ECE 421: Regulating Transformers

kVA := kW \quad \text{pu} := 1

Regulating Transformer Example:

Two transformers in parallel connect two voltage sources together. Transformer has a series impedance of 0.01 + j0.1 pu. Transformer 2 has a series impedance of 0.015 + j0.15pu. Transformer 2 is a tap changing transformer.

\[ Z_{T1} := (0.01 + j\cdot0.1)\text{pu} \quad Z_{T2} := (0.015 + j\cdot0.15)\text{pu} \]

\[ V_1 := 1.0e^{-j\cdot0\text{deg}} \quad V_2 := 1.0e^{-j\cdot10\text{deg}} \]

A. Find real and reactive power delivered to the source at Bus 2 if the transformer tap is normal (1.0).

\[ C_{tr} := 1 \]

\[ I_{T1} := \frac{V_1 - V_2}{Z_{T1}} \quad |I_{T1}| = 1.73\cdot\text{pu} \quad \text{arg}(I_{T1}) = 0.71\cdot\text{deg} \]

\[ I_{T2} := \frac{V_1 - V_2}{Z_{T2}} \quad |I_{T2}| = 1.16\cdot\text{pu} \quad \text{arg}(I_{T2}) = 0.71\cdot\text{deg} \]

\[ I_2 := I_{T1} + I_{T2} \quad |I_2| = 2.89\cdot\text{pu} \quad \text{arg}(I_2) = 0.71\cdot\text{deg} \]

\[ S_{T1} := V_2 \cdot \overline{I_{T1}} \quad P_{T1} := \text{Re}(S_{T1}) \quad P_{T1} = 1.7\cdot\text{pu} \]

\[ Q_{T1} := \text{Im}(S_{T1}) \quad Q_{T1} = -0.32\cdot\text{pu} \]

\[ S_{T2} := V_2 \cdot \overline{I_{T2}} \quad P_{T2} := \text{Re}(S_{T2}) \quad P_{T2} = 1.14\cdot\text{pu} \]

\[ Q_{T2} := \text{Im}(S_{T2}) \quad Q_{T2} = -0.21\cdot\text{pu} \]

B. Repeat if the transformer tap is set to be 0.95, on the side facing Bus 2

\[ C_{tr_b} := \frac{1}{1 - 0.05} \quad C_{tr_b} = 1.05 \]
Option 1: Using two port matrix approach:

\[
\begin{pmatrix}
    I_1 \\
    I_2
\end{pmatrix} =
\begin{pmatrix}
    Y_{11} & Y_{12} \\
    Y_{21} & Y_{22}
\end{pmatrix}
\begin{pmatrix}
    V_1 \\
    V_2
\end{pmatrix}
\]

We know \( V_1 \) and \( V_2 \)

\[ Y_{11} := \frac{1}{Z_{T1}} + \frac{1}{Z_{T2}} \quad Y_{11} = (1.65 - 16.5i) \cdot \text{pu} \]

\[ Y_{22} := \frac{1}{Z_{T1}} + \left[ \frac{|C_{tr_b}|^2}{Z_{T2}} \right] \quad Y_{22} = (1.72 - 17.21i) \cdot \text{pu} \]

- Since we have parallel impedances, there are two admittances in parallel in the off diagonal term:

\[ Y_{12} := -\frac{1}{Z_{T1}} + C_{tr_b} \left( \frac{-1}{Z_{T2}} \right) \quad Y_{12} = (-1.68 + 16.85i) \cdot \text{pu} \]

\[ Y_{21} := -\frac{1}{Z_{T1}} + C_{tr_b} \left( \frac{-1}{Z_{T2}} \right) \quad Y_{21} = (-1.68 + 16.85i) \cdot \text{pu} \]

- We care about the load current, \( I_2 \), at the moment:

\[ I_{2_b} := -(Y_{21} \cdot V_1 + Y_{22} \cdot V_2) \quad |I_{2_b}| = 3.01 \cdot \text{pu} \quad \text{arg}(I_{2_b}) = 7.71 \cdot \text{deg} \]

Minus sign since current is going into the source

- The current through transformer 1 will not change

\[ |I_{T1}| = 1.73 \cdot \text{pu} \quad \text{arg}(I_{T1}) = 0.71 \cdot \text{deg} \]

- KCL equation to find the new transformer 2 current

\[ I_{T2_b} := I_{2_b} - I_{T1} \quad |I_{T2_b}| = 1.3 \cdot \text{pu} \quad \text{arg}(I_{T2_b}) = 17.05 \cdot \text{deg} \]

\[ S_{T2_b} := V_2 \cdot I_{T2_b} \quad P_{T2_b} := \text{Re}(S_{T2_b}) \quad P_{T2_b} = 1.16 \cdot \text{pu} \]

\[ Q_{T2_b} := \text{Im}(S_{T2_b}) \quad Q_{T2_b} = -0.59 \cdot \text{pu} \]
Option 2: Using KVL:

We know

\[ V_1 = V_2 + I_{T1} \cdot Z_{T1} \]

and we also know

\[ V_1 = C_{tr\_b} \cdot V_2 + \frac{I_{T2}}{C_{tr\_b}} \cdot Z_{T2} \]

where \( V_2 \) and \( I_{L2} \) are the voltage and the current on the secondary side of the transformer and need to be referred to the primary side.

\[ I_{T2\_B\_alt2} := (V_1 - C_{tr\_b} \cdot V_2) \cdot \frac{C_{tr\_b}}{Z_{T2}} \]

\[ |I_{T2\_B\_alt2}| = 1.3 \cdot \text{pu} \quad \text{arg}(I_{T2\_B\_alt2}) = 17.05\text{-deg} \quad \text{Same as above} \]

Option 3: Using the circuit of Figure 3.25(b) from the textbook:

The current going to the load through T2 is:

\[ I_{T2\_B\_alt3} := (V_1 - V_2) \cdot \frac{C_{tr\_b}}{Z_{T2}} - V_2 \cdot \left( \frac{(|C_{tr\_b}|)^2 - C_{tr\_b}}{Z_{T2}} \right) \]

\[ |I_{T2\_B\_alt3}| = 1.3 \cdot \text{pu} \quad \text{arg}(I_{T2\_B\_alt3}) = 17.05\text{-deg} \]

C. Repeat if the transformer tap is set to create a phase shift such that:

\[ C_{tr\_c} := 1.0e^{j \cdot 2.5\text{deg}} \]

Option 1: Using two port matrix approach:

\[
\begin{pmatrix}
I_1 \\
I_2
\end{pmatrix} =
\begin{pmatrix}
Y_{11} & Y_{12} \\
Y_{21} & Y_{22}
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2
\end{pmatrix}
\]

We know \( V_1 \) and \( V_2 \)
\[
Y_{11_c} := \frac{1}{Z_{T1}} + \frac{1}{Z_{T2}} \quad Y_{11_c} = (1.65 - 16.5i) \cdot \text{pu}
\]
\[
Y_{22_c} := \frac{1}{Z_{T1}} + \left[ \frac{|C_{tr_c}|^2}{Z_{T2}} \right] \quad Y_{22_c} = (1.65 - 16.5i) \cdot \text{pu}
\]

- Since we have parallel impedances, there are two admittances in parallel in the off diagonal term:
\[
Y_{12_c} := \frac{-1}{Z_{T1}} + C_{tr_c} \left[ \frac{-1}{Z_{T2}} \right] \quad Y_{12_c} = (-1.94 + 16.47i) \cdot \text{pu}
\]
\[
Y_{21_c} := \frac{-1}{Z_{T1}} + C_{tr_c} \left[ \frac{-1}{Z_{T2}} \right] \quad Y_{21_c} = (-1.36 + 16.52i) \cdot \text{pu}
\]

- We care about the load current, \(I_2\), at the moment:
\[
I_{2_c} := -(Y_{21_c} \cdot V_1 + Y_{22_c} \cdot V_2) \quad |I_{2_c}| = 2.6 \cdot \text{pu} \quad \arg(I_{2_c}) = 0.29 \cdot \text{deg}
\]
Minus sign since current is going into the source

- The current through transformer 1 will not change
\[
|I_{T1}| = 1.73 \cdot \text{pu} \quad \arg(I_{T1}) = 0.71 \cdot \text{deg}
\]

- KCL equation to find the new transformer 2 current
\[
I_{T2_c} := I_{2_c} - I_{T1} \quad |I_{T2_c}| = 0.87 \cdot \text{pu} \quad \arg(I_{T2_c}) = -0.54 \cdot \text{deg}
\]
\[
S_{T2_c} := V_{2} \cdot I_{T2_c} \quad P_{T2_c} := \text{Re}(S_{T2_c}) \quad P_{T2_c} = 0.86 \cdot \text{pu}
\]
\[
Q_{T2_c} := \text{Im}(S_{T2_c}) \quad Q_{T2_c} = -0.14 \cdot \text{pu}
\]

Option 2: Using KVL:

We know
\[
V_1 = V_2 + I_{T1} \cdot Z_{T1}
\]
and we also know

\[ V_1 = \frac{I_{T2}}{C_{tr,c}} \cdot V_2 + \frac{I_{T2}}{C_{tr,c}} \cdot Z_{T2} \]

where \( V_2 \) and \( I_{T2} \) are the voltage and the current on the secondary side of the transformer and need to be referred to the primary side.

\[ I_{T2,C_{alt2}} := (V_1 - \frac{C_{tr,c} \cdot V_2}{Z_{T2}}) \cdot \frac{C_{tr,c}}{Z_{T2}} \]

\[ |I_{T2,C_{alt2}}| = 0.87 \cdot \text{pu} \quad \arg(I_{T2,C_{alt2}}) = -0.54 \cdot \text{deg} \quad \text{Same as above} \]

- Note that Option 3 from part B does not apply, since the equivalent circuit given in the figure is for a real \( C \) (so not a phase shift).