Line Constants for Untransposed and Transposed Lines

A. Original Case

AC Resistance from table:

$R_{ac} := 0.278 \frac{ohm}{mi}$ at 25 C and $freq := 60Hz$

$\mu_0 := 4 \cdot \pi \cdot 10^{-7} \frac{H}{m}$ $\rho := 100 ohm \cdot m$

Conductor GMR from table:

$D_s := 0.01668 ft$

$CarsonsResistConst := 9.869 \times 10^{-7} \frac{ohm}{m \cdot Hz}$

$R_d := CarsonsResistConst \cdot freq$ $R_d = 0.0953 \cdot \frac{ohm}{mi}$

$R_{self} := R_{ac} + R_d$ $R_{self} = 0.3733 \cdot \frac{ohm}{mi}$

$$R_{perlength} := \begin{pmatrix} R_{self} & R_d & R_d \\ R_d & R_{self} & R_d \\ R_d & R_d & R_{self} \end{pmatrix}$$

$$R_{perlength} = \begin{pmatrix} 0.3733 & 0.0953 & 0.0953 \\ 0.0953 & 0.3733 & 0.0953 \\ 0.0953 & 0.0953 & 0.3733 \end{pmatrix} \cdot \frac{ohm}{mi}$$

$$D_{e\_const} := 2160 \cdot \frac{ft \cdot Hz^{0.5}}{(ohm \cdot m)^{0.5}}$$

$$D_e := D_{e\_const} \cdot \sqrt{\frac{\rho}{freq}} \qquad D_e = 2.7885 \times 10^3 \cdot ft$$

$$D_{ab} := 10ft \qquad D_{ac} := 20ft \qquad D_{bc} := 10ft$$

$$L_{perlength} := \frac{\mu_0}{2 \cdot \pi} \cdot \begin{pmatrix} \ln\left(\frac{D_e}{D_s}\right) & \ln\left(\frac{D_e}{D_{ab}}\right) & \ln\left(\frac{D_e}{D_{ac}}\right) \\ \ln\left(\frac{D_e}{D_{ab}}\right) & \ln\left(\frac{D_e}{D_s}\right) & \ln\left(\frac{D_e}{D_{bc}}\right) \\ \ln\left(\frac{D_e}{D_{ac}}\right) & \ln\left(\frac{D_e}{D_{bc}}\right) & \ln\left(\frac{D_e}{D_s}\right) \end{pmatrix}$$

$$L_{perlength} = \begin{pmatrix} 3.8711 & 1.8123 & 1.5892 \\ 1.8123 & 3.8711 & 1.8123 \\ 1.5892 & 1.8123 & 3.8711 \end{pmatrix} \cdot \frac{mH}{mi}$$

$$Z' := R_{perlength} + j \cdot 2 \cdot \pi \cdot freq \cdot L_{perlength}$$

$$Z' = \begin{pmatrix} 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.599i \\ 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.683i \\ 0.095 + 0.599i & 0.095 + 0.683i & 0.373 + 1.459i \end{pmatrix} \cdot \frac{ohm}{mi}$$

If length = 40 miles: $Z_{line} := Z' \cdot 40mi$

$$Z_{line} = \begin{pmatrix} 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 23.965i \\ 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 27.33i \\ 3.812 + 23.965i & 3.812 + 27.33i & 14.932 + 58.374i \end{pmatrix} \Omega$$

$$a := 1 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}}$$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$$Z_{012} := A_{012}^{-1} \cdot Z_{line} \cdot A_{012}$$

$$Z_{012} = \begin{pmatrix} 22.5555 + 110.7903i & 0.9712 - 0.5607i & -0.9712 - 0.5607i \\ -0.9712 - 0.5607i & 11.12 + 32.1661i & -1.9424 + 1.1214i \\ 0.9712 - 0.5607i & 1.9424 + 1.1214i & 11.12 + 32.1661i \end{pmatrix} \Omega$$

By comparsion:

$$D_m := \left(D_{ab} \cdot D_{bc} \cdot D_{ac}\right)^{\frac{1}{3}}$$

$$Z_1 := \left(R_{ac} + j \cdot 2 \cdot \pi \cdot freq \cdot \frac{\mu_0}{2 \cdot \pi} \cdot \ln\left(\frac{D_m}{D_s}\right)\right) \cdot 40mi \qquad Z_1 = (11.12 + 32.1661i)\,\Omega$$

- ***Transposition***

$$Rp := \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Example 1:

$f1 := 0.2 \quad f2 := 0.3 \quad f3 := 0.5$

$$Z_{net} := f1 \cdot Z_{line} + f2 \cdot Rp^{-1} \cdot Z_{line} \cdot Rp + f3 \cdot Rp \cdot Z_{line} \cdot Rp^{-1}$$

$$Z_{net} = \begin{pmatrix} 14.9318 + 58.3742i & 3.8118 + 25.6473i & 3.8118 + 26.6566i \\ 3.8118 + 25.6473i & 14.9318 + 58.3742i & 3.8118 + 26.3202i \\ 3.8118 + 26.6566i & 3.8118 + 26.3202i & 14.9318 + 58.3742i \end{pmatrix} \Omega$$

$$Z_{0121} := A_{012}^{-1} \cdot Z_{net} \cdot A_{012}$$

$$Z_{0121} = \begin{pmatrix} 22.5555 + 110.7903i & -0.2914 - 0.0561i & 0.2914 - 0.0561i \\ 0.2914 - 0.0561i & 11.12 + 32.1661i & 0.5827 + 0.1121i \\ -0.2914 - 0.0561i & -0.5827 + 0.1121i & 11.12 + 32.1661i \end{pmatrix} \Omega$$

Example 2

$f16 := \frac{1}{3} \qquad f26 := \frac{1}{3} \qquad f36 := \frac{1}{3}$

$$Z_{net6} := f16 \cdot Z_{line} + f26 \cdot Rp^{-1} \cdot Z_{line} \cdot Rp + f36 \cdot Rp \cdot Z_{line} \cdot Rp^{-1}$$

$$Z_{net6} = \begin{pmatrix} 14.9318 + 58.3742i & 3.8118 + 26.2081i & 3.8118 + 26.2081i \\ 3.8118 + 26.2081i & 14.9318 + 58.3742i & 3.8118 + 26.2081i \\ 3.8118 + 26.2081i & 3.8118 + 26.2081i & 14.9318 + 58.3742i \end{pmatrix} \Omega$$

$$Z_{0126} := A_{012}^{-1} \cdot Z_{net6} \cdot A_{012}$$

$$Z_{0126} = \begin{pmatrix} 22.555 + 110.79i & 0 & 0 \\ 0 & 11.12 + 32.166i & 0 \\ 0 & 0 & 11.12 + 32.166i \end{pmatrix} \Omega$$

Double Circuit Line:

Two parallel lines, 100 feet apart (center to center). Each has flat spacing

A. Resistance Matrix

AC Resistance from table:

$$Rac := 0.278 \frac{ohm}{mi} \qquad \text{at 25 C and} \qquad freq := 60Hz$$

$$CarsonsResistConst := 9.869 \times 10^{-7} \frac{ohm}{m \cdot Hz}$$

$$Rd := CarsonsResistConst \cdot freq \qquad Rd = 0.0953 \cdot \frac{ohm}{mi}$$

$$Rself := Rac + Rd \qquad Rself = 0.3733 \cdot \frac{ohm}{mi}$$

$$R' := \begin{pmatrix} Rself & Rd & Rd & Rd & Rd & Rd \\ Rd & Rself & Rd & Rd & Rd & Rd \\ Rd & Rd & Rself & Rd & Rd & Rd \\ Rd & Rd & Rd & Rself & Rd & Rd \\ Rd & Rd & Rd & Rd & Rself & Rd \\ Rd & Rd & Rd & Rd & Rd & Rself \end{pmatrix}$$

$$R' = \begin{pmatrix} 0.3733 & 0.0953 & 0.0953 & 0.0953 & 0.0953 & 0.0953 \\ 0.0953 & 0.3733 & 0.0953 & 0.0953 & 0.0953 & 0.0953 \\ 0.0953 & 0.0953 & 0.3733 & 0.0953 & 0.0953 & 0.0953 \\ 0.0953 & 0.0953 & 0.0953 & 0.3733 & 0.0953 & 0.0953 \\ 0.0953 & 0.0953 & 0.0953 & 0.0953 & 0.3733 & 0.0953 \\ 0.0953 & 0.0953 & 0.0953 & 0.0953 & 0.0953 & 0.3733 \end{pmatrix} \cdot \frac{ohm}{mi}$$

B. Inductance Matrix

$\mu o := 4 \cdot \pi \cdot 10^{-7} \frac{H}{m}$ $\qquad$ $\rho := 100 ohm \cdot m$

Calculate GMR from conductor diameter

$dia := 0.528 in$ $\qquad$ $GMR := e^{\frac{-1}{4}} \cdot \frac{dia}{2}$ $\qquad$ $GMR = 0.01713 \cdot ft$

Conductor GMR from table:

$Ds := 0.01668 ft$

$$De := 2160 \cdot \frac{ft \cdot Hz^{0.5}}{(ohm \cdot m)^{0.5}}$$

$De := De \cdot \sqrt{\frac{\rho}{freq}}$ $\qquad$ $De = 2.7885 \times 10^{3} \cdot ft$

$Da1b1 := 10ft$ $Da1c1 := 20ft$ $Db1c1 := 10ft$ $Da1a2 := 100ft$ $Db1b2 := 100ft$ $Dc1c2 := 100ft$

$Da2b2 := 10ft$ $Da2c2 := 20ft$ $Db2c2 := 10ft$ $Da1b2 := 110ft$ $Da1c2 := 120ft$ $Db1a2 := 90ft$

$Db1c2 := 100ft$ $Dc1a2 := 80ft$ $Dc1b2 := 90ft$

$$L' := \frac{\mu o}{2\cdot\pi}\cdot\begin{pmatrix} \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Da1b1}\right) & \ln\left(\frac{De}{Da1c1}\right) & \ln\left(\frac{De}{Da1a2}\right) & \ln\left(\frac{De}{Da1b2}\right) & \ln\left(\frac{De}{Da1c2}\right) \\ \ln\left(\frac{De}{Da1b1}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Db1c1}\right) & \ln\left(\frac{De}{Db1a2}\right) & \ln\left(\frac{De}{Db1b2}\right) & \ln\left(\frac{De}{Db1c2}\right) \\ \ln\left(\frac{De}{Da1c1}\right) & \ln\left(\frac{De}{Db1c1}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Dc1a2}\right) & \ln\left(\frac{De}{Dc1b2}\right) & \ln\left(\frac{De}{Dc1c2}\right) \\ \ln\left(\frac{De}{Da1a2}\right) & \ln\left(\frac{De}{Db1a2}\right) & \ln\left(\frac{De}{Dc1a2}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Da2b2}\right) & \ln\left(\frac{De}{Da2c2}\right) \\ \ln\left(\frac{De}{Da1b2}\right) & \ln\left(\frac{De}{Db1b2}\right) & \ln\left(\frac{De}{Dc1b2}\right) & \ln\left(\frac{De}{Da2b2}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Db2c2}\right) \\ \ln\left(\frac{De}{Da1c2}\right) & \ln\left(\frac{De}{Db1c2}\right) & \ln\left(\frac{De}{Dc1c2}\right) & \ln\left(\frac{De}{Da2c2}\right) & \ln\left(\frac{De}{Db2c2}\right) & \ln\left(\frac{De}{Ds}\right) \end{pmatrix}$$

$$L' = \begin{pmatrix} 3.8711 & 1.8123 & 1.5892 & 1.0712 & 1.0405 & 1.0125 \\ 1.8123 & 3.8711 & 1.8123 & 1.1051 & 1.0712 & 1.0712 \\ 1.5892 & 1.8123 & 3.8711 & 1.143 & 1.1051 & 1.0712 \\ 1.0712 & 1.1051 & 1.143 & 3.8711 & 1.8123 & 1.5892 \\ 1.0405 & 1.0712 & 1.1051 & 1.8123 & 3.8711 & 1.8123 \\ 1.0125 & 1.0712 & 1.0712 & 1.5892 & 1.8123 & 3.8711 \end{pmatrix}\cdot\frac{mH}{mi}$$

$$Z' := R' + j\cdot 2\cdot\pi\cdot freq\cdot L'$$

$$Z' = \begin{pmatrix} 0.373+1.459i & 0.095+0.683i & 0.095+0.599i & 0.095+0.404i & 0.095+0.392i & 0.095+0.382i \\ 0.095+0.683i & 0.373+1.459i & 0.095+0.683i & 0.095+0.417i & 0.095+0.404i & 0.095+0.404i \\ 0.095+0.599i & 0.095+0.683i & 0.373+1.459i & 0.095+0.431i & 0.095+0.417i & 0.095+0.404i \\ 0.095+0.404i & 0.095+0.417i & 0.095+0.431i & 0.373+1.459i & 0.095+0.683i & 0.095+0.599i \\ 0.095+0.392i & 0.095+0.404i & 0.095+0.417i & 0.095+0.683i & 0.373+1.459i & 0.095+0.683i \\ 0.095+0.382i & 0.095+0.404i & 0.095+0.404i & 0.095+0.599i & 0.095+0.683i & 0.373+1.459i \end{pmatrix}\cdot\frac{ohm}{mi}$$

If length = 40 miles:

$$Zline := Z'\cdot 40mi$$

$$Zline = \begin{pmatrix} 14.932+58.374i & 3.812+27.33i & 3.812+23.965i & 3.812+16.154i & 3.812+15.691i & 3.812+15.269i \\ 3.812+27.33i & 14.932+58.374i & 3.812+27.33i & 3.812+16.665i & 3.812+16.154i & 3.812+16.154i \\ 3.812+23.965i & 3.812+27.33i & 14.932+58.374i & 3.812+17.237i & 3.812+16.665i & 3.812+16.154i \\ 3.812+16.154i & 3.812+16.665i & 3.812+17.237i & 14.932+58.374i & 3.812+27.33i & 3.812+23.965i \\ 3.812+15.691i & 3.812+16.154i & 3.812+16.665i & 3.812+27.33i & 14.932+58.374i & 3.812+27.33i \\ 3.812+15.269i & 3.812+16.154i & 3.812+16.154i & 3.812+23.965i & 3.812+27.33i & 14.932+58.374i \end{pmatrix}\Omega$$

$$a := 1\cdot e^{j\cdot\frac{2\cdot\pi}{3}}$$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$Za := \text{submatrix}(Zline, 0, 2, 0, 2)$ $Zb := \text{submatrix}(Zline, 0, 2, 3, 5)$

$Zc := \text{submatrix}(Zline, 3, 5, 0, 2)$ $Zd := \text{submatrix}(Zline, 3, 5, 3, 5)$

$Za012 := A_{012}^{-1} \cdot Za \cdot A_{012}$ $Zb012 := A_{012}^{-1} \cdot Zb \cdot A_{012}$

$Zc012 := A_{012}^{-1} \cdot Zc \cdot A_{012}$ $Zd012 := A_{012}^{-1} \cdot Zd \cdot A_{012}$

Build the matrix by stacking and augmenting submatrices

$Z012left := \text{stack}(Za012, Zc012)$

$Z012right := \text{stack}(Zb012, Zd012)$

$Z_{012} := \text{augment}(Z012left, Z012right)$

Note the off-diagonal subblocks and their coupling.

$$Z_{012} = \begin{pmatrix} 22.555 + 110.79i & 0.971 - 0.561i & -0.971 - 0.561i & 11.435 + 48.713i & 0.27 + 0.671i & -0.27 + 0.671i \\ -0.971 - 0.561i & 11.12 + 32.166i & -1.942 + 1.121i & 0.313 - 0.8i & 0.139 - 0.126i & 0.043 + 0.129i \\ 0.971 - 0.561i & 1.942 + 1.121i & 11.12 + 32.166i & -0.313 - 0.8i & -0.043 + 0.129i & -0.139 - 0.126i \\ 11.435 + 48.713i & -0.313 - 0.8i & 0.313 - 0.8i & 22.555 + 110.79i & 0.971 - 0.561i & -0.971 - 0.561i \\ -0.27 + 0.671i & -0.139 - 0.126i & 0.043 + 0.129i & -0.971 - 0.561i & 11.12 + 32.166i & -1.942 + 1.121i \\ 0.27 + 0.671i & -0.043 + 0.129i & 0.139 - 0.126i & 0.971 - 0.561i & 1.942 + 1.121i & 11.12 + 32.166i \end{pmatrix} \Omega$$

Now try transposing one or both of the lines

$$Rp := \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

First, use the exact same scheme on both lines, so the Zb and Zc are kept simpler.

$$f1 := \frac{1}{3} \quad f2 := \frac{1}{3} \quad f3 := \frac{1}{3}$$

$$Zanet := f1 \cdot Za + f2 \cdot Rp^{-1} \cdot Za \cdot Rp + f3 \cdot Rp \cdot Za \cdot Rp^{-1}$$

$$Zbnet := f1 \cdot Zb + f2 \cdot Rp^{-1} \cdot Zb \cdot Rp + f3 \cdot Rp \cdot Zb \cdot Rp^{-1}$$

$$Zcnet := f1 \cdot Zc + f2 \cdot Rp^{-1} \cdot Zc \cdot Rp + f3 \cdot Rp \cdot Zc \cdot Rp^{-1}$$

$$Zdnet := f1 \cdot Zd + f2 \cdot Rp^{-1} \cdot Zd \cdot Rp + f3 \cdot Rp \cdot Zd \cdot Rp^{-1}$$

$$Zanet = \begin{pmatrix} 14.932 + 58.374i & 3.812 + 26.208i & 3.812 + 26.208i \\ 3.812 + 26.208i & 14.932 + 58.374i & 3.812 + 26.208i \\ 3.812 + 26.208i & 3.812 + 26.208i & 14.932 + 58.374i \end{pmatrix} \Omega$$

$$Zbnet = \begin{pmatrix} 3.812 + 16.154i & 3.812 + 16.36i & 3.812 + 16.199i \\ 3.812 + 16.199i & 3.812 + 16.154i & 3.812 + 16.36i \\ 3.812 + 16.36i & 3.812 + 16.199i & 3.812 + 16.154i \end{pmatrix} \Omega$$

$$Zcnet = \begin{pmatrix} 3.812 + 16.154i & 3.812 + 16.199i & 3.812 + 16.36i \\ 3.812 + 16.36i & 3.812 + 16.154i & 3.812 + 16.199i \\ 3.812 + 16.199i & 3.812 + 16.36i & 3.812 + 16.154i \end{pmatrix} \Omega$$

$$Zdnet = \begin{pmatrix} 14.932 + 58.374i & 3.812 + 26.208i & 3.812 + 26.208i \\ 3.812 + 26.208i & 14.932 + 58.374i & 3.812 + 26.208i \\ 3.812 + 26.208i & 3.812 + 26.208i & 14.932 + 58.374i \end{pmatrix} \Omega$$

$$Za0121 := A_{012}^{-1} \cdot Zanet \cdot A_{012}$$

$$Zb0121 := A_{012}^{-1} \cdot Zbnet \cdot A_{012}$$

$$Zc0121 := A_{012}^{-1} \cdot Zcnet \cdot A_{012}$$

$$Zd0121 := A_{012}^{-1} \cdot Zdnet \cdot A_{012}$$

Build the matrix by stacking and augmenting submatrices

$Z012left1 := \text{stack}(Za0121, Zc0121)$

$Z012right1 := \text{stack}(Zb0121, Zd0121)$

$Z_{0121} := \text{augment}(Z012left1, Z012right1)$

$$Z_{0121} = \begin{pmatrix} 22.555 + 110.79i & 0 & 0 & 11.435 + 48.713i & 0 & 0 \\ 0 & 11.12 + 32.166i & 0 & 0 & 0.139 - 0.126i & 0 \\ 0 & 0 & 11.12 + 32.166i & 0 & 0 & -0.139 - 0.126i \\ 11.435 + 48.713i & 0 & 0 & 22.555 + 110.79i & 0 & 0 \\ 0 & -0.139 - 0.126i & 0 & 0 & 11.12 + 32.166i & 0 \\ 0 & 0 & 0.139 - 0.126i & 0 & 0 & 11.12 + 32.166i \end{pmatrix} \Omega$$

- Notice what this has done to the cross-coupling terms. The positive and negative sequence coupling between the two lines is very small relative to the zero sequence cross-coupling.