Unbalanced Fault Analysis Examples

Define units: $MVA := 1000kW$ $\quad MVAR := MVA$

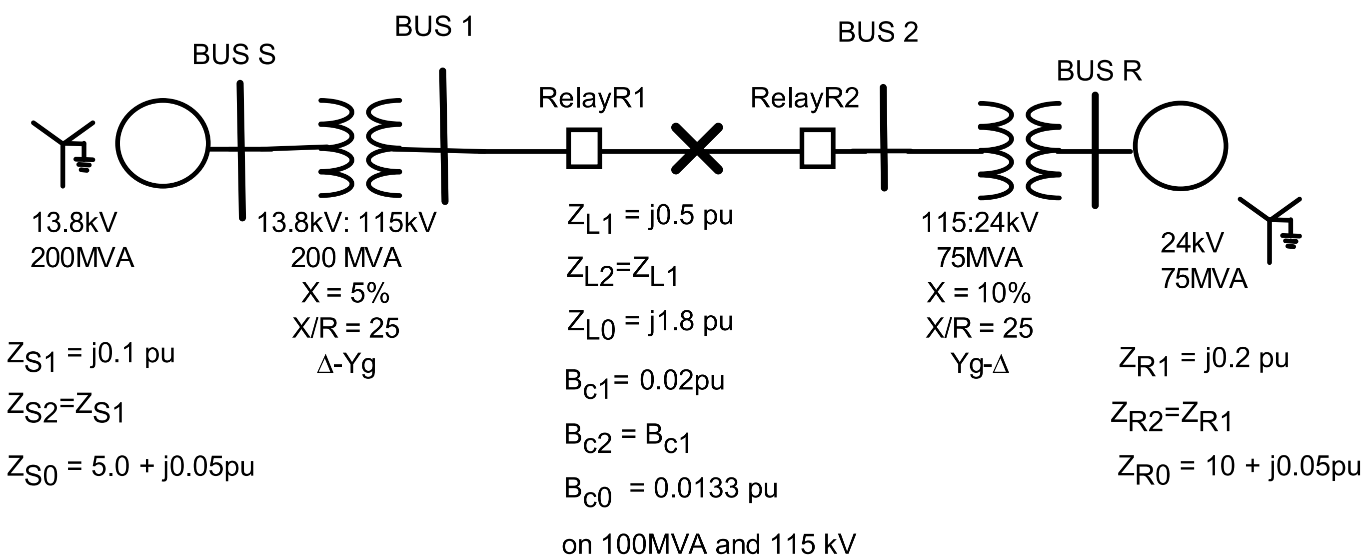
$MW := MVA$ $\quad pu := 1$

$a := 1 \cdot e^{j \cdot 120deg}$

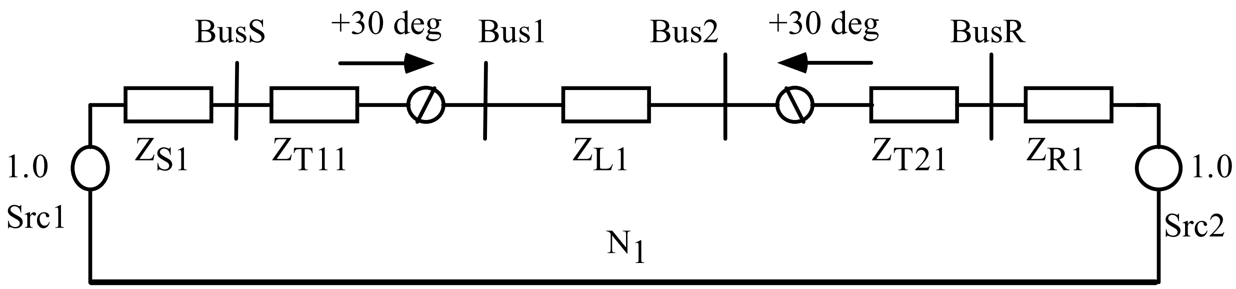
$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$$\angle(mag, ang) := mag \cdot \cos(ang \cdot deg) + j \cdot mag \cdot \sin(ang \cdot deg)$$

System Description:



Per Unit Change of Base Calculations (hidden, to view double click on the arrow)



- Positive sequence Ybus

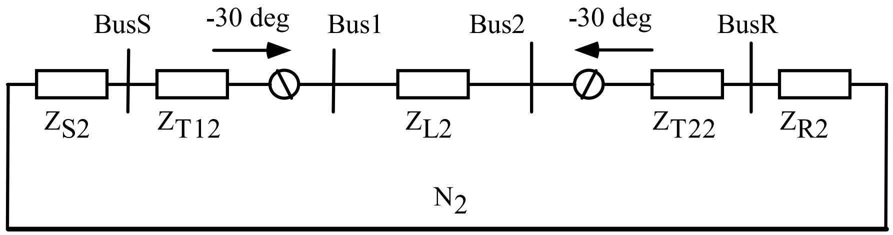
$$Y_1(M) := \begin{bmatrix} \frac{1}{jX_{G11}} + \frac{1}{Z_{T11}} & \frac{-1\cdot e^{-j\cdot 30deg}}{Z_{T11}} & 0 & 0 & 0 \\ \frac{-1\cdot e^{j\cdot 30deg}}{Z_{T11}} & \frac{1}{Z_{T11}} + \frac{1}{M\cdot Z_{L11}} & 0 & 0 & \frac{-1}{M\cdot Z_{L11}} \\ 0 & 0 & \frac{1}{Z_{T21}} + \frac{1}{(1-M)\cdot Z_{L11}} & \frac{-1\cdot e^{j\cdot 30deg}}{Z_{T21}} & \frac{-1}{(1-M)\cdot Z_{L11}} \\ 0 & 0 & \frac{-1\cdot e^{-j\cdot 30deg}}{Z_{T21}} & \frac{1}{Z_{T21}} + \frac{1}{jX_{G21}} & 0 \\ 0 & \frac{-1}{M\cdot Z_{L11}} & \frac{-1}{(1-M)\cdot Z_{L11}} & 0 & \frac{1}{M\cdot Z_{L11}} + \frac{1}{(1-M)\cdot Z_{L11}} \end{bmatrix}$$

$$Z_1(M) := Y_1(M)^{-1}$$

$V_S := 1.0pu$ $V_R := 1.0pu$ $V_f := 1.0pu$

Negative Sequence Ybus

- All of the impedance values are the same in this case.
- Only change is in the Δ-Y phase shifts.
- Equivalent circuit looks the same, but no sources.

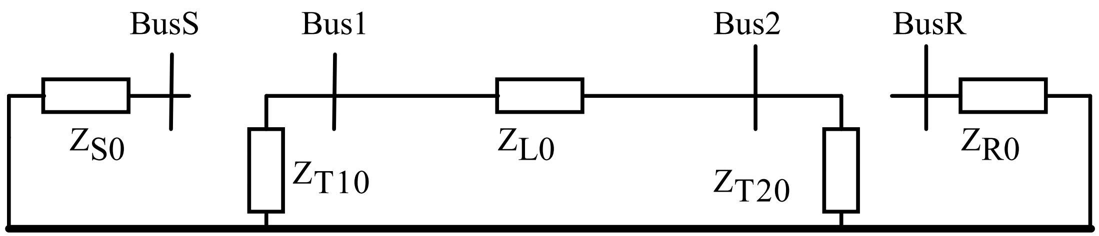


$$Y_2(M) := \begin{bmatrix} \frac{1}{jX_{G12}} + \frac{1}{Z_{T12}} & \frac{-1\cdot e^{j\cdot 30deg}}{Z_{T12}} & 0 & 0 & 0 \\ \frac{-1\cdot e^{-j\cdot 30deg}}{Z_{T12}} & \frac{1}{Z_{T12}} + \frac{1}{M\cdot Z_{L12}} & 0 & 0 & \frac{-1}{M\cdot Z_{L12}} \\ 0 & 0 & \frac{1}{Z_{T22}} + \frac{1}{(1-M)\cdot Z_{L12}} & \frac{-1\cdot e^{-j\cdot 30deg}}{Z_{T22}} & \frac{-1}{(1-M)\cdot Z_{L12}} \\ 0 & 0 & \frac{-1\cdot e^{j\cdot 30deg}}{Z_{T22}} & \frac{1}{Z_{T22}} + \frac{1}{jX_{G22}} & 0 \\ 0 & \frac{-1}{M\cdot Z_{L12}} & \frac{-1}{(1-M)\cdot Z_{L12}} & 0 & \frac{1}{M\cdot Z_{L12}} + \frac{1}{(1-M)\cdot Z_{L12}} \end{bmatrix}$$

$$Z_2(M) := Y_2(M)^{-1}$$

Zero Sequence Ybus

- Impedance values differ
- Model the transformer zero sequence connections
- Different looking equivalent circuit.



$$Y_0(M) := \begin{bmatrix} \frac{1}{Z_{G10}} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{Z_{T10}} + \frac{1}{M \cdot Z_{L10}} & 0 & 0 & \frac{-1}{M \cdot Z_{L10}} \\ 0 & 0 & \frac{1}{Z_{T20}} + \frac{1}{(1-M) \cdot Z_{L10}} & 0 & \frac{-1}{(1-M) \cdot Z_{L10}} \\ 0 & 0 & 0 & \frac{1}{Z_{G20}} & 0 \\ 0 & \frac{-1}{M \cdot Z_{L10}} & \frac{-1}{(1-M) \cdot Z_{L10}} & 0 & \frac{1}{M \cdot Z_{L10}} + \frac{1}{(1-M) \cdot Z_{L10}} \end{bmatrix}$$

$$Z_0(M) := Y_0(M)^{-1}$$

SLG Fault:

$$I_{0\_SLG}(M,R_f) := \frac{V_f}{Z_1(M)_{4,4} + Z_2(M)_{4,4} + Z_0(M)_{4,4} + 3\cdot R_f}$$

$$\left|I_{0\_SLG}(0.5,0)\right| = 1.0853$$

$$\arg\left(I_{0\_SLG}(0.5,0)\right) = -90\cdot deg$$

$$I_{1\_SLG}(M,R_f) := I_{0\_SLG}(M,R_f)$$

$$I_{2\_SLG}(M,R_f) := I_{0\_SLG}(M,R_f)$$

$$I_{ABC\_SLG}(M,R_f) := A_{012}\cdot\begin{pmatrix} I_{0\_SLG}(M,R_f) \\ I_{1\_SLG}(M,R_f) \\ I_{2\_SLG}(M,R_f) \end{pmatrix}$$

$$\overrightarrow{\left|I_{ABC\_SLG}(0.5,0)\right|} = \begin{pmatrix} 3.2558 \\ 0 \\ 0 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_SLG}(0.5,0)\right)} = \begin{pmatrix} -90 \\ 18.4349 \\ 18.4349 \end{pmatrix}\cdot deg$$

- Angles meaningless when magnitude is 0

- Now find voltages in each sequence component)

$$\Delta V1_{SLG}(M,R_f) := Z_1(M)\cdot\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_{1\_SLG}(M,R_f) \end{pmatrix}$$

$$\overrightarrow{\left|\Delta V1_{SLG}(0.5,0)\right|} = \begin{pmatrix} 0.0362 \\ 0.0543 \\ 0.1447 \\ 0.0965 \\ 0.2351 \end{pmatrix} \qquad \overrightarrow{\arg\left(\Delta V1_{SLG}(0.5,0)\right)} = \begin{pmatrix} 150 \\ 180 \\ -180 \\ 150 \\ 180 \end{pmatrix}\cdot deg$$

- Now when we find V1 we need to *include the transformer phase shift* in the prefault voltages

$$V1_{SLG}(M,R_f) := \begin{pmatrix} 1.0e^{-j\cdot 30deg} \\ 1.0 \\ 1.0 \\ 1.0\cdot e^{-j\cdot 30deg} \\ 1.0 \end{pmatrix} + \Delta V1_{SLG}(M,R_f) \qquad \overrightarrow{\left|V1_{SLG}(0.5,0)\right|} = \begin{pmatrix} 0.9638 \\ 0.9457 \\ 0.8553 \\ 0.9035 \\ 0.7649 \end{pmatrix} \qquad \overrightarrow{\arg\left(V1_{SLG}(0.5,0)\right)} = \begin{pmatrix} -30 \\ 0 \\ 0 \\ -30 \\ 0 \end{pmatrix}\cdot deg$$

$$\Delta V2_{SLG}(M,R_f) := Z_2(M)\cdot\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_{2\_SLG}(M,R_f) \end{pmatrix} \qquad \overrightarrow{\left|\Delta V2_{SLG}(0.5,0)\right|} = \begin{pmatrix} 0.0362 \\ 0.0543 \\ 0.1447 \\ 0.0965 \\ 0.2351 \end{pmatrix} \qquad \overrightarrow{\arg\left(\Delta V2_{SLG}(0.5,0)\right)} = \begin{pmatrix} -150 \\ -180 \\ -180 \\ -150 \\ -180 \end{pmatrix}\cdot deg$$

Note the -30 degree shift...

$$V2_{SLG}(M,R_f) := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \Delta V2_{SLG}(M,R_f)$$

- No prefault voltage, so enter 0.

$$\Delta V0_{SLG}(M,R_f) := Z_0(M)\cdot\begin{pmatrix}0\\0\\0\\0\\-I_{0\_SLG}(M,R_f)\end{pmatrix} \quad \Delta V0_{SLG}(0.5,0) = \begin{pmatrix}0\\-0.0143\\-0.0683\\0\\-0.5297\end{pmatrix} \quad V0_{SLG}(M,R_f) := \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix} + \Delta V0_{SLG}(M,R_f)$$

Again, no prefault voltage

- ABC Voltages at Bus 1

$$V_{ABCB1.SLG}(M,R_f) := A_{012}\cdot\begin{pmatrix}V0_{SLG}(M,R_f)_1\\V1_{SLG}(M,R_f)_1\\V2_{SLG}(M,R_f)_1\end{pmatrix} \quad \left|\overrightarrow{V_{ABCB1.SLG}(0.5,0)}\right| = \begin{pmatrix}0.8772\\0.9806\\0.9806\end{pmatrix}\cdot pu \quad arg\left(V_{ABCB1.SLG}(0.5,0)\right) = \begin{pmatrix}0\\-117.9783\\117.9783\end{pmatrix}\cdot deg$$

- ABC Voltages at Bus S

$$V_{ABCBS.SLG}(M,R_f) := A_{012}\cdot\begin{pmatrix}V0_{SLG}(M,R_f)_0\\V1_{SLG}(M,R_f)_0\\V2_{SLG}(M,R_f)_0\end{pmatrix} \quad \left|\overrightarrow{V_{ABCBS.SLG}(0.5,0)}\right| = \begin{pmatrix}0.9463\\0.9463\\1\end{pmatrix}\cdot pu \quad arg\left(V_{ABCBS.SLG}(0.5,0)\right) = \begin{pmatrix}-31.8973\\-148.1027\\90\end{pmatrix}\cdot deg$$

- Branch currents
  - Relay 1 currents

$$I_{1\_B1.SLG}(M,R_f) := \frac{V1_{SLG}(M,R_f)_1 - V1_{SLG}(M,R_f)_4}{M \cdot Z_{L11}} \qquad \left|I_{1\_B1.SLG}(0.5,0)\right| = 0.7235 \qquad \arg\left(I_{1\_B1.SLG}(0.5,0)\right) = -90 \cdot deg$$

$$I_{2\_B1.SLG}(M,R_f) := \frac{V2_{SLG}(M,R_f)_1 - V2_{SLG}(M,R_f)_4}{M \cdot Z_{L12}} \qquad \left|I_{2\_B1.SLG}(0.5,0)\right| = 0.7235 \qquad \arg\left(I_{2\_B1.SLG}(0.5,0)\right) = -90 \cdot deg$$

$$I_{0\_B1.SLG}(M,R_f) := \frac{V0_{SLG}(M,R_f)_1 - V0_{SLG}(M,R_f)_4}{M \cdot Z_{L10}} \qquad \left|I_{0\_B1.SLG}(0.5,0)\right| = 0.5727 \qquad \arg\left(I_{0\_B1.SLG}(0.5,0)\right) = -90 \cdot deg$$

$$I_{ABC\_B1.SLG}(M,R_f) := A_{012} \cdot \begin{pmatrix} I_{0\_B1.SLG}(M,R_f) \\ I_{1\_B1.SLG}(M,R_f) \\ I_{2\_B1.SLG}(M,R_f) \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_B1.SLG}(0.5,0)\right|} = \begin{pmatrix} 2.0197 \\ 0.1509 \\ 0.1509 \end{pmatrix} \cdot pu \qquad \arg\left(I_{ABC\_B1.SLG}(0.5,0)\right) = \begin{pmatrix} -90 \\ 90 \\ 90 \end{pmatrix} \cdot deg$$

- LVside of transformer (Bus S)

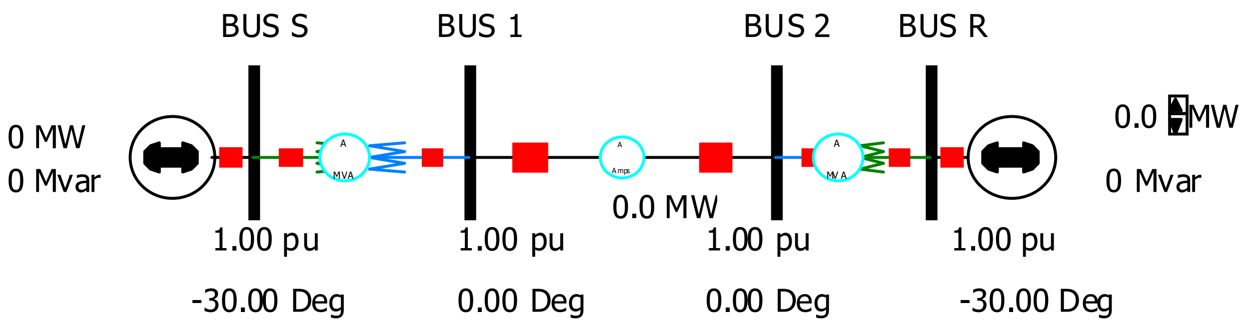
$$I_{1\_BS.SLG}(M,R_f) := \frac{V_S \cdot e^{-j \cdot 30deg} - V1_{SLG}(M,R_f)_0}{jX_{G11}} \qquad \left|I_{1\_BS.SLG}(0.5,0)\right| = 0.7235 \cdot pu \qquad \arg\left(I_{1\_BS.SLG}(0.5,0)\right) = -120 \cdot deg$$

$$I_{2\_BS.SLG}(M,R_f) := \frac{0 - V2_{SLG}(M,R_f)_0}{jX_{G12}} \qquad \left|I_{2\_BS.SLG}(0.5,0)\right| = 0.7235 \cdot pu \qquad \arg\left(I_{2\_BS.SLG}(0.5,0)\right) = -60 \cdot deg$$

$$I_{0\_BS.SLG} := 0$$

$$I_{ABC\_BS.SLG}(M,R_f) := A_{012} \cdot \begin{pmatrix} I_{0\_BS.SLG} \\ I_{1\_BS.SLG}(M,R_f) \\ I_{2\_BS.SLG}(M,R_f) \end{pmatrix} \quad \overrightarrow{\left|I_{ABC\_BS.SLG}(0.5,0)\right|} = \begin{pmatrix} 1.2532 \\ 1.2532 \\ 0 \end{pmatrix} \cdot pu \quad \arg\left(I_{ABC\_BS.SLG}(0.5,0)\right) = \begin{pmatrix} -90 \\ 90 \\ 38.6598 \end{pmatrix} \cdot deg$$

- **Circuit implementation in Powerworld**



- **Powerworld Results**

$I_{AFSLG} := 3.256 \angle (-90deg)$

Fault Data - Buses						
Name	Phase Volt A	Phase Ang A	Phase Volt B	Phase Ang B	Phase Volt C	Phase Ang C
BUS S	0.94625	-31.9	0.94625	-148.1	1	90
BUS 1	0.87716	0	0.98064	-117.98	0.98064	117.98
BUS 2	0.64224	0	0.96409	-116.07	0.96409	116.07
BUS R	0.85937	-35.58	0.85937	-144.42	1	90
FaultPt	0	0	1.1753	-132.54	1.1753	132.54

Fault Data - Lines							
From Name	To Name	Phase Cur A From	Phase Ang A From	Phase Cur B From	Phase Ang B From	Phase Cur C From	Phase Ang C From
BUS S	BUS 1	1.25318	-90	1.25318	90	0	0
BUS 1	BUS 2	0	0	0	0	0	0
BUS 1	FaultPt	2.0197	-90	0.15086	90	0.15086	90
BUS R	BUS 2	0.62659	-90	0.62659	90	0	0
FaultPt	BUS 2	1.23615	90	0.15086	90	0.15086	90

Good match with calculations above.

LL Fault:

$$I_{1\_LL}(M,R_f) := \frac{V_f}{Z_1(M)_{4,4} + Z_2(M)_{4,4} + R_f}$$

$$\left|I_{1\_LL}(0.5,0)\right| = 2.3077$$

$$\arg\left(I_{1\_LL}(0.5,0)\right) = -90\cdot deg$$

$$I_{2\_LL}(M,R_f) := -I_{1\_LL}(M,R_f)$$

$I_{0\_LL} := 10^{-15}$ Approximately zero, works better with MathCAD.

$$I_{ABC\_LL}(M,R_f) := A_{012}\cdot\begin{pmatrix} I_{0\_LL} \\ I_{1\_LL}(M,R_f) \\ I_{2\_LL}(M,R_f) \end{pmatrix}$$

$$\overrightarrow{\left|I_{ABC\_LL}(0.5,0)\right|} = \begin{pmatrix} 0 \\ 3.997 \\ 3.997 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_LL}(0.5,0)\right)} = \begin{pmatrix} 0 \\ 180 \\ 0 \end{pmatrix}\cdot deg$$

Note: $\dfrac{I_{ABC\_LL}(0.5,0)_1}{I_{1\_LL}(0.5,0)} = -1.7321i$

- Now find voltages in each sequence component)

$$\Delta V1_{LL}(M,R_f) := Z_1(M)\cdot\begin{pmatrix}0\\0\\0\\0\\-I_{1\_LL}(M,R_f)\end{pmatrix} \qquad \overrightarrow{\left|\Delta V1_{LL}(0.5,0)\right|} = \begin{pmatrix}0.0769\\0.1154\\0.3077\\0.2051\\0.5\end{pmatrix} \qquad \overrightarrow{\arg\left(\Delta V1_{LL}(0.5,0)\right)} = \begin{pmatrix}150\\180\\-180\\150\\180\end{pmatrix}\cdot deg$$

- Again, when we find V1 we need to *include the transformer phase shift* in the prefault voltages

$$V1_{LL}(M,R_f) := \begin{pmatrix}1.0e^{-j\cdot 30deg}\\1.0\\1.0\\1.0\cdot e^{-j\cdot 30deg}\\1.0\end{pmatrix} + \Delta V1_{LL}(M,R_f) \qquad \overrightarrow{\left|V1_{LL}(0.5,0)\right|} = \begin{pmatrix}0.9231\\0.8846\\0.6923\\0.7949\\0.5\end{pmatrix} \qquad \overrightarrow{\arg\left(V1_{LL}(0.5,0)\right)} = \begin{pmatrix}-30\\0\\0\\-30\\0\end{pmatrix}\cdot deg$$

$$\Delta V2_{LL}(M,R_f) := Z_2(M)\cdot\begin{pmatrix}0\\0\\0\\0\\-I_{2\_LL}(M,R_f)\end{pmatrix}$$

$$\overrightarrow{\left|\Delta V2_{LL}(0.5,0)\right|} = \begin{pmatrix}0.0769\\0.1154\\0.3077\\0.2051\\0.5\end{pmatrix}$$

$$\overrightarrow{\arg\left(\Delta V2_{LL}(0.5,0)\right)} = \begin{pmatrix}30\\0\\0\\30\\0\end{pmatrix}\cdot deg$$

Note the -30 degree shift...

$$V2_{LL}(M,R_f) := \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix} + \Delta V2_{LL}(M,R_f)$$

$$\Delta V0_{LL}(M,R_f) := Z_0(M)\cdot\begin{pmatrix}0\\0\\0\\0\\-I_{0\_LL}\end{pmatrix}$$

$$\Delta V0_{LL}(0.5,0) = \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix}$$

$$V0_{LL}(M,R_f) := \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix} + \Delta V0_{LL}(M,R_f)$$

- ABC Voltages at Bus 1

$$V_{ABCB1.LL}(M,R_f) := A_{012}\cdot\begin{pmatrix} V0_{LL}(M,R_f)_1 \\ V1_{LL}(M,R_f)_1 \\ V2_{LL}(M,R_f)_1 \end{pmatrix} \qquad \overrightarrow{\left|V_{ABCB1.LL}(0.5,0)\right|} = \begin{pmatrix} 1 \\ 0.8329 \\ 0.8329 \end{pmatrix}\cdot pu \qquad \arg\left(V_{ABCB1.LL}(0.5,0)\right) = \begin{pmatrix} 0 \\ -126.8903 \\ 126.8903 \end{pmatrix}\cdot deg$$

- ABC Voltages at Bus S

$$V_{ABCBS.LL}(M,R_f) := A_{012}\cdot\begin{pmatrix} V0_{LL}(M,R_f)_0 \\ V1_{LL}(M,R_f)_0 \\ V2_{LL}(M,R_f)_0 \end{pmatrix} \qquad \overrightarrow{\left|V_{ABCBS.LL}(0.5,0)\right|} = \begin{pmatrix} 0.9638 \\ 0.9638 \\ 0.8462 \end{pmatrix}\cdot pu \qquad \arg\left(V_{ABCBS.LL}(0.5,0)\right) = \begin{pmatrix} -26.0368 \\ -153.9632 \\ 90 \end{pmatrix}\cdot deg$$

- Branch currents

- Relay 1 currents

$$I_{1\_B1.LL}(M,R_f) := \frac{V1_{LL}(M,R_f)_1 - V1_{LL}(M,R_f)_4}{M\cdot Z_{L11}} \qquad \left|I_{1\_B1.LL}(0.5,0)\right| = 1.5385 \qquad \arg\left(I_{1\_B1.LL}(0.5,0)\right) = -90\cdot deg$$

$$I_{2\_B1.LL}(M,R_f) := \frac{V2_{LL}(M,R_f)_1 - V2_{LL}(M,R_f)_4}{M\cdot Z_{L12}} \qquad \left|I_{2\_B1.LL}(0.5,0)\right| = 1.5385 \qquad \arg\left(I_{2\_B1.LL}(0.5,0)\right) = 90\cdot deg$$

$$I_{0\_B1.LL}(M,R_f) := \frac{V0_{LL}(M,R_f)_1 - V0_{LL}(M,R_f)_4}{M \cdot Z_{L10}}$$ $\left|I_{0\_B1.LL}(0.5,0)\right| = 0$ $\arg\left(I_{0\_B1.LL}(0.5,0)\right) = 0 \cdot deg$

$$I_{ABC\_B1.LL}(M,R_f) := A_{012} \cdot \begin{pmatrix} I_{0\_B1.LL}(M,R_f) \\ I_{1\_B1.LL}(M,R_f) \\ I_{2\_B1.LL}(M,R_f) \end{pmatrix}$$ $\overrightarrow{\left|I_{ABC\_B1.LL}(0.5,0)\right|} = \begin{pmatrix} 0 \\ 2.6647 \\ 2.6647 \end{pmatrix} \cdot pu$ $\arg\left(I_{ABC\_B1.LL}(0.5,0)\right) = \begin{pmatrix} 21.2839 \\ 180 \\ 0 \end{pmatrix} \cdot deg$

- LVside of transformer (Bus S)

$$I_{1\_BS.LL}(M,R_f) := \frac{V_S \cdot e^{-j \cdot 30deg} - V1_{LL}(M,R_f)_0}{jX_{G11}}$$ $\left|I_{1\_BS.LL}(0.5,0)\right| = 1.5385 \cdot pu$ $\arg\left(I_{1\_BS.LL}(0.5,0)\right) = -120 \cdot deg$

$$I_{2\_BS.LL}(M,R_f) := \frac{0 - V2_{LL}(M,R_f)_0}{jX_{G12}}$$ $\left|I_{2\_BS.LL}(0.5,0)\right| = 1.5385 \cdot pu$ $\arg\left(I_{2\_BS.LL}(0.5,0)\right) = 120 \cdot deg$

$$I_{0\_BS.LL} := 0$$

$$I_{ABC\_BS.LL}(M,R_f) := A_{012} \cdot \begin{pmatrix} I_{0\_BS.LL} \\ I_{1\_BS.LL}(M,R_f) \\ I_{2\_BS.LL}(M,R_f) \end{pmatrix}$$ $\overrightarrow{\left|I_{ABC\_BS.LL}(0.5,0)\right|} = \begin{pmatrix} 1.5385 \\ 1.5385 \\ 3.0769 \end{pmatrix} \cdot pu$ $\arg\left(I_{ABC\_BS.LL}(0.5,0)\right) = \begin{pmatrix} 180 \\ 180 \\ 0 \end{pmatrix} \cdot deg$

- **Powerworld Results**

$I_{BFLL} := 3.997 \angle (180\text{deg})$

Fault Data - Buses						
Name	Phase Volt A	Phase Ang A	Phase Volt B	Phase Ang B	Phase Volt C	Phase Ang C
BUS S	0.96384	-26.04	0.96384	-153.96	0.84615	90
BUS 1	1	0	0.83294	-126.89	0.83294	126.89
BUS 2	1	0	0.60079	-146.33	0.60079	146.33
BUS R	0.91485	-18.8	0.91485	-161.2	0.58974	90
FaultPt	1	0	0.5	180	0.5	-180

Fault Data - Lines							
From Name	To Name	Phase Cur A From	Phase Ang A From	Phase Cur B From	Phase Ang B From	Phase Cur C From	Phase Ang C From
BUS S	BUS 1	1.53846	-180	1.53846	-180	3.07692	0
BUS 1	BUS 2	0	0	0	0	0	0
BUS 1	FaultPt	0	0	2.66469	-180	2.66469	0
BUS R	BUS 2	0.76923	-180	0.76923	-180	1.53846	0
FaultPt	BUS 2	0	0	1.33235	0	1.33235	-180

DLG Fault:

$$I_{1\_DLG}(M,R_f) := \frac{V_f}{\left(Z_1(M)_{4,4}\right) + \left[\frac{1}{Z_2(M)_{4,4}} + \frac{1}{\left(Z_0(M)_{4,4} + 3\cdot R_f\right)}\right]^{-1}}$$

$$\left|I_{1\_DLG}(0.5,0)\right| = 2.7269 \qquad \arg\left(I_{1\_DLG}(0.5,0)\right) = -90\cdot deg$$

Current dividers for I_0 and I_2

$$I_{0\_DLG}(M,R_f) := -I_{1\_DLG}(M,R_f)\cdot\left[\frac{Z_2(M)_{4,4}}{Z_2(M)_{4,4} + \left(Z_0(M)_{4,4} + 3\cdot R_f\right)}\right]$$

$$I_{2\_DLG}(M,R_f) := -I_{1\_DLG}(M,R_f)\cdot\left[\frac{Z_0(M)_{4,4} + 3\cdot R_f}{Z_2(M)_{4,4} + \left(Z_0(M)_{4,4} + 3\cdot R_f\right)}\right]$$

$$I_{ABC\_DLG}(M,R_f) := A_{012}\cdot\begin{pmatrix} I_{0\_DLG}(M,R_f) \\ I_{1\_DLG}(M,R_f) \\ I_{2\_DLG}(M,R_f) \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_DLG}(0.5,0)\right|} = \begin{pmatrix} 0 \\ 4.1902 \\ 4.1902 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_DLG}(0.5,0)\right)} = \begin{pmatrix} -90 \\ 162.5359 \\ 17.4641 \end{pmatrix}\cdot deg$$

- Angles meaningless when magnitude is 0

- Now find voltages in each sequence component)

$$\Delta V1_{DLG}\left(M,R_f\right) := Z_1(M)\cdot\begin{pmatrix}0\\0\\0\\0\\-I_{1\_DLG}\left(M,R_f\right)\end{pmatrix} \qquad \overrightarrow{\left|\Delta V1_{DLG}(0.5,0)\right|} = \begin{pmatrix}0.0909\\0.1363\\0.3636\\0.2424\\0.5908\end{pmatrix} \qquad \overrightarrow{\arg\left(\Delta V1_{DLG}(0.5,0)\right)} = \begin{pmatrix}150\\180\\-180\\150\\180\end{pmatrix}\cdot deg$$

- Now when we find V1 we need to *include the transformer phase shift* in the prefault voltages

$$V1_{DLG}\left(M,R_f\right) := \begin{pmatrix}1.0e^{-j\cdot 30deg}\\1.0\\1.0\\1.0\cdot e^{-j\cdot 30deg}\\1.0\end{pmatrix} + \Delta V1_{DLG}\left(M,R_f\right) \qquad \overrightarrow{\left|V1_{DLG}(0.5,0)\right|} = \begin{pmatrix}0.9091\\0.8637\\0.6364\\0.7576\\0.4092\end{pmatrix} \qquad \overrightarrow{\arg\left(V1_{DLG}(0.5,0)\right)} = \begin{pmatrix}-30\\0\\0\\-30\\0\end{pmatrix}\cdot deg$$

$$\Delta V2_{DLG}(M,R_f) := Z_2(M)\cdot\begin{pmatrix}0\\0\\0\\0\\-I_{2\_DLG}(M,R_f)\end{pmatrix}$$

$$\overrightarrow{\left|\Delta V2_{DLG}(0.5,0)\right|} = \begin{pmatrix}0.063\\0.0944\\0.2518\\0.1679\\0.4092\end{pmatrix}$$

$$\overrightarrow{\arg\left(\Delta V2_{DLG}(0.5,0)\right)} = \begin{pmatrix}30\\0\\0\\30\\0\end{pmatrix}\cdot deg$$

$$V2_{DLG}(M,R_f) := \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix} + \Delta V2_{DLG}(M,R_f)$$

- No prefault voltage, so enter 0.

$$\Delta V0_{DLG}(M,R_f) := Z_0(M)\cdot\begin{pmatrix}0\\0\\0\\0\\-I_{0\_DLG}(M,R_f)\end{pmatrix}$$

$$\Delta V0_{DLG}(0.5,0) = \begin{pmatrix}0\\0.0111\\0.0528\\0\\0.4092\end{pmatrix}$$

$$V0_{DLG}(M,R_f) := \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix} + \Delta V0_{DLG}(M,R_f)$$

Again, no prefault voltage

- ABC Voltages at Bus 1

$$V_{ABCB1.DLG}(M,R_f) := A_{012}\cdot\begin{pmatrix} V0_{DLG}(M,R_f)_1 \\ V1_{DLG}(M,R_f)_1 \\ V2_{DLG}(M,R_f)_1 \end{pmatrix}$$

$$\overrightarrow{\left|V_{ABCB1.DLG}(0.5,0)\right|} = \begin{pmatrix} 0.9691 \\ 0.8141 \\ 0.8141 \end{pmatrix}\cdot pu \quad \arg\left(V_{ABCB1.DLG}(0.5,0)\right) = \begin{pmatrix} 0 \\ -125.0878 \\ 125.0878 \end{pmatrix}\cdot deg$$

- ABC Voltages at Bus S

$$V_{ABCBS.DLG}(M,R_f) := A_{012}\cdot\begin{pmatrix} V0_{DLG}(M,R_f)_0 \\ V1_{DLG}(M,R_f)_0 \\ V2_{DLG}(M,R_f)_0 \end{pmatrix}$$

$$\overrightarrow{\left|V_{ABCBS.DLG}(0.5,0)\right|} = \begin{pmatrix} 0.9422 \\ 0.9422 \\ 0.8462 \end{pmatrix}\cdot pu \quad \arg\left(V_{ABCBS.DLG}(0.5,0)\right) = \begin{pmatrix} -26.6828 \\ -153.3172 \\ 90 \end{pmatrix}\cdot deg$$

- Branch currents

  - Relay 1 currents

$$I_{1\_B1.DLG}(M,R_f) := \frac{V1_{DLG}(M,R_f)_1 - V1_{DLG}(M,R_f)_4}{M\cdot Z_{L11}} \qquad \left|I_{1\_B1.DLG}(0.5,0)\right| = 1.8179 \qquad \arg\left(I_{1\_B1.DLG}(0.5,0)\right) = -90\cdot deg$$

$$I_{2\_B1.DLG}(M,R_f) := \frac{V2_{DLG}(M,R_f)_1 - V2_{DLG}(M,R_f)_4}{M\cdot Z_{L12}} \qquad \left|I_{2\_B1.DLG}(0.5,0)\right| = 1.259 \qquad \arg\left(I_{2\_B1.DLG}(0.5,0)\right) = 90\cdot deg$$

$$I_{0\_B1.DLG}(M,R_f) := \frac{V0_{DLG}(M,R_f)_1 - V0_{DLG}(M,R_f)_4}{M \cdot Z_{L10}} \qquad |I_{0\_B1.DLG}(0.5,0)| = 0.4424 \qquad \arg(I_{0\_B1.DLG}(0.5,0)) = 90 \cdot deg$$

$$I_{ABC\_B1.DLG}(M,R_f) := A_{012} \cdot \begin{pmatrix} I_{0\_B1.DLG}(M,R_f) \\ I_{1\_B1.DLG}(M,R_f) \\ I_{2\_B1.DLG}(M,R_f) \end{pmatrix} \qquad \overrightarrow{|I_{ABC\_B1.DLG}(0.5,0)|} = \begin{pmatrix} 0.1165 \\ 2.7607 \\ 2.7607 \end{pmatrix} \cdot pu \qquad \arg(I_{ABC\_B1.DLG}(0.5,0)) = \begin{pmatrix} -90 \\ 164.8436 \\ 15.1564 \end{pmatrix} \cdot deg$$

- LVside of transformer (Bus S)

$$I_{1\_BS.DLG}(M,R_f) := \frac{V_S \cdot e^{-j \cdot 30deg} - V1_{DLG}(M,R_f)_0}{jX_{G11}} \qquad |I_{1\_BS.DLG}(0.5,0)| = 1.8179 \cdot pu \qquad \arg(I_{1\_BS.DLG}(0.5,0)) = -120 \cdot deg$$

$$I_{2\_BS.DLG}(M,R_f) := \frac{0 - V2_{DLG}(M,R_f)_0}{jX_{G12}} \qquad |I_{2\_BS.DLG}(0.5,0)| = 1.259 \cdot pu \qquad \arg(I_{2\_BS.DLG}(0.5,0)) = 120 \cdot deg$$

$$I_{0\_BS.DLG} := 0$$

$$I_{ABC\_BS.DLG}(M,R_f) := A_{012} \cdot \begin{pmatrix} I_{0\_BS.DLG} \\ I_{1\_BS.DLG}(M,R_f) \\ I_{2\_BS.DLG}(M,R_f) \end{pmatrix} \qquad \overrightarrow{|I_{ABC\_BS.DLG}(0.5,0)|} = \begin{pmatrix} 1.6128 \\ 1.6128 \\ 3.0769 \end{pmatrix} \cdot pu \qquad \arg(I_{ABC\_BS.DLG}(0.5,0)) = \begin{pmatrix} -162.5359 \\ 162.5359 \\ 0 \end{pmatrix} \cdot deg$$

- **Powerworld Results**

Powerworld does not give a useful number for the fault current.

Fault Data - Buses						
Name	Phase Volt A	Phase Ang A	Phase Volt B	Phase Ang B	Phase Volt C	Phase Ang C
BUS S	0.94216	-26.68	0.94216	-153.32	0.84615	90
BUS 1	0.96914	0	0.81412	-125.09	0.81412	125.09
BUS 2	0.94102	0	0.51388	-139.6	0.51388	139.6
BUS R	0.85401	-20.2	0.85401	-159.8	0.58974	90
FaultPt	1.22754	0	0	0	0	0

Fault Data - Lines							
From Name	To Name	Phase Cur A From	Phase Ang A From	Phase Cur B From	Phase Ang B From	Phase Cur C From	Phase Ang C From
BUS S	BUS 1	1.6128	-162.54	1.6128	162.54	3.07692	0
BUS 1	BUS 2	0	0	0	0	0	0
BUS 1	FaultPt	0.11653	-90	2.76072	164.84	2.76072	15.16
BUS R	BUS 2	0.8064	-162.54	0.8064	162.54	1.53846	0
FaultPt	BUS 2	0.11653	-90	1.43601	-21.9	1.43601	-158.1