Example

Define Units: $kVA := kW \quad kVAR := kW$

A 3-ϕ load draws 200kW at a PF of 0.707 lagging from a 440V line. In parallel is a 3-ϕ capacitor bank that supplies 50kVAR. Find the resultant power factor and current (magnitude) into the parallel combination.

$P_{Load} := 200kW \qquad PF := 0.707 \qquad \phi_{PF} := acos(0.707) \qquad \phi_{PF} = 45.009\,deg$

$V_{Line} := 440V \qquad Q_{Cap} := 50kVAR$

$Sload := \frac{P_{Load}}{PF} \qquad Sload = 282.885\,kVA$

$Qload := Sload \cdot sin(\phi_{PF}) \qquad Qload = 200.06\,kVAR$

Or in one step:

$Q_{Load} := P_{Load} \cdot tan(\phi_{PF}) \qquad Q_{Load} = 200.06\,kVAR$

The total reactive power drawn form the source will be

$Q_{Source} := Q_{Load} - Q_{Cap} \qquad Q_{Source} = 150.06\,kVAR$

Assuming the active power drawn by the load remains constant the apparent power drawn by the load will be

$S_{Load} := P_{Load} + j \cdot (Q_{Source}) \qquad S_{Load} = 200 + 150.06i\,kVA$

The resultant power factor angle will b $\phi_{PFnew} := arg(S_{Load})$

$\phi_{PFnew} = 36.881\,deg$ Lagging $\qquad PF_{new} := cos(\phi_{PFnew}) \qquad PF_{new} = 0.8$

The current magnitude into the parallel combination will be

$I_{new} := \frac{\overline{S_{Load}}}{\overline{\sqrt{3} \cdot V_{Line}}}$

$I_{new} = 262.432 - 196.903i\,A \qquad |I_{new}| = 328.087\,A \qquad arg(I_{new}) = -36.881\,deg$

The current magnitude into the load without the parallel capacitor bank is

$S_{Load\_old} := P_{Load} + j \cdot Q_{Load}$

$$I_{Load\_old} := \overline{\left(\frac{S_{Load\_old}}{\sqrt{3} \cdot \overline{V_{Line}}}\right)}$$

$I_{Load\_old} = 262.432 - 262.511i\,A$

$\left|I_{Load\_old}\right| = 371.191\,A$

$\arg\left(I_{Load\_old}\right) = -45.009\,deg$

Note: Comparing the magnitudes of the currents and power factors in both the cases (ie with and without capacitor bank connected in parallel with the load), we see that the load current is higher in the case without capacitor bank and this current is supplied by the source. Since the transmission lines connecting the source and load will have impedance, larger load currents will result in higher losses and larger voltage drops along the line and will ultimately result in poor voltage regulation. So if a capacitor is connected, in parallel with the load, it will meet some of reactive power required by the load and there by reduce the magnitude of the total current (or reactive power) that should be supplied by the source and hence the voltage drop in the line. Thus it provides better voltage regulation in heavily loaded lines.

The same can be explained in terms of power factor. Loads operating at lower power factors will draw large reactive powers. So connecting a capacitor bank in parallel with the load will support some of the reactive power and there by improve the overall load (overall load is parallel combination of load and capacitor) power factor and reduce the total load current.

Problem 2.8: In the system shown in Figure P2.8, find I_a, I_b and I_c if

(a) $Z_a = Z_b = j1.0$, $Z_c = j\,0.9$

(b) $Z_a = Z_b = Z_c = j\,1.0$

Part (a): $Z_a := j \cdot 1.0 ohm$ $\quad Z_b := Z_a$ $\quad Z_c := j \cdot 0.9 ohm$

$Va := 1 \cdot e^{j \cdot 0deg} V \qquad Vb := 1 \cdot e^{-j \cdot 120deg} V \qquad Vc := 1 \cdot e^{j \cdot 120deg} V$

Since the system is not a balanced system we cannot apply per phase analysis. Applying KVL along different loops we have:

$$Va - I_a(j\cdot 0.1) - I_a\cdot Z_a - I_n\cdot(j\cdot 0.1) = 0$$

$$Vb - I_b(j\cdot 0.1) - I_b\cdot Z_b - I_n\cdot(j\cdot 0.1) = 0$$

$$Vc - I_c(j\cdot 0.1) - I_c\cdot Z_c - I_n\cdot(j\cdot 0.1) = 0$$

Substituting $I_a + I_b + I_c = I_n$ we have

$$Va - I_a(j\cdot 0.1) - I_a\cdot Z_a - \left(I_a + I_b + I_c\right)\cdot(j\cdot 0.1) = 0$$

$$Vb - I_b(j\cdot 0.1) - I_b\cdot Z_b - \left(I_a + I_b + I_c\right)\cdot(j\cdot 0.1) = 0$$

$$Vc - I_c(j\cdot 0.1) - I_c\cdot Z_c - \left(I_a + I_b + I_c\right)\cdot(j\cdot 0.1) = 0$$

Collecting terms we are left with:

$$Va - I_a\left(j\cdot 0.2 + Z_a\right) - I_b\cdot(j\cdot 0.1) - I_c\cdot(j\cdot 0.1) = 0$$

$$Vb - I_a(j\cdot 0.1) - I_b\cdot\left(j\cdot 0.2 + Z_b\right) - I_c\cdot(j\cdot 0.1) = 0$$

$$Vc - I_a(j\cdot 0.1) - I_b\cdot(j\cdot 0.1) - I_c\cdot\left(j\cdot 0.2 + Z_c\right) = 0$$

Three equations and three unknowns. Put this into matrix form:

$$\begin{pmatrix} j\cdot 0.2\text{ohm} + Z_a & j\cdot 0.1\text{ohm} & j\cdot 0.1\text{ohm} \\ j\cdot 0.1\text{ohm} & j\cdot 0.2\text{ohm} + Z_b & j\cdot 0.1\text{ohm} \\ j\cdot 0.1\text{ohm} & j\cdot 0.1\text{ohm} & j\cdot 0.2\text{ohm} + Z_c \end{pmatrix} \cdot \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = \begin{pmatrix} Va \\ Vb \\ Vc \end{pmatrix}$$

$$\begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} := \begin{pmatrix} j\cdot 0.2\text{ohm} + Z_a & j\cdot 0.1\text{ohm} & j\cdot 0.1\text{ohm} \\ j\cdot 0.1\text{ohm} & j\cdot 0.2\text{ohm} + Z_b & j\cdot 0.1\text{ohm} \\ j\cdot 0.1\text{ohm} & j\cdot 0.1\text{ohm} & j\cdot 0.2\text{ohm} + Z_c \end{pmatrix}^{-1} \cdot \begin{pmatrix} Va \\ Vb \\ Vc \end{pmatrix}$$

$$\begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = \begin{pmatrix} -5.584 \times 10^{-3} - 0.912i \\ -0.793 + 0.451i \\ 0.86 + 0.496i \end{pmatrix} A$$

$|I_a| = 0.912\,A$ $\quad \arg(I_a) = -90.351\,deg$

$|I_b| = 0.912\,A$ $\quad \arg(I_b) = 150.351\,deg$

$|I_c| = 0.993\,A$ $\quad \arg(I_c) = 30\,deg$

Notice that the currents are still fairly close to balanced three phase.

$I_n := I_a + I_b + I_c \qquad |I_n| = 0.071\,A \qquad \arg(I_n) = 30\,deg$

Voltage from N' to N:

$Vnp\_n := j \cdot 0.1ohm \cdot I_n \qquad |Vnp\_n| = 7.092 \times 10^{-3}\,V$

Roughly 0.71% of source voltage.

Part (b): $Z_{a\_2} := j \cdot 1.0ohm \qquad Z_{b\_2} := j \cdot 1.0ohm \qquad Z_{c\_2} := j \cdot 1.0ohm$

Applying per phase analysis we have

$I_{a\_2} := \dfrac{Va}{Z_{a\_2} + j \cdot 0.1ohm}$ $\quad |I_{a\_2}| = 0.909\,A$ $\quad \arg(I_{a\_2}) = -90\,deg$

$I_{b\_2} := I_{a\_2} \cdot e^{-j \cdot 120deg}$ $\quad |I_{b\_2}| = 0.909\,A$ $\quad \arg(I_{b\_2}) = 150\,deg$

$I_{c\_2} := I_{a\_2} \cdot e^{j \cdot 120deg}$ $\quad |I_{c\_2}| = 0.909\,A$ $\quad \arg(I_{c\_2}) = 30\,deg$

As a check, use the matrix equations set up for part A:

$$\begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} := \begin{pmatrix} j \cdot 0.2ohm + Z_{a\_2} & j \cdot 0.1ohm & j \cdot 0.1ohm \\ j \cdot 0.1ohm & j \cdot 0.2ohm + Z_{b\_2} & j \cdot 0.1ohm \\ j \cdot 0.1ohm & j \cdot 0.1ohm & j \cdot 0.2ohm + Z_{c\_2} \end{pmatrix}^{-1} \cdot \begin{pmatrix} Va \\ Vb \\ Vc \end{pmatrix}$$

$$\begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = \begin{pmatrix} -0.909i \\ -0.787 + 0.455i \\ 0.787 + 0.455i \end{pmatrix} A$$

$|I_a| = 0.909\,A$ $\quad \arg(I_a) = -90\,deg$

$|I_b| = 0.909\,A$ $\quad \arg(I_b) = 150\,deg$

$|I_c| = 0.909\,A$ $\quad \arg(I_c) = 30\,deg$