CS/ECE 444/544: Analog to Digital Conversion

$f := 10\text{Hz}$ $\quad\quad \phi := 10\text{deg}$

$T_s := \frac{1}{f}$ $\quad\quad T_s = 0.1\,\text{s}$

Time vector for plotting: use 32 points per cycle

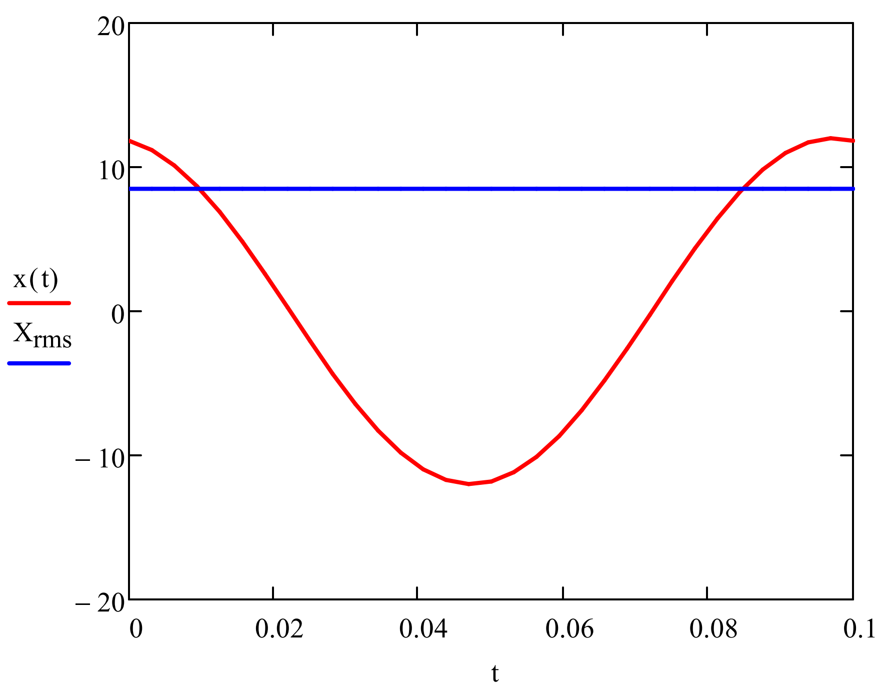
$t := 0\text{sec}, \frac{T_s}{32} .. T_s$

$x(t) := 12\cdot\cos(2\cdot\pi\cdot f\cdot t + \phi)$

- Peak amplitude of x(t): $X_m := 12$
- For a single frequency sinusoidal function

$X_{rms} := \frac{X_m}{\sqrt{2}}$ $\quad\quad X_{rms} = 8.49$

Figure 1



1. Plot the output versus time for an analog-to-digital (A/D) converter applied to the waveform for (x(t).
   Assume the following
   a) you sampling at 8 samples per 10 Hz cycle
   b) You have a 3 bit A/D where the most significant bit is a sign bit (MSB). Use the signed magnitude to represent the negative number (0 is positive and 1 is negative). In your plot, simply put the negative number below zero. Assume full scale for you're A/D converter is -15 to + 15.

- Sampling rate: $RS := 8$ samples per cycle
- Sample period $t_{sample} := \frac{1}{RS \cdot 10Hz}$ $t_{sample} = 12.5 \cdot ms$
- Analog scale: Maximum positive and negative value has magnitude of 14
- We have 2^n combinations of the magnitude bits, where n=2
- in this case two of which are 0, that step up as:

$n := 2$ $\frac{15}{2^2 - 1} = 5$

-Mapping digital bit combinations to analog values (MSB is sign bit, the next two are the magnitude bits).

Table 1:

			Value
0	0	0	0
0	0	1	5
0	1	0	10
0	1	1	15
1	0	0	−0
1	0	1	−5
1	1	0	−10
1	1	1	−15

Values of x(t) at the sample points:

- Put A/D results in a vector for plotting (round absolute value down to the nearest number from the value column of Table 1

$x(0 \cdot t_{sample}) = 11.82$ A/D output $x_{AD\_part1_0} := 10A$

$x(1 \cdot t_{sample}) = 6.88$ A/D output $x_{AD\_part1_1} := 5A$

$x(2 \cdot t_{sample}) = -2.08$ A/D output $x_{AD\_part1_2} := 0A$

note the significant rounding error in most of these.

$x(3 \cdot t_{sample}) = -9.83$ A/D output $x_{AD\_part1_3} := -5A$

$x(4 \cdot t_{sample}) = -11.82$ A/D output $x_{AD\_part1_4} := -10A$

$x(5 \cdot t_{sample}) = -6.88$ A/D output $x_{AD\_part1_5} := -5A$

$x(6 \cdot t_{sample}) = 2.08$ A/D output $x_{AD\_part1_6} := 0A$

$x(7 \cdot t_{sample}) = 9.83$ A/D output $x_{AD\_part1_7} := 5A$

- Put in one extra sample for plotting

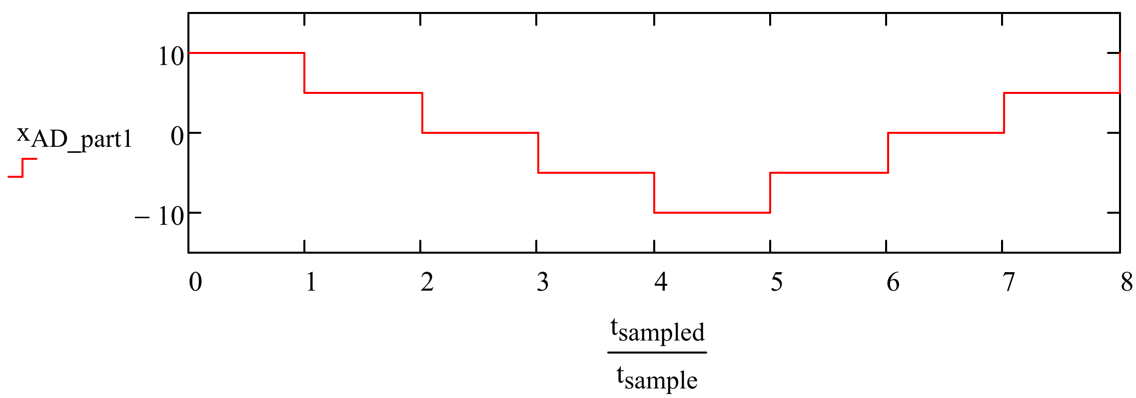
$x(8 \cdot t_{sample}) = 11.82$ A/D output $x_{AD\_part1_8} := 10A$

Now plot the i_{AD_part1} values versus time

$i := 0, 1 .. 9$ $t_{sampled_i} := i \cdot t_{sample}$

Figure 2

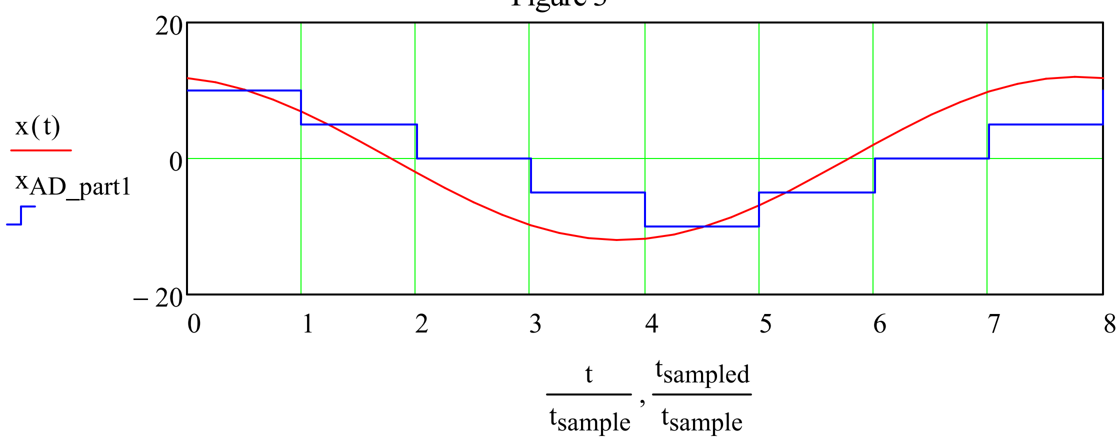
Sample and hold output



- Now compare this to the original signal

Figure 3

Notice that the low sampling rate causes a bit of a phase shift



- The delay in the blue waveform is more apparent because of the low sample rate and the low number of magnitude bits.
- A higher sampling rate will improve the waveform significantly. For example, the red line is plotted at 32 samples per cycle
- And has much much higher resolution numbers (so no rounding)
- But if all measurements have same delay, they cancel

2. Calculate the RMS current magnitudes for problem 4 parts a) and c) and compare with your original magnitudes.

$$X_{RMS} = \sqrt{\frac{1}{T}\cdot\int_0^T \left(i(t)^2\right) dt}$$

$$T_{10} := 8\cdot t_{sample}$$

- Original waveform:

$$X_{s\_RMS} := \sqrt{\frac{1}{T_{10}}\cdot\int_0^{T_{10}} \left(x(t)^2\right) dt} \qquad X_{s\_RMS} = 8.49$$

recall that originally we used the shortcut equation and had:

$$\left|X_{rms}\right| = 8.49$$

Now repeat this calculation over each of the sample and hold intervals of the waveform from the plot of Figure 2

$$A1 := \int_0^{1\cdot t_{sample}} \left(x_{AD\_part1_0}\right)^2 dt_{sample}$$

$$A2 := \int_{1\cdot t_{sample}}^{2\cdot t_{sample}} \left(x_{AD\_part1_1}\right)^2 dt_{sample}$$

$$A3 := \int_{2\cdot t_{sample}}^{3\cdot t_{sample}} \left(x_{AD\_part1_2}\right)^2 dt_{sample}$$

$$A4 := \int_{3 \cdot t_{sample}}^{4 \cdot t_{sample}} \left(x_{AD\_part1_3}\right)^2 \, dt_{sample}$$

$$A5 := \int_{4 \cdot t_{sample}}^{5 \cdot t_{sample}} \left(x_{AD\_part1_4}\right)^2 \, dt_{sample}$$

$$A6 := \int_{6 \cdot t_{sample}}^{7 \cdot t_{sample}} \left(x_{AD\_part1_5}\right)^2 \, dt_{sample}$$

$$A7 := \int_{6 \cdot t_{sample}}^{7 \cdot t_{sample}} \left(x_{AD\_part1_6}\right)^2 \, dt_{sample}$$

$$A8 := \int_{7 \cdot t_{sample}}^{8 \cdot t_{sample}} \left(x_{AD\_part1_7}\right)^2 \, dt_{sample}$$

$$X_{AD\_RMS} := \sqrt{\frac{1}{T_{10}} \cdot (A1 + A2 + A3 + A4 + A5 + A6 + A7 + A8)}$$

$$X_{AD\_RMS} = 6.12\,A$$

Quite a bit smaller than correct value due to quantization error.