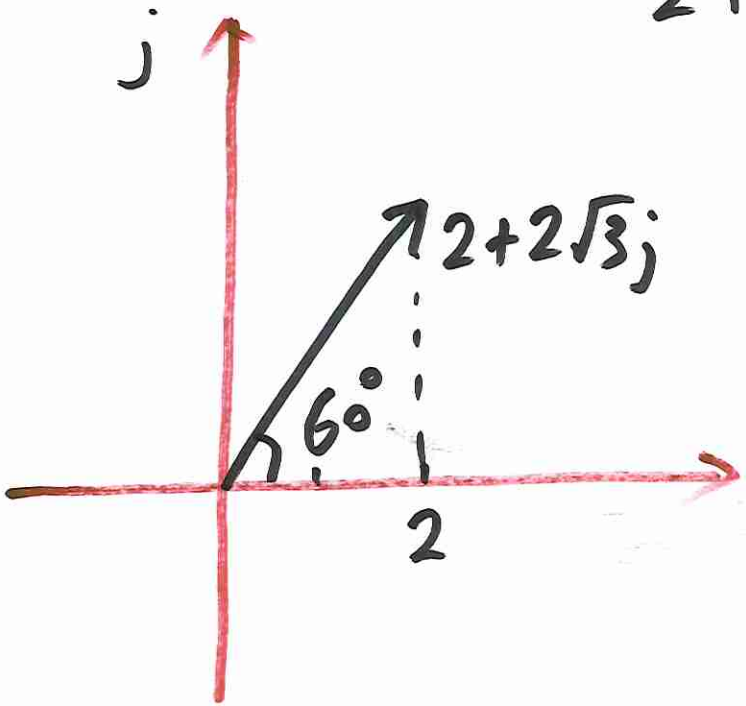
ECE 420

Complex Numbers

~~$2+2\sqrt{3}i$~~

$2+2\sqrt{3}\,j$ — rectangular form

$= 4\angle 60^\circ$ — polar form



$j^2 = -1$

$j = 1\angle 90^\circ$

$-j = (-1)\angle 90^\circ = 1\angle -90^\circ$

Complex Number Computation

Addition

$$Z_1 = x_1 + jy_1 \qquad Z_2 = x_2 + jy_2$$

$$Z_1 + Z_2 = x_1 + jy_1 + x_2 + jy_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtraction

$$Z_1 = x_1 + jy_1 \qquad Z_2 = x_2 + jy_2$$

$$Z_1 - Z_2 = x_1 + jy_1 - (x_2 + jy_2)$$

$$= (x_1 - x_2) + j(y_1 - y_2)$$

$j^2 = -1$

Multiplication

$$Z_1 = x_1 + jy_1 \qquad Z_2 = x_2 + jy_2$$

$$Z_1 \cdot Z_2 = (x_1 + jy_1)(x_2 + jy_2)$$

$$= x_1 x_2 + j x_2 y_1 + j x_1 y_2 + j^2 y_1 y_2$$

$$= (x_1 x_2 - y_1 y_2) + j(x_2 y_1 + x_1 y_2)$$

For Example:

If $Z_1 = 4 \angle 60^\circ \qquad Z_2 = 2 \angle 15^\circ$

$$Z_1 \cdot Z_2 = 4 \times 2 \angle 60^\circ + 15^\circ = 8 \angle 75^\circ$$

Division

$$Z_1 = A_1 \angle \theta_1 \qquad Z_2 = A_2 \angle \theta_2$$

$$\frac{Z_1}{Z_1} = \frac{A_1}{A_2} \angle \theta_1 - \theta_2$$

For Example

$$Z_1 = 4 \angle 60^\circ \qquad Z_2 = 2 \angle 15^\circ$$

$$\frac{Z_1}{Z_2} = \frac{4}{2} \angle 60^\circ - 15^\circ = 2 \angle 45^\circ$$

$i(t) = 200\sqrt{2}\cos(240\pi t + 60^\circ)$ A

peak value : $200\sqrt{2}$ A

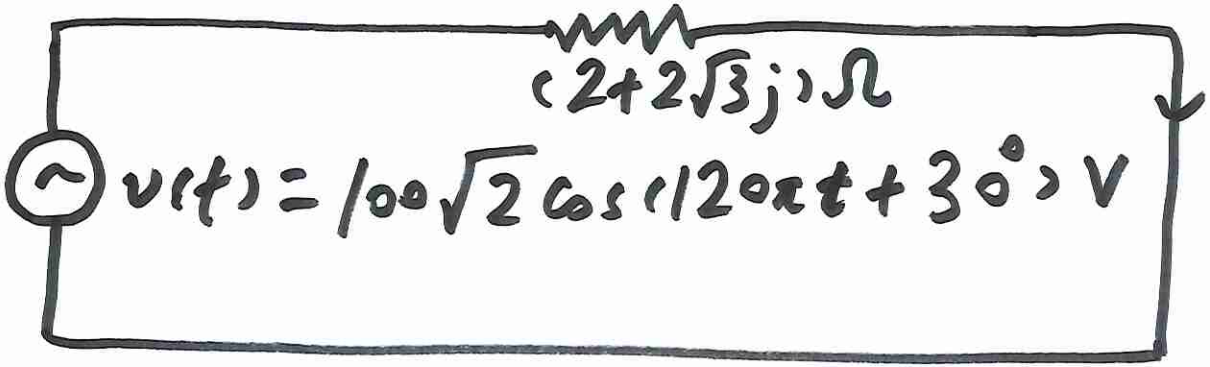
RMS value (effective value) : 200 A

ω : 240π $\omega = 2\pi f$

frequency : 120 Hz

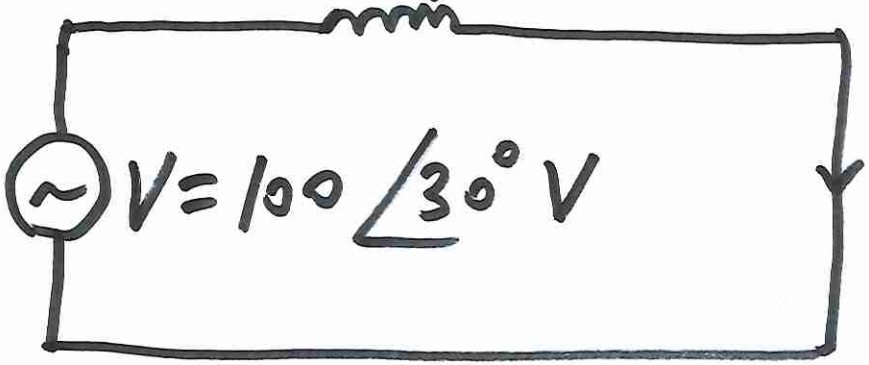
phase angle : 60°

Phasor Representation : $I = 200\angle 60^\circ$ A



$$\Downarrow$$

$Z = (2+2\sqrt{3}j)\ \Omega$



$$I = \frac{V}{Z} = \frac{100\angle 30^\circ}{4\angle 60^\circ}$$

$$= 25\angle -30^\circ\ A$$

$$i(t) = 25\sqrt{2}\cos(120\pi t - 30^\circ)\ A$$