ECE: 421 Line Models

$MVA := MW$

1. Given a line as follows:

$Length := 200mi$

$r' := 0.21 \cdot \frac{ohm}{mi}$ $\quad x' := 1.89 \frac{ohm}{mi}$ $\quad b' := 9.42 \cdot 10^{-6} \frac{mho}{mi}$

$z_{1'} := r' + j \cdot x'$ $\quad y_{1'} := j \cdot b'$

$\frac{x'}{r'} = 9$

$Z_{c1} := \sqrt{\frac{z_{1'}}{y_{1'}}}$ $\quad Z_{c1} = (448.614 - 24.847i)\,\Omega$ $\quad |Z_{c1}| = 449.301\,\Omega$ $\quad \arg(Z_{c1}) = -3.17 \cdot deg$

$\gamma_1 := \sqrt{z_{1'} \cdot y_{1'}}$ $\quad \gamma_1 = \left(2.341 \times 10^{-4} + 0.004i\right) \cdot \frac{1}{mi}$ $\quad |\gamma_1| = 0.0042 \cdot \frac{1}{mi}$ $\quad \arg(\gamma_1) = 86.83 \cdot deg$

$\alpha_1 := Re(\gamma_1)$ $\quad \alpha_1 = 2.341 \times 10^{-4} \cdot \frac{1}{mi}$

$\beta_1 := Im(\gamma_1)$ $\quad \beta_1 = 0.0042 \cdot \frac{1}{mi}$

- Neglecting g', if r' << ωL' then phase constant β should be approximately equal to ω* sqrt(L'C').

$\omega := 2\pi \cdot 60Hz$

$L_{1'} := \frac{Im(z_{1'})}{\omega}$ $\quad L_{1'} = 5.013 \cdot \frac{mH}{mi}$

$C_{1'} := \frac{Im(y_{1'})}{\omega}$ $\quad C_{1'} = 24.987 \cdot \frac{nF}{mi}$

$\frac{1}{\omega \cdot \sqrt{L_{1'} \cdot C_{1'}}} = 381.41 \cdot km$ $\quad \frac{1}{\beta_1} = 380.825 \cdot km$

2. Given an EHV three phase line with series impedance and shunt admittance below, find the characteristic impedance, the propagation constant, attenuation constant and the phase constant.

$$z' := 0.06\,\frac{ohm}{mi} + j\cdot 0.54\,\frac{ohm}{mi} \qquad y' := j\cdot 7.8\cdot 10^{-6}\,\frac{mho}{mi}$$

$$\frac{Im(z')}{Re(z')} = 9$$ much higher ratio than above.

$$Z_c := \sqrt{\frac{z'}{y'}} \qquad Z_c = (263.522 - 14.595i)\,\Omega \qquad |Z_c| = 263.926\,\Omega \qquad \arg(Z_c) = -3.17\cdot deg$$

$$\gamma := \sqrt{z'\cdot y'} \qquad \gamma = \left(1.138\times 10^{-4} + 0.002i\right)\cdot\frac{1}{mi} \qquad |\gamma| = 0.0021\cdot\frac{1}{mi}$$

$$\arg(\gamma) = 86.83\cdot deg$$

$$\alpha := Re(\gamma) \qquad \alpha = 1.138\times 10^{-4}\cdot\frac{1}{mi}$$

$$\beta := Im(\gamma) \qquad \beta = 0.0021\cdot\frac{1}{mi}$$

$$L' := \frac{Im(z')}{\omega} \qquad L' = 1.432\cdot\frac{mH}{mi}$$

$$C' := \frac{Im(y')}{\omega} \qquad C' = 20.69\cdot\frac{nF}{mi}$$

$$\frac{1}{\omega\cdot\sqrt{L'\cdot C'}} = 784.16\cdot km \qquad \frac{1}{\beta} = 782.956\cdot km$$ Much close match

3. Suppose the three phase line in the second example above is 400 mi long and delivers 1000MW at 750kV at 95% power factor lagging. Find the sending end voltage and the sending end current and the transmission efficiency. Use the long line model. (also find the pi model)

$Length := 400mi$

$Z := z' \cdot Length$ $\quad Z = (24 + 216i)\,\Omega$ $\quad \arg(Z) = 83.66 \cdot deg$ $\quad |Z| = 217.329\,\Omega$

$Y := y' \cdot Length$ $\quad Y = 0.003i \cdot mho$

Exct pi model:

$$Z_p := Z \cdot \frac{\sinh(\gamma \cdot Length)}{\gamma \cdot Length}$$

$Z_p = (18.874 + 192.814i)\,\Omega$ $\quad |Z_p| = 193.735\,\Omega$

compared to Z, we see that the real and imaginary parts are smaller. While the angle is slightly larger

$\arg(Z_p) = 84.409 \cdot deg$

$$Y_{p_over_2} := \frac{\cosh(\gamma \cdot Length) - 1}{Z_c \cdot \sinh(\gamma \cdot Length)} \qquad Y_{p_over_2} = \left(1.119 \times 10^{-5} + 0.002i\right) \cdot mho$$

$Y_p := 2 \cdot Y_{p_over_2}$ $\quad Y_p = \left(2.239 \times 10^{-5} + 0.003i\right) \cdot mho$

$|Y_p| = 0.003 \cdot mho$ $\quad \arg(Y_p) = 89.612 \cdot deg$

compared to the original value of $Y = 0.003i \cdot mho$ at 90 degrees

Load conditions:

$V_{Rll} := 750kV$

$P_{load} := 1000MW$

$pf := 0.95$ $\quad \theta_{load} := acos(pf)$ $\quad \theta_{load} = 18.195 \cdot deg$

$$S_{load} := \frac{P_{load} \cdot e^{j \cdot \theta_{load}}}{pf} \qquad |S_{load}| = 1052.632 \cdot MVA$$

Use V2 as reference angle

$\theta 2 := 0deg$

Convert to phase voltages:

$$V_R := \frac{V_{Rll}}{\sqrt{3}} \cdot e^{j \cdot \theta 2} \qquad V_R = 433.013 \cdot kV$$

Find IR (use S = 3 Vp I*)

$$I_R := \overline{\left(\frac{S_{load}}{3 \cdot V_R}\right)} \qquad I_R = (769.8 - 253.021i)\ A \qquad |I_R| = 810.316\ A$$

$$\arg(I_R) = -18.195 \cdot deg$$

Now find IS and VS. There are two options.

1) Use the pi model derived above:

$$I_{capR} := \frac{Y_p}{2} \cdot V_R \qquad I_{capR} = (4.847 + 716.141i)\ A$$

$$I_{line} := I_R + I_{capR} \qquad I_{line} = (774.647 + 463.119i)\ A$$

$$|I_{line}| = 902.529\ A$$

Notice that current through the line is significantly larger than the load current due to the capacitor current.

$$V_S := V_R + I_{line} \cdot Z_p \qquad V_S = (358.338 + 158.104i) \cdot kV \qquad \theta 1 := \arg(V_S)$$

$$|V_S| = 391.667 \cdot kV \qquad \theta 1 = 23.808 \cdot deg$$

$$V_{SLL} := \sqrt{3} \cdot V_S \qquad |V_{SLL}| = 678.387 \cdot kV$$

$$I_{capS} := \frac{Y_p}{2} \cdot V_S \qquad I_{capS} = (-257.47 + 594.409i)\ A$$

$$I_S := I_{line} + I_{capS} \qquad |I_S| = 1.177 \cdot kA$$

$$\theta_{IS} := \arg(I_S) \qquad \theta_{IS} = 63.939 \cdot deg$$

2) Two port network equations:

$$V_{S1} := V_R \cdot \cosh(\gamma \cdot Length) + Z_c \cdot I_R \cdot \sinh(\gamma \cdot Length)$$

$$|V_{S1}| = 391.667 \cdot kV \qquad \arg(V_{S1}) = 23.808 \cdot deg$$

$$I_{S1} := I_R \cdot \cosh(\gamma \cdot Length) + \frac{V_R}{Z_c} \cdot \sinh(\gamma \cdot Length)$$

$$|I_{S1}| = 1.177 \cdot kA \qquad \arg(I_{S1}) = 63.939 \cdot deg$$

same as above.

Calculate efficiency:

$$S_S := 3 \cdot V_S \cdot \overline{I_S} \qquad P_S := Re(S_S) \qquad P_S = 1057.57 \cdot MW$$

$$\eta := \frac{P_{load}}{P_S} \qquad \boxed{\eta = 94.556 \cdot \%}$$

Repeat using the nominal pi equivalent circuit.

$$I_{capRb} := \frac{Y}{2} \cdot V_R \qquad I_{capRb} = 675.5i\ A$$

$$I_{lineb} := I_R + I_{capRb} \qquad I_{lineb} = (769.8 + 422.479i)\ A$$

$$|I_{lineb}| = 878.112\ A$$

$$V_{Sb} := V_R + I_{lineb} \cdot Z \qquad V_{Sb} = (360.233 + 176.416i) \cdot kV \qquad \theta 1b := \arg(V_{Sb})$$

$$\boxed{|V_{Sb}| = 401.111 \cdot kV} \qquad \boxed{\theta 1b = 26.092 \cdot deg}$$

$$V_{SLLb} := \sqrt{3} \cdot V_{Sb} \qquad |V_{SLLb}| = 694.745 \cdot kV$$

$$I_{capSb} := \frac{Y}{2} \cdot V_{Sb} \qquad I_{capSb} = (-275.21 + 561.963i)\ A$$

$$I_{Sb} := I_{lineb} + I_{capSb} \qquad \boxed{|I_{Sb}| = 1.102 \cdot kA} \qquad \theta_I1b := \arg(I_{Sb}) \qquad \boxed{\theta_I1b = 63.325 \cdot deg}$$

I1 is slightly smaller, since the capacitance is smaller than it was in long line model.

Calculate efficiency:

$S_{Sb} := 3 \cdot V_{Sb} \cdot \overline{I_{Sb}}$ $\quad P_{Sb} := \mathrm{Re}(S_{Sb})$ $\quad P_{Sb} = 1055.518 \cdot MW$

$\eta_b := \dfrac{P_{load}}{P_{Sb}}$ $\quad \boxed{\eta_b = 94.74 \cdot \%}$ $\quad$ Efficiency is slightly higher

4. A wind farm operator wants to transmit 2500 MW from a rural local to a load center located 300 km away. Determine the number of three-phase, uncompensated transmission lines required to transmit this power with one line out of service (this is to be stable under a n-1 contingency condition) for the following options for lines:

(a) 345-kV lines, $Z_c = 320\,\Omega$,
(b) 500-kV lines, $Z_c = 275\,\Omega$,
(c) 765-kV lines, $Z_c = 260\,\Omega$.

Assume that $V_S = 1.0$ per unit, $V_R = 0.95$ per unit, and $\delta_{max} = 40$ deg, and a λ of 5000 km

$pu := 1$

$P_{trans} := 2500MW$ $\quad Length := 300km$

$V_S := 1.0pu$ $\quad V_R := 0.95pu$ $\quad \delta_{max} := 40deg$

$\lambda := 5000km$

(a) 345-kV lines, $Z_c = 320\,\Omega$

$V_{LLa} := 345kV$

$Z_{c_a} := 320\Omega$

$P_{SIL_A} := \dfrac{V_{LLa}^2}{Z_{c_a}}$ $\quad P_{SIL_A} = 371.953 \cdot MW$

$P_{limit_A} := \dfrac{V_S \cdot V_R \cdot P_{SIL_A} \cdot \sin(\delta_{max})}{\sin\left(\dfrac{2 \cdot \pi \cdot Length}{\lambda}\right)}$ $\quad P_{limit_A} = 616.999 \cdot MW$

$$No345kVLines := ceil\left(\frac{P_{trans}}{P_{limit_A}}\right) + 1$$

$$\boxed{No345kVLines = 6}$$

- The "ceil" function rounds up to nearest integer

(b) 500-kV lines, $Z_c = 275\,\Omega$

$$V_{LLb} := 500kV$$

$$Z_{c_b} := 275\Omega$$

$$P_{SIL_B} := \frac{V_{LLb}^2}{Z_{c_b}} \qquad P_{SIL_B} = 909.091 \cdot MW$$

$$P_{limit_B} := \frac{V_S \cdot V_R \cdot P_{SIL_B} \cdot \sin(\delta_{max})}{\sin\left(\frac{2 \cdot \pi \cdot Length}{\lambda}\right)} \qquad P_{limit_B} = 1508.008 \cdot MW$$

$$No500kVLines := ceil\left(\frac{P_{trans}}{P_{limit_B}}\right) + 1$$

$$\boxed{No500kVLines = 3}$$

(c) 765-kV lines, $Z_c = 260\,\Omega$

$$V_{LLc} := 765kV$$

$$Z_{c_c} := 260\Omega$$

$$P_{SIL_C} := \frac{V_{LLc}^2}{Z_{c_c}} \qquad P_{SIL_C} = 2250.865 \cdot MW$$

$$P_{limit_C} := \frac{V_S \cdot V_R \cdot P_{SIL_C} \cdot \sin(\delta_{max})}{\sin\left(\frac{2 \cdot \pi \cdot Length}{\lambda}\right)} \qquad P_{limit_C} = 3733.755 \cdot MW$$

$$No765kVLines := ceil\left(\frac{P_{trans}}{P_{limit_C}}\right) + 1$$

$$\boxed{No765kVLines = 2}$$

- Minimum possible to meet the (n-1) criteria to be able to operate with one line out