University of Idaho

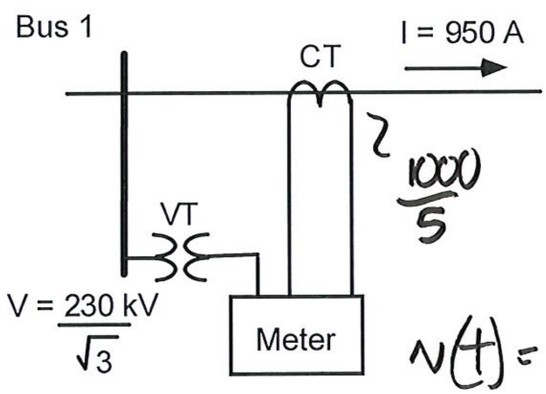
ECE 444 / ECE 544 / CS 444 / CS 544 Supervisory Control and Critical Infrastructure Systems

Session 5

CS/ECE 444/544: Homework #1

Due: Session 9 (February 8)

The 60 Hz line current on the transmission line below is 950 A RMS at an angle $\varphi_i = -13$ degrees, and the line to neutral bus voltage is (230/SQRT(3)) kV RMS at an angle $\varphi_v = 5$ degrees.



$i(t) = \sqrt{2} \cdot 950 \cos(\omega t + \phi_i)$ — peak amplitude; line to neutral; $2\pi \cdot 60Hz$

$v(t) = \sqrt{2}\left(\frac{230kV}{\sqrt{3}}\right)\cos(\omega t + \phi_v)$

The current transformer ratio is 1000:5 and the voltage transformer ratio is 230,000:120

Complete the following steps:

1. Calculate the secondary (scaled) values of the RMS voltage and current

2. Calculate the real power transfer using in primary values and secondary values using the equation below. ***544 students:*** If they don't match, comment on why that is the case.

$$P_{3ph} = 3 \cdot |V_{LN}| \cdot |I| \cdot \cos(\phi_v - \phi_i)$$

3. Plot the secondary values of *v(t)* and *i(t)* versus time for one 60 Hz cycle (you don't have to use Mathcad you can use Matlab, write your own program, or use a spreadsheet.

4. Plot the output versus time for an analog-to-digital (A/D) converter applied to the current waveform from part 3 for the following options. Assume 8 samples per 60 Hz cycle.

   a) You have a 4 bit A/D where the most significant bit is a sign bit (MSB). Use the signed magnitude to represent the negative number (0 is positive and 1 is negative). In your plot, simply put the negative number below zero. Assume full scale for you're A/D converter is -7A to + 7A. Compare your result to the original waveform.

   b) Repeat part a) with the full scale range changed to -35A to +35A.

   c) Repeat parts a) and b) with a 5 bit A/D (again, MSB is the sign bit).

L5 1/16

5. **CS and ECE 544 students only**: Plot the output versus time for an analog-to-digital (A/D) converter applied to the voltage waveform from part 3 for the following options. Assume 8 samples per 60 Hz cycle.

a) You have a 4 bit A/D where the most significant bit is a sign bit (MSB). Use the signed magnitude to represent the negative number (0 is positive and 1 is negative). In your plot, simply put the negative number below zero. Assume full scale for you're A/D converter is -150V to + 150V.

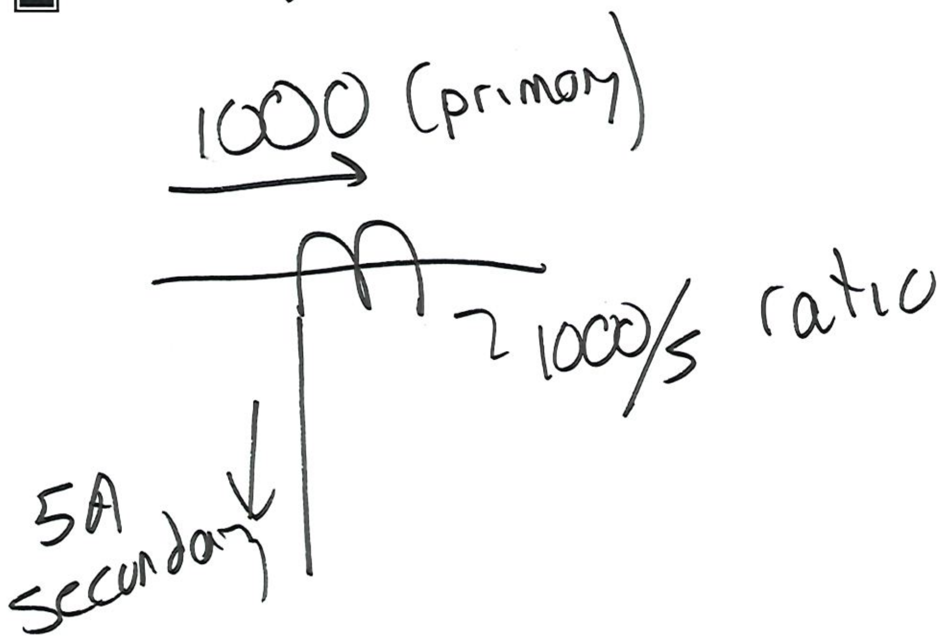
b) Repeat 5 a) with a 5 bit A/D (again, MSB is the sign bit) and the same full scale

c) Calculate the RMS current magnitudes for problem 4 parts a) and c) and compare with your original magnitudes. Use the following formula (you will need to break the intergral over the the flat line segments)

$$I_{RMS} = \sqrt{\frac{1}{T} \cdot \int_0^T \left(i(t)^2\right) dt}$$

57

2/16


1000 (primary)
1000/5 ratio
5A secondary

CS/ECE 444/544: Analog to Digital Conversion

$f := 10Hz$ $\quad\quad$ $\phi := 10deg$

$T_s := \frac{1}{f}$ $\quad\quad$ $T_s = 0.1\,s$

Time vector for plotting: use 32 points per cycle

$t := 0sec, \frac{T_s}{32} .. T_s$

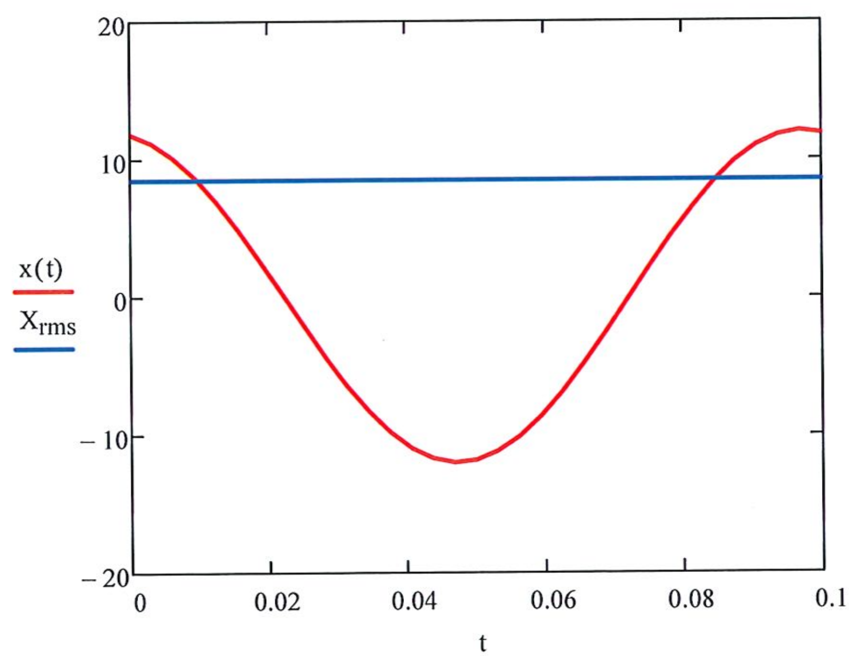
0 Δt end time — peak amplitude

$x(t) := 12\cdot\cos(2\cdot\pi\cdot f\cdot t + \phi)$ → $x(t) = X_m \cos(2\pi f t + \phi)$ ω ω

- Peak amplitude of x(t): $\quad X_m := 12$
- For a single frequency sinusoidal function

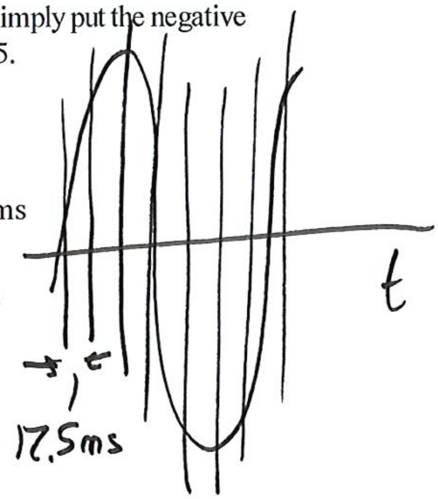
$X_{rms} := \frac{X_m}{\sqrt{2}}$ - simple approximation if perfect sinusoid

Figure 1



L5 4/16

1. Plot the output versus time for an analog-to-digital (A/D) converter applied to the waveform for (x(t).
   Assume the following
   a) you sampling at 8 samples per 10 Hz cycle
   b) You have a 4 bit A/D where the most significant bit is a sign bit (MSB). Use the signed magnitude to represent the negative number (0 is positive and 1 is negative). In your plot, simply put the negative number below zero. Assume full scale for you're A/D converter is -15 to + 15.



- Sampling rate: $RS := 8$ samples per cycle
- Sample period $t_{sample} := \frac{1}{RS \cdot 10Hz}$ $t_{sample} = 12.5 \cdot ms$
- Analog scale: Maximum positive and negative value has magnitude of 14
- We have 2^n combinations of the magnitude bits, where n=2
- in this case two of which are 0, that step up as:

$n := 2$ $\frac{15}{2^2 - 1} = 5$

-Mapping digital bit combinations to analog values (MSB is sign bit, the next two are the magnitude bits).

Table 1:

			Value
0	0	0	0
0	0	1	5
0	1	0	10
0	1	1	15
1	0	0	−0
1	0	1	−5
1	1	0	−10
1	1	1	−15

Values of x(t) at the sample points:

- Put A/D results in a vector for plotting (round absolute value down to the nearest number from the value column of Table 1

$x(0 \cdot t_{sample}) = 11.82$ A/D output $x_{AD\_part1_0} := 10A$

$x(1 \cdot t_{sample}) = 6.88$ A/D output $x_{AD\_part1_1} := 5A$

$x(2 \cdot t_{sample}) = -2.08$ A/D output $x_{AD\_part1_2} := 0A$

note the significant rounding error in most of these.

$x(3 \cdot t_{sample}) = -9.83$ A/D output $x_{AD\_part1_3} := -5A$

$x(4 \cdot t_{sample}) = -11.82$ A/D output $x_{AD\_part1_4} := -10A$

1 cycle 8 samples

$x(5 \cdot t_{sample}) = -6.88$ A/D output $x_{AD\_part1_5} := -5A$

$x(6 \cdot t_{sample}) = 2.08$ A/D output $x_{AD\_part1_6} := 0A$

$x(7 \cdot t_{sample}) = 9.83$ A/D output $x_{AD\_part1_7} := 5A$

- Put in one extra sample for plotting

$x(8 \cdot t_{sample}) = 11.82$ A/D output $x_{AD\_part1_8} := 10A$

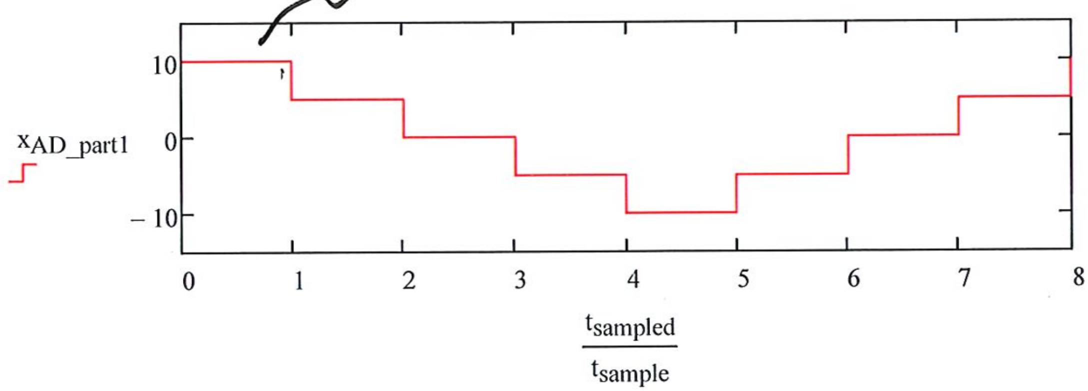
Now plot the i_{AD_part1} values versus time

$i := 0, 1 .. 9$ $t_{sampled_i} := i \cdot t_{sample}$

sample & hold

Figure 2

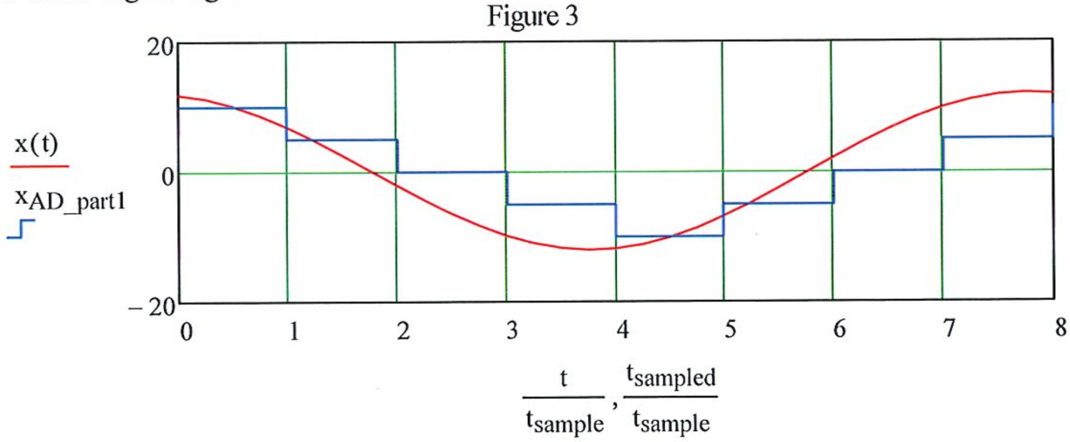
Sample and hold output



- Now compare this to the original signal

Figure 3

Notice that the low sampling rate causes a bit of a phase shift



L5 9/16

- The delay in the blue waveform is more apparent because of the low sample rate and the low number of magnitude bits.
- A higher sampling rate will improve the waveform significantly. For example, the red line is plotted at 32 samples per cycle
- And has much much higher resolution numbers (so no rounding)
- But if all measurements have same delay, they cancel

2. Calculate the RMS current magnitudes for problem 4 parts a) and c) and compare with your original magnitudes.

$$X_{RMS} = \sqrt{\left(\frac{1}{T}\right)\cdot\int_0^T \left(i(t)^2\right)dt}$$

$$T_{10} := 8\cdot t_{sample}$$

T = 1/10Hz

- Original waveform:

$$X_{s\_RMS} := \sqrt{\frac{1}{T_{10}}\cdot\int_0^{T_{10}} \left(x(t)^2\right)dt} \qquad X_{s\_RMS} = 8.49$$

recall that originally we used the shortcut equation and had:

$$\frac{12}{\sqrt{2}} \rightarrow \left|X_{rms}\right| = 8.49$$

Now repeat this calculation over each of the sample and hold intervals of the waveform from the plot of Figure 2

$$A1 := \int_0^{1\cdot t_{sample}} \left(x_{AD\_part1_0}\right)^2 dt_{sample}$$

$$A2 := \int_{1\cdot t_{sample}}^{2\cdot t_{sample}} \left(x_{AD\_part1_1}\right)^2 dt_{sample}$$

$$A3 := \int_{2\cdot t_{sample}}^{3\cdot t_{sample}} \left(x_{AD\_part1_2}\right)^2 dt_{sample}$$

L5 7/16

$$A4 := \int_{3 \cdot t_{sample}}^{4 \cdot t_{sample}} \left(x_{AD\_part1_3}\right)^2 \, dt_{sample}$$

$$A5 := \int_{4 \cdot t_{sample}}^{5 \cdot t_{sample}} \left(x_{AD\_part1_4}\right)^2 \, dt_{sample}$$

$$A6 := \int_{6 \cdot t_{sample}}^{7 \cdot t_{sample}} \left(x_{AD\_part1_5}\right)^2 \, dt_{sample}$$

$$A7 := \int_{6 \cdot t_{sample}}^{7 \cdot t_{sample}} \left(x_{AD\_part1_6}\right)^2 \, dt_{sample}$$

$$A8 := \int_{7 \cdot t_{sample}}^{8 \cdot t_{sample}} \left(x_{AD\_part1_7}\right)^2 \, dt_{sample}$$

$$X_{AD\_RMS} := \sqrt{\frac{1}{T_{10}} \cdot (A1 + A2 + A3 + A4 + A5 + A6 + A7 + A8)}$$

$X_{AD\_RMS} = 6.12\,A$

Quite a bit smaller than correct value due to quantization error.

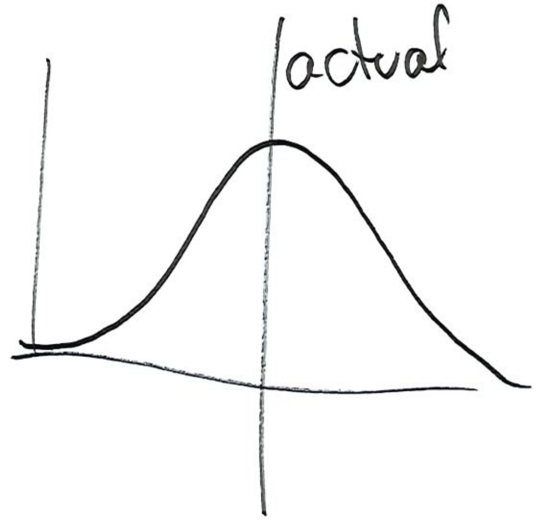
L5 8/16

measurement error

1. Sensors (CT, VT, etc)
2. Quantization (A/D conversion)

→ statistical accuracy of measurements

⇒ often assume Gaussian distribut



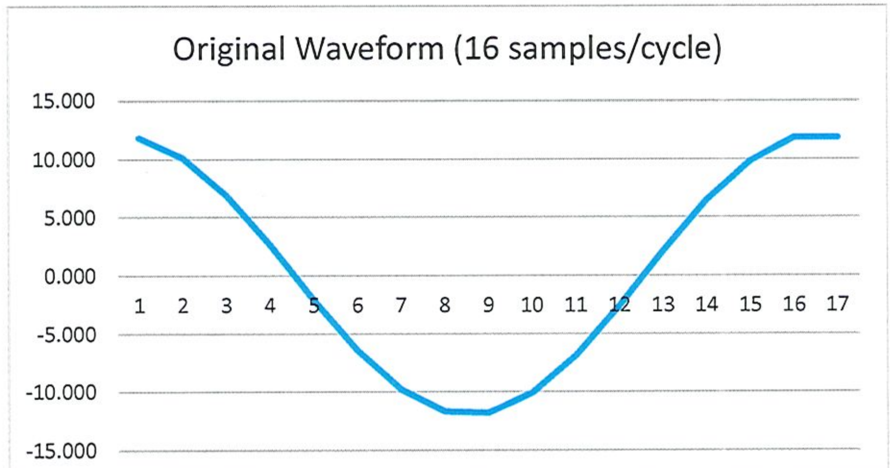
L5 10/16

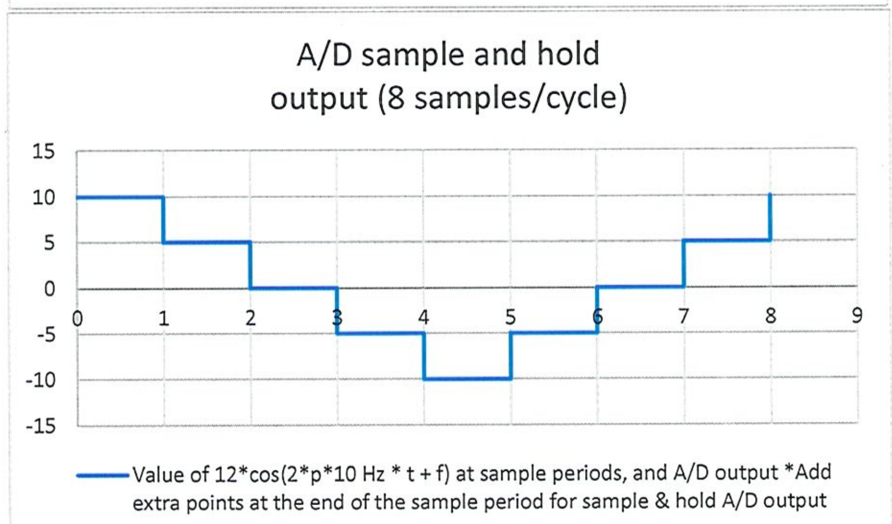
Value of 12*cos(2*π*10 Hz * t + ϕ) at sample periods, and A/D output

*Add extra points at the end of the sample period for sample & hold

for plotting

Sample number	Time	Value	A/D output
0	0.000	11.818	10
0.99999	0.012	11.818	10
1	0.013	6.883	5
1.99999	0.025	6.883	5
2	0.025	-2.084	0
2.99999	0.037	-2.084	0
3	0.038	-9.830	-5
3.99999	0.050	-9.830	-5
4	0.050	-11.818	-10
4.99999	0.062	-11.818	-10
5	0.063	-6.883	-5
5.99999	0.075	-6.883	-5
6	0.075	2.084	0
6.99999	0.087	2.084	0
7	0.088	9.830	5
7.99999	0.100	9.830	5
8	0.100	11.818	10



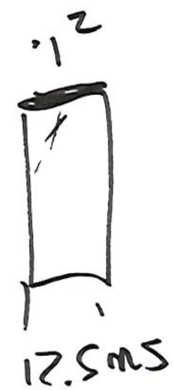


L5 11/16

Now calculate RMS current

* Implement the following formula in pieces
* Remember that integal is a sum of areas

$$X_{RMS} = \sqrt{\frac{1}{T} \cdot \int_0^T \left(i(t)^2\right) dt}$$



Sample num	Current from A/D	Integrals by parts (t.sample * v^2)
0	10	1.25
1	5	0.3125
2	0	0
3	-5	0.3125
4	-10	1.25
5	-5	0.3125
6	0	0
7	5	0.3125
	Sum	3.75

	Sum/T	37.5	Where T is the period
Take SQRT	X_RMS_a/d	6.124	

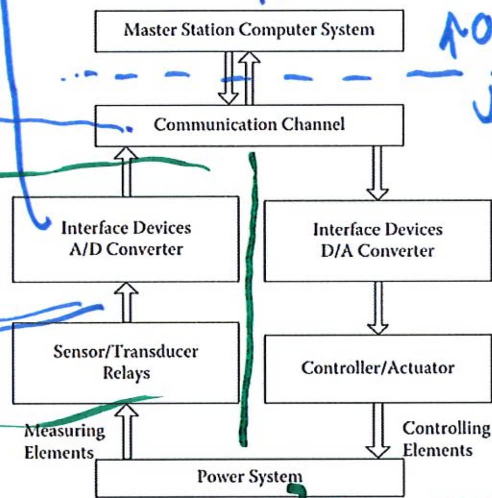
Compare to original waveform

Xrms= 12/SQRT(2	8.49	Significant difference due to quantization

Monitoring/Controlling Architecture

CS/ECE 444/544

Lecture 5



Human

↑outside ↓inside a facility

RTU

often current or a voltage

processing, signal conditioning

moniter

Actuator/control

4-20mA

linear scale 4-20mA

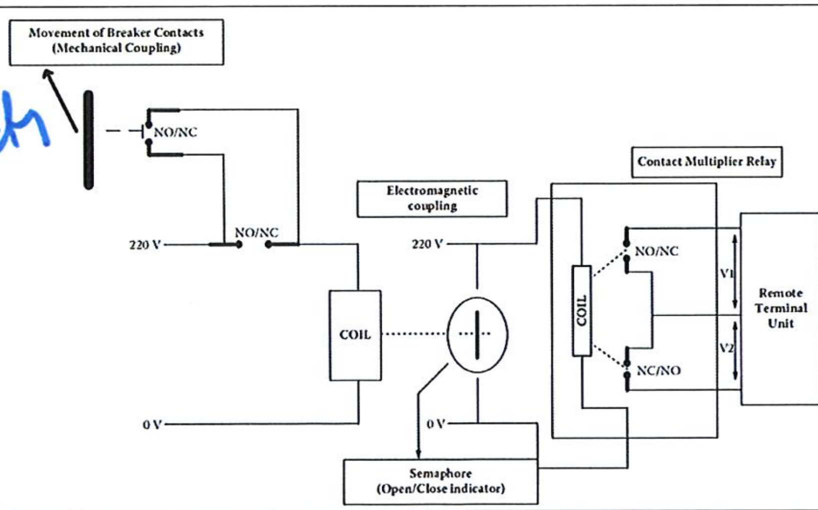
2 process

1

Example Digital Measurement

CS/ECE 444/544

Lecture 5



0 ↑ don't use 0mA to differentiate failed connection

2

L5 S7 12/16

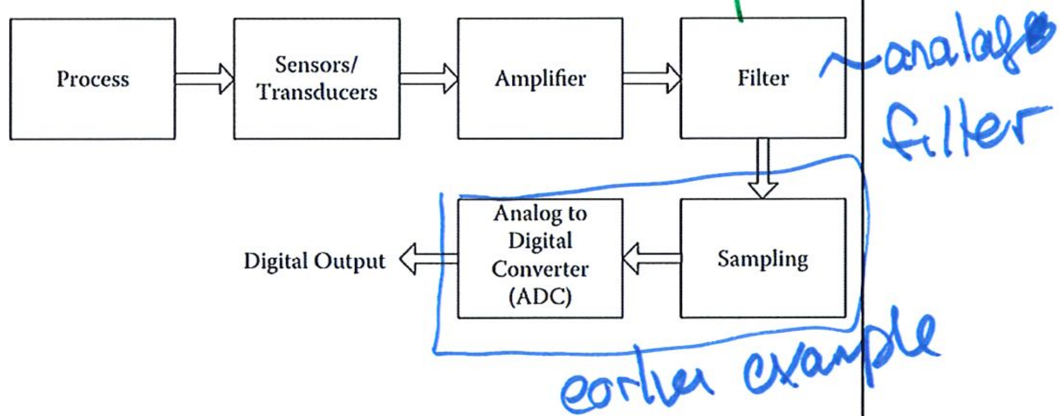
- in our example we had 80 samples per second
- Nyquist - anything above 40 Hz in signal is lost

Antialiasing filter (low pass filter)

Analog measurement – process to digital measurement data

CS/ECE 444/544

Lecture 5



Processing Voltage and Current Measurements

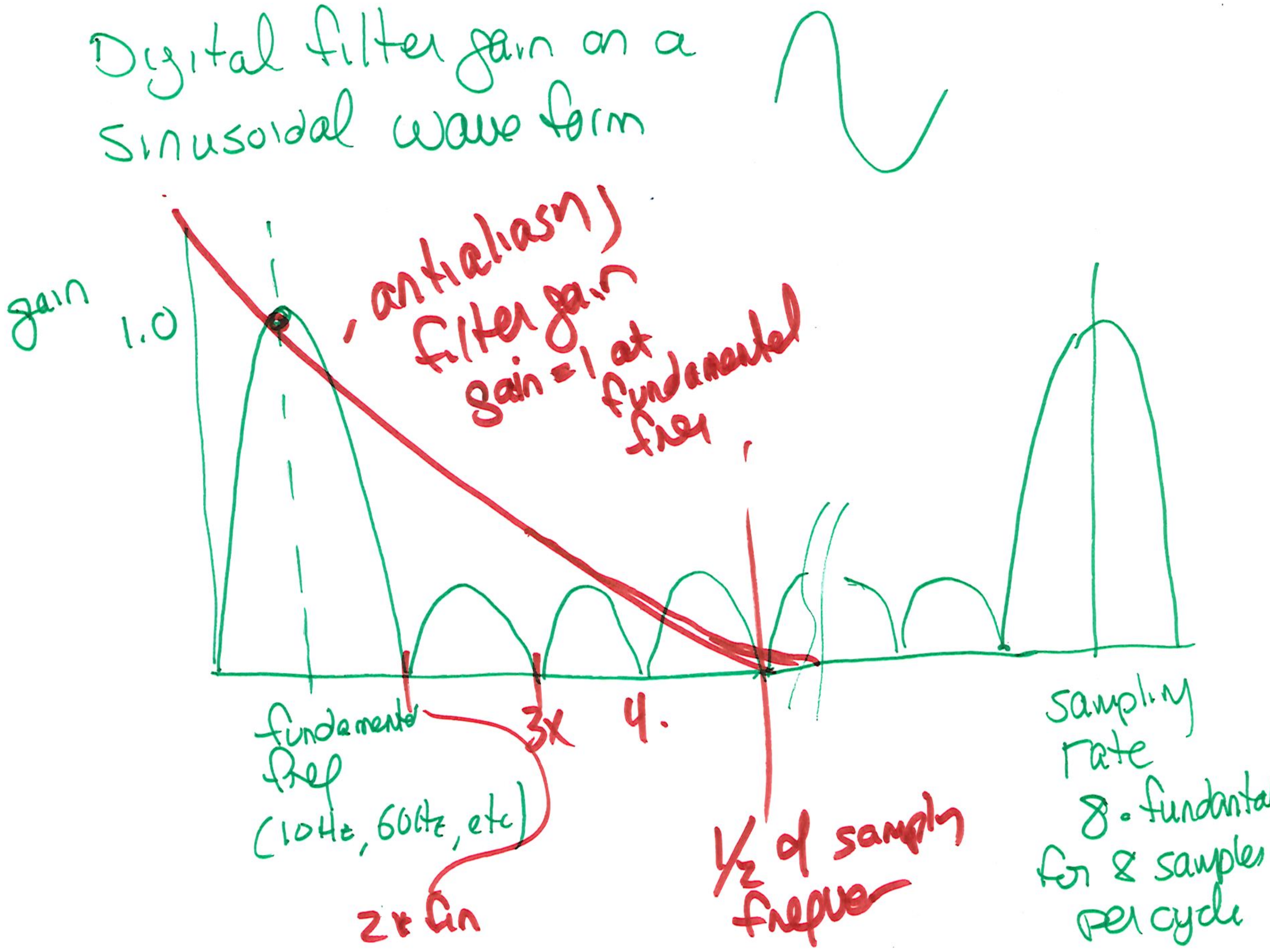
CS/ECE 444/544

Lecture 5

- Measurement transducers scale amplitude
- Provide isolation
- Options
  - » Iron transformers
  - » Optical transducers

L5 57 1/13/16

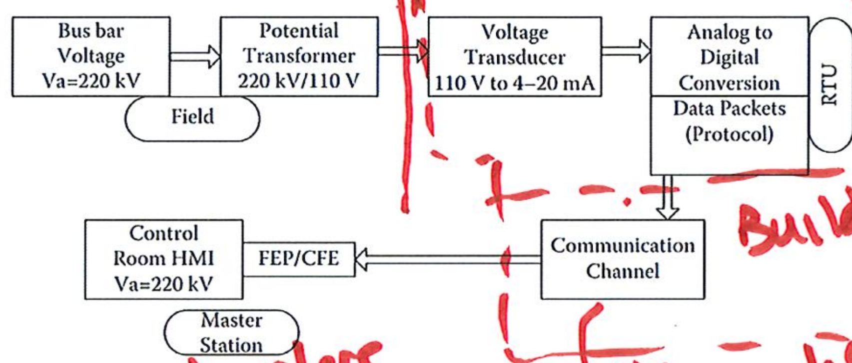
Digital filter gain on a sinusoidal wave form



Historical Power System Implementation

CS/ECE 444/544

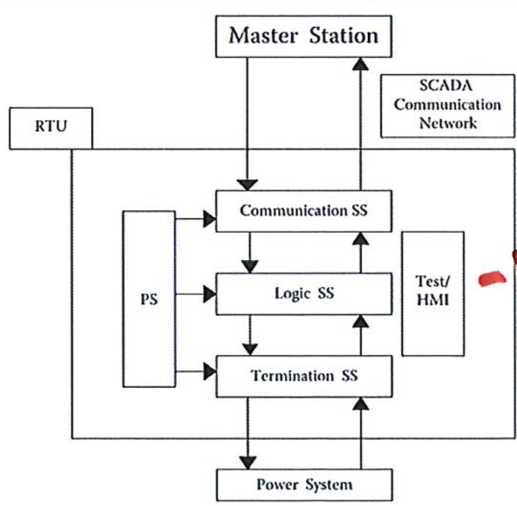
Lecture 5



Remote Terminal Unit

CS/ECE 444/544

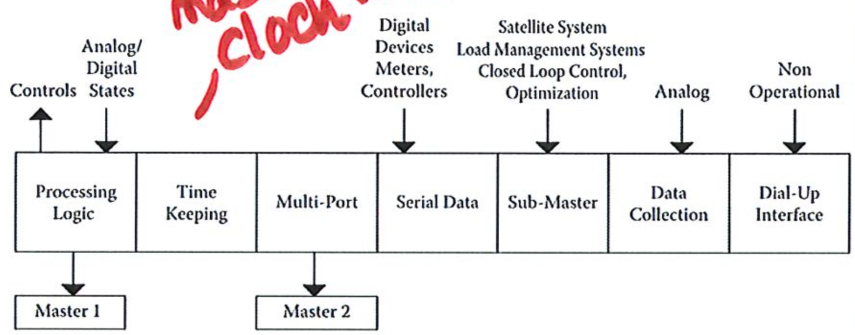
Lecture 5



Remote Terminal Unit

CS/ECE 444/544

Lecture 5



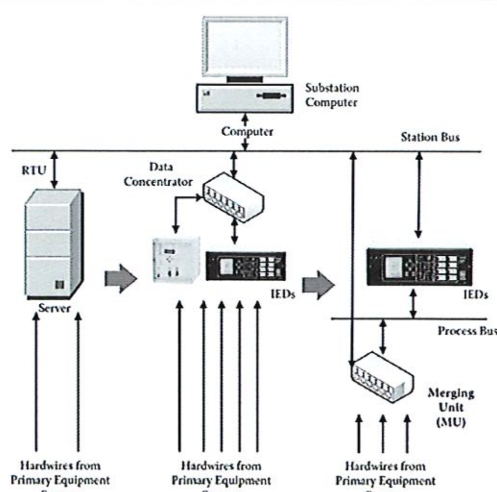
master clock in station

IRIG-B
PTP (precision time protocol) / ethernet

Modern Analog Measurement Options

CS/ECE 444/544

Lecture 5



L5 9/16