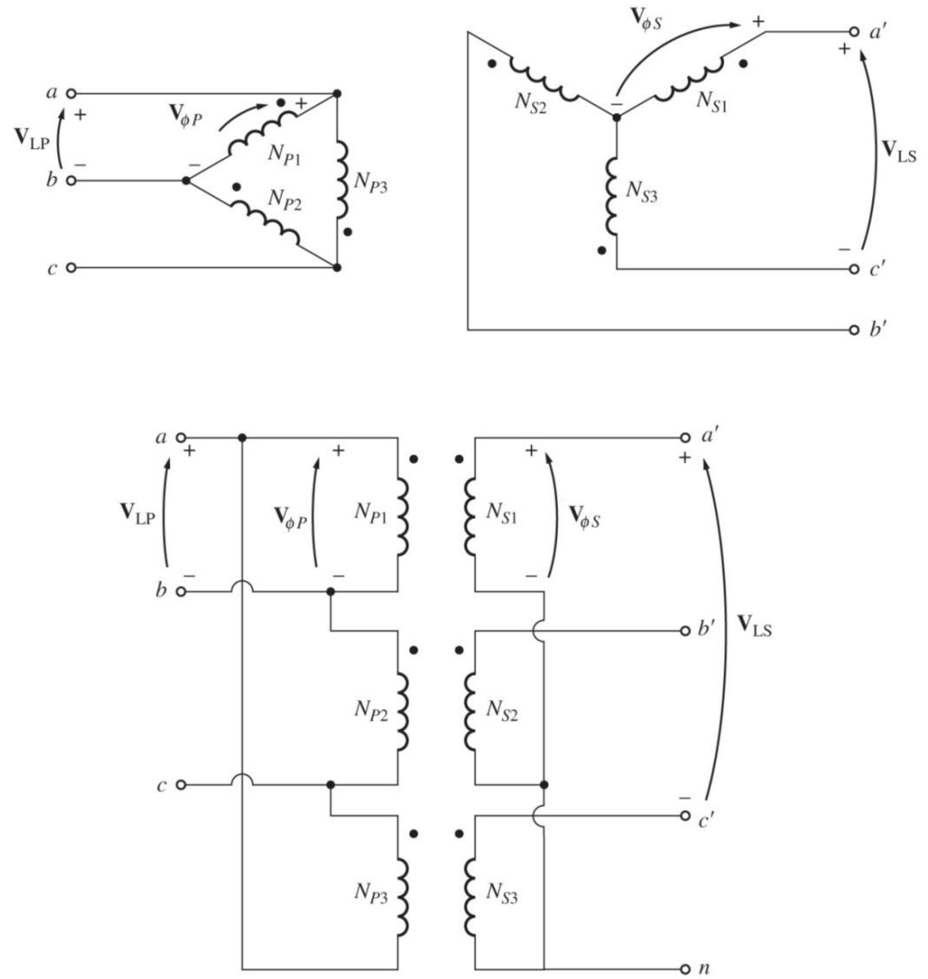


Figure 2-37(c)

Three-phase transformer Δ-Y connections and wiring diagram.

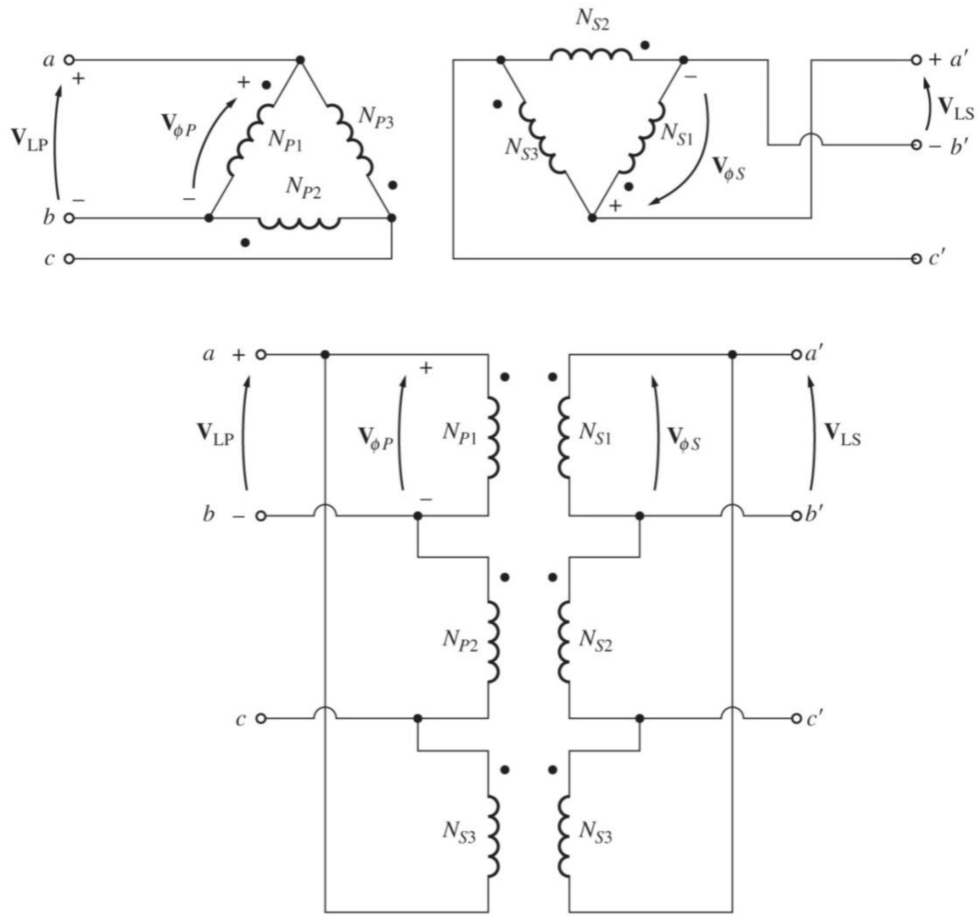


Figure 2-37(d)

Three-phase transformer Δ- Δ connections and wiring diagram.