ECE 523
Symmetrical Components

Session 10

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$\rightarrow Z[i,j]$

ORIGIN := 0

$Z_{i,j}$

$Z_{1\text{thev}}(\underline{0.5}) = 0.2167i\,\text{pu}$ $\qquad Z_1(\underline{0.5})_{4,4} = 0.2167i\,\text{pu}$

- The diagonal elements on the bus impedance matrix are the Thevenin impedance looking out from that node/bus in the system

Three Phase Fault

prefault volt at fault location $1\angle 0$

STEP1

$$I_{f3ph}(M) := \frac{V_f}{Z_1(M)_{\underline{4,4}}} \qquad \left|I_{f3ph}(0.5)\right| = 4.6154\cdot\text{pu} \qquad \arg\left(I_{f3ph}(0.5)\right) = -90\cdot\text{deg}$$

m=0.5

From above: $\quad I_{3ph}(0.5) = -4.6154i$

STEP2

$$\underline{\Delta V1(M)} := Z_1(M)\cdot\begin{array}{cl}\left(\begin{array}{c}0\\0\\0\\0\\-I_{f3ph}(M)\end{array}\right) & \begin{array}{l}\text{Bus 1}\\\text{Bus 2}\\\text{Bus 3}\\\text{Bus 4}\\\text{Bus F}\end{array}\end{array} \qquad \Delta V1(0.5) = \begin{array}{cl}\left(\begin{array}{c}-0.1332+0.0769i\\-0.2308\\-0.6154\\-0.3553+0.2051i\\-1\end{array}\right)\cdot\text{pu} & \begin{array}{l}\text{Bus 1}\\\text{Bus 2}\\\text{Bus}\\\text{Bus 4}\\\text{Bus F}\end{array}\end{array}$$

fault current injected

$$\overrightarrow{\left|\Delta V1(0.5)\right|} = \left(\begin{array}{c}0.1538\\0.2308\\0.6154\\0.4103\\1\end{array}\right)\cdot\text{pu} \qquad \overrightarrow{\arg(\Delta V1(0.5))} = \left(\begin{array}{c}150\\180\\-180\\150\\-180\end{array}\right)\cdot\text{deg}$$

$\rightarrow$

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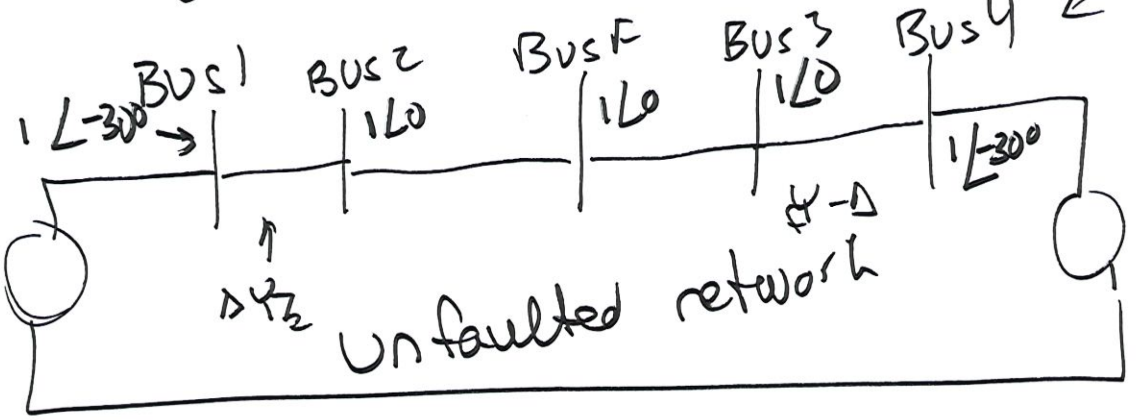
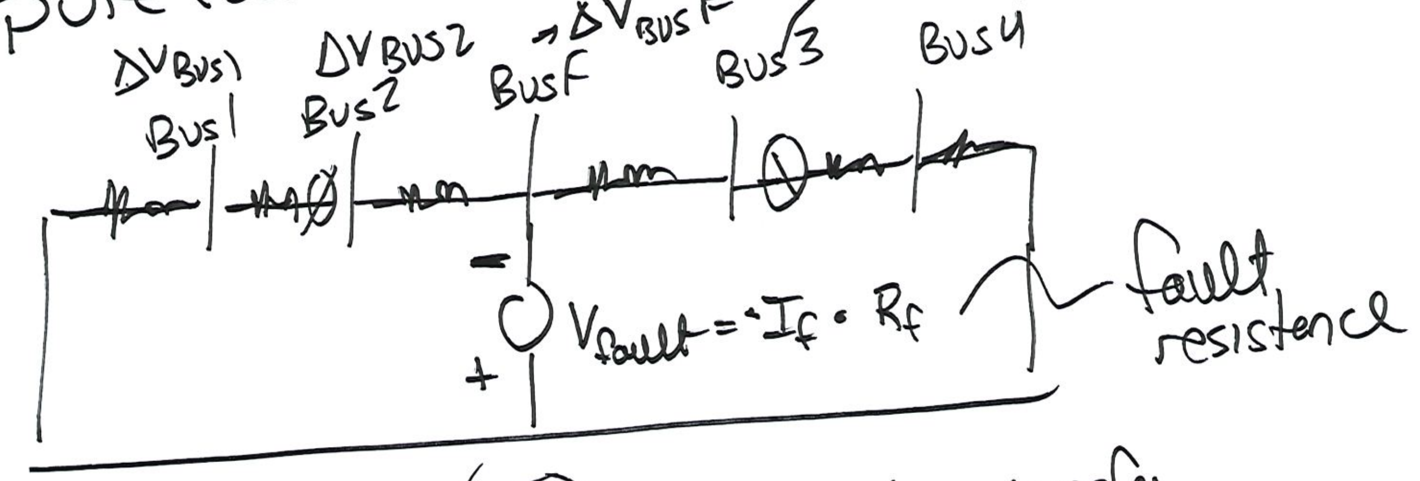
Prefault (unfaulted network)



← prefault voltage

(No load current)

Pure faulted network



$\bar{V}_{1new} = \bar{V}_{1prefault} + \boxed{\Delta V_1}$ → Depend on transfer impedances (off diagonal Z_{BUS})

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Easiest Approach is to Create Ybus Matrix Modified for Fault Calculations: $\}$ Z_{BUS} Matrix Approach

Transformer Phase Shift:

$$Y_1(M) := \begin{array}{cccccc} \text{BUS1} & \text{BUS2} & \text{BUS3} & \text{BUS4} & \text{BUSF} & \\ \left[\begin{array}{c} \frac{1}{jX_{G11}}+\frac{1}{Z_{T11}} \\ \frac{-1\cdot e^{j\cdot 30deg}}{Z_{T11}} \\ 0 \\ 0 \\ 0 \end{array}\right. & \begin{array}{c} \frac{-1\cdot e^{-j\cdot 30deg}}{Z_{T11}} \\ \frac{1}{Z_{T11}}+\frac{1}{M\cdot Z_{L11}} \\ 0 \\ 0 \\ \frac{-1}{M\cdot Z_{L11}} \end{array} & \begin{array}{c} 0 \\ 0 \\ \frac{1}{Z_{T21}}+\frac{1}{(1-M)\cdot Z_{L11}} \\ \frac{-1\cdot e^{-j\cdot 30deg}}{Z_{T21}} \\ \frac{-1}{(1-M)\cdot Z_{L11}} \end{array} & \begin{array}{c} 0 \\ 0 \\ \frac{-1\cdot e^{j\cdot 30deg}}{Z_{T21}} \\ \frac{1}{Z_{T21}}+\frac{1}{jX_{G21}} \\ 0 \end{array} & \left.\begin{array}{c} 0 \\ \frac{-1}{M\cdot Z_{L11}} \\ \frac{-1}{(1-M)\cdot Z_{L11}} \\ 0 \\ \frac{1}{M\cdot Z_{L11}}+\frac{1}{(1-M)\cdot Z_{L11}} \end{array}\right] & \begin{array}{c} \text{BUS 1} \\ \text{BUS 2} \\ \text{BUS 3} \\ \text{BUS 4} \\ \text{BUS F} \end{array} \end{array}$$

$[\underline{I}] = [Y][\underline{V}]$

$$Z_1(M) := Y_1(M)^{-1}$$

$$Z_1(0.5) = \begin{pmatrix} 0.0474i & 0.0231+0.04i & 0.0103+0.0178i & 0.0137i & 0.0167+0.0289i \\ -0.0231+0.04i & 0.0692i & 0.0308i & -0.0103+0.0178i & 0.05i \\ -0.0103+0.0178i & 0.0308i & 0.2359i & -0.0786+0.1362i & 0.1333i \\ 0.0137i & 0.0103+0.0178i & 0.0786+0.1362i & 0.1937i & 0.0444+0.077i \\ -0.0167+0.0289i & 0.05i & 0.1333i & -0.0444+0.077i & 0.2167i \end{pmatrix}$$

$\underline{V} = [Z]\,I$

$Z_{BUS\,4,4}$

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prefault

- For now, assume initially flat voltage profile, but with transformer phase shifts added

$$V1(M) := \begin{pmatrix} 1.0e^{-j\cdot 30deg} \\ 1.0 \\ 1.0 \\ 1.0\cdot e^{-j\cdot 30deg} \\ 1.0 \end{pmatrix} + \underline{\Delta V1(M)} \qquad V1(0.5) = \begin{pmatrix} 0.7328 - 0.4231i \\ 0.7692 \\ 0.3846 \\ 0.5107 - 0.2949i \\ 0 \end{pmatrix}$$

low side

$V_{B1}(M) := V1(M)_0$ $\qquad \left|V_{B1}(0.5)\right| = 0.8462\cdot pu$ $\qquad \arg\left(V_{B1}(0.5)\right) = -30\cdot deg$

$V_{B4}(M) := V1(M)_3$ $\qquad \left|V_{B4}(0.5)\right| = 0.5897\cdot pu$ $\qquad \arg\left(V_{B4}(0.5)\right) = -30\cdot deg$

- Both balanced 3 phase.

of xfms

From above:

$\left|V_{bus1}(0.5)\right| = 0.8462$ $\qquad \arg\left(V_{bus1}(0.5)\right) = -30\cdot deg$

$\left|V_{bus4}(0.5)\right| = 0.5897$ $\qquad \arg\left(V_{bus4}(0.5)\right) = -30\cdot deg$

$$I = \frac{\bar{V}_i - V_j}{Z_{ij}}$$

STEP3

$$I_S(M) := \frac{V_S\cdot e^{-j\cdot 30deg} - V1(M)_0}{jX_{G11}} \qquad \left|I_S(0.5)\right| = 3.0769\cdot pu \qquad \arg\left(I_S(0.5)\right) = -120\cdot deg$$

$$I_R(M) := \frac{V_R\cdot e^{-j\cdot 30deg} - V1(M)_3}{jX_{G21}} \qquad \left|I_R(0.5)\right| = 1.5385\cdot pu \qquad \arg\left(I_R(0.5)\right) = -120\cdot deg$$

Can
calculate
current in any branch

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$$\left|I_{SrcS}(0.5)\right| = 3.0769 \qquad \arg\left(I_{SrcS}(0.5)\right) = -120\cdot deg$$

$$\left|I_{SrcR}(0.5)\right| = 1.5385 \qquad \arg\left(I_{SrcR}(0.5)\right) = -120\cdot deg$$

Alternate:

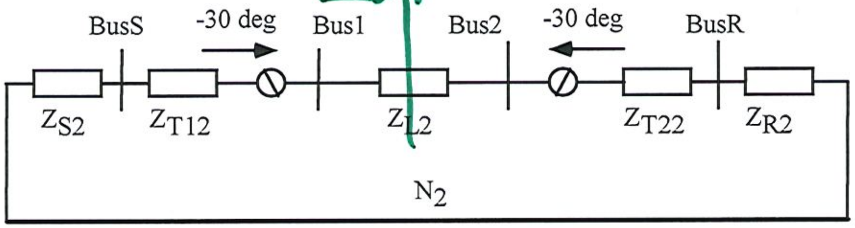
$$I_{SA}(M) := \frac{V1(M)_0 - V1(M)_1\cdot e^{-j\cdot 30deg}}{Z_{T11}} \qquad \left|I_{SA}(0.5)\right| = 3.0769\cdot pu \qquad \arg\left(I_{SA}(0.5)\right) = -120\cdot deg$$

$$I_{RA}(M) := \frac{V1(M)_3 - V1(M)_2\cdot e^{-j\cdot 30deg}}{Z_{T21}} \qquad \left|I_{RA}(0.5)\right| = 1.5385\cdot pu \qquad \arg\left(I_{RA}(0.5)\right) = -120\cdot deg$$

Negative Sequence Ybus

- All of the impedance values are the same in this case.
- Only change is in the Δ-Y phase shifts.
- Equivalent circuit looks the same, but no sources.

  - Negative sequence per unit, per phase, equivalent circuit:



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opposite of pos seq.

$$Y_2(M) := \begin{bmatrix} \frac{1}{jX_{G12}} + \frac{1}{Z_{T12}} & \frac{-1\cdot e^{j\cdot 30deg}}{Z_{T12}} & 0 & 0 & 0 \\ \frac{-1\cdot e^{-j\cdot 30deg}}{Z_{T12}} & \frac{1}{Z_{T12}} + \frac{1}{M\cdot Z_{L12}} & 0 & 0 & \frac{-1}{M\cdot Z_{L12}} \\ 0 & 0 & \frac{1}{Z_{T22}} + \frac{1}{(1-M)\cdot Z_{L12}} & \frac{-1\cdot e^{-j\cdot 30deg}}{Z_{T22}} & \frac{-1}{(1-M)\cdot Z_{L12}} \\ 0 & 0 & \frac{-1\cdot e^{j\cdot 30deg}}{Z_{T22}} & \frac{1}{Z_{T22}} + \frac{1}{jX_{G22}} & 0 \\ 0 & \frac{-1}{M\cdot Z_{L12}} & \frac{-1}{(1-M)\cdot Z_{L12}} & 0 & \frac{1}{M\cdot Z_{L12}} + \frac{1}{(1-M)\cdot Z_{L12}} \end{bmatrix}$$

$$Z_2(M) := Y_2(M)^{-1}$$

$$Z_2(0.5) = \begin{pmatrix} 0.0474i & -0.0231+0.04i & -0.0103+0.0178i & 0.0137i & -0.0167+0.0289i \\ 0.0231+0.04i & 0.0692i & 0.0308i & 0.0103+0.0178i & 0.05i \\ 0.0103+0.0178i & 0.0308i & 0.2359i & 0.0786+0.1362i & 0.1333i \\ 0.0137i & -0.0103+0.0178i & -0.0786+0.1362i & 0.1937i & -0.0444+0.077i \\ 0.0167+0.0289i & 0.05i & 0.1333i & 0.0444+0.077i & 0.2167i \end{pmatrix}$$

↗

Don't do $Z_2(0)$ or $Z_2(1)$

$M=1$ or $m=0$ causes a divide by 0

$m = 10^{-10}$

$m = 0.9999..$

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$$\underline{Y_0(M)} := \begin{bmatrix} \frac{1}{Z_{G10}} & \textcircled{0} & 0 & 0 & 0 \\ \textcircled{0} & \frac{1}{Z_{T10}} + \frac{1}{M \cdot Z_{L10}} & 0 & 0 & \frac{-1}{M \cdot Z_{L10}} \\ 0 & 0 & \frac{1}{Z_{T20}} + \frac{1}{(1-M) \cdot Z_{L10}} & 0 & \frac{-1}{(1-M) \cdot Z_{L10}} \\ 0 & 0 & 0 & \frac{1}{Z_{G20}} & 0 \\ 0 & \frac{-1}{M \cdot Z_{L10}} & \frac{-1}{(1-M) \cdot Z_{L10}} & 0 & \frac{1}{M \cdot Z_{L10}} + \frac{1}{(1-M) \cdot Z_{L10}} \end{bmatrix}$$

$$Z_0(M) := Y_0(M)^{-1}$$

$$Z_0(0.5) = \begin{pmatrix} 2.5 + 0.025i & 0 & 0 & 0 & 0 \\ 0 & 0.0247i & 0.0017i & 0 & 0.0132i \\ 0 & 0.0017i & 0.1243i & 0 & 0.063i \\ 0 & 0 & 0 & 13.3333 + 0.0667i & 0 \\ 0 & 0.0132i & 0.063i & 0 & 0.4881i \end{pmatrix}$$

$Z_{0thev}(0.5) = 0.4881i\,pu$ $\qquad$ $Z_0(0.5)_{4,4} = 0.4881i\,pu$

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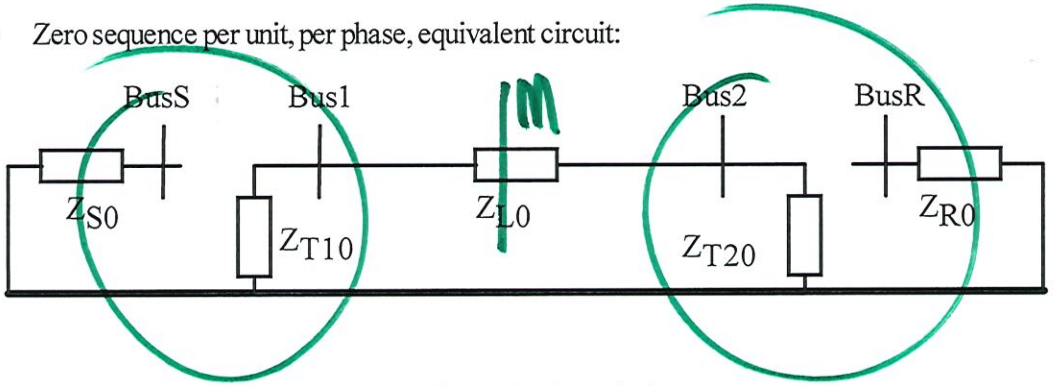
Zero Sequence Ybus

- Impedance values differ
- Model the transformer zero sequence connections
- Different looking equivalent circuit.
  - Zero sequence per unit, per phase, equivalent circuit:



- Thevenin equivalent impedance by brute force circuit analysis:

$$Z_{0left}(M) := Z_{T10} + Z_{L10} \cdot M$$

- Note that generator impedances are not present due to transformer connections

$$Z_{0right}(M) := Z_{L10} \cdot (1 - M) + Z_{T20}$$

$$Z_{0equiv}(M) := \left[\frac{1}{Z_{T10} + Z_{L10} \cdot M} + \frac{1}{Z_{L10} \cdot (1 - M) + Z_{T20}} \right]^{-1}$$

- More compactly: $Z_{0thev}(M) := \left(\frac{1}{Z_{0left}(M)} + \frac{1}{Z_{0right}(M)} \right)^{-1}$

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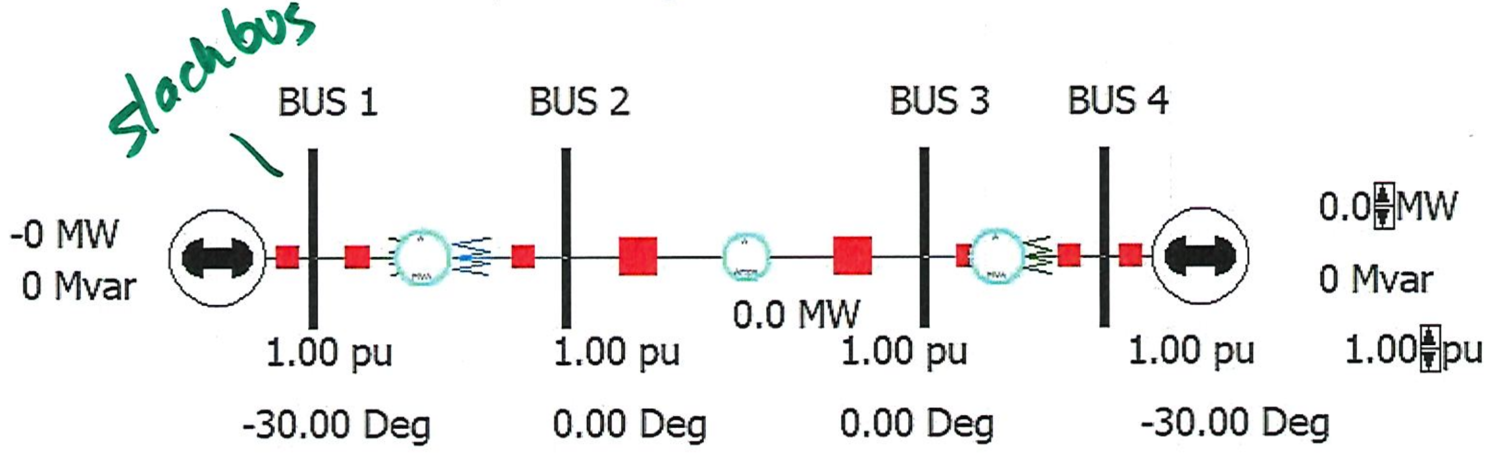
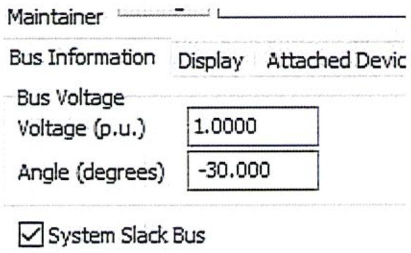
ECE 523: Fault Analysis Using Powerworld



- Set slack bus angle such that the faulted line is at 30 deg



Voltage (p.u.) 1.0000

Angle (degrees) -30.000

☑ System Slack Bus

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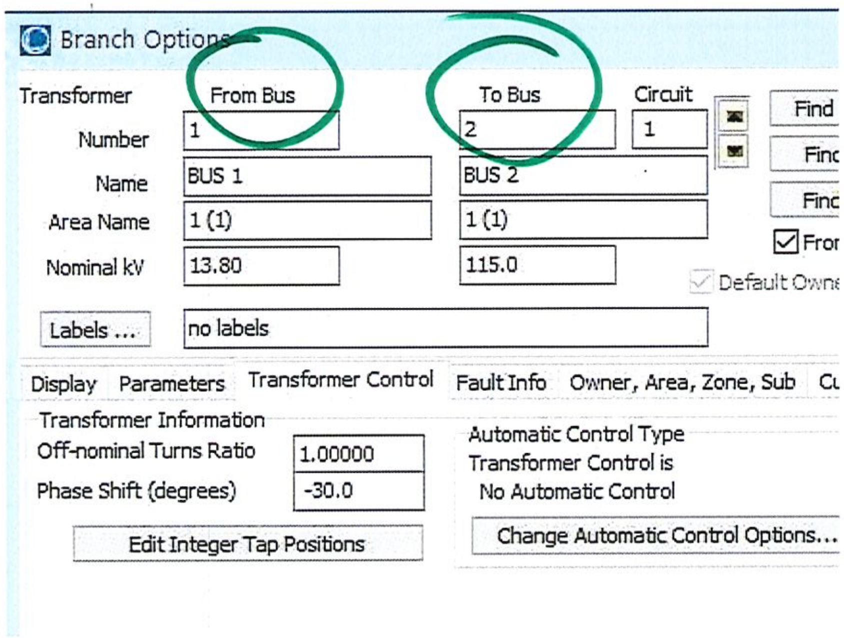
- Transformer phase shift



Transformer	From Bus	To Bus	Circuit
Number	1	2	1
Name	BUS 1	BUS 2	
Area Name	1 (1)	1 (1)	
Nominal kV	13.80	115.0	

☑ Fror
☑ Default Own

Labels: no labels

Transformer Information
Off-nominal Turns Ratio: 1.00000
Phase Shift (degrees): -30.0

Automatic Control Type
Transformer Control is
No Automatic Control

- Transformer connection

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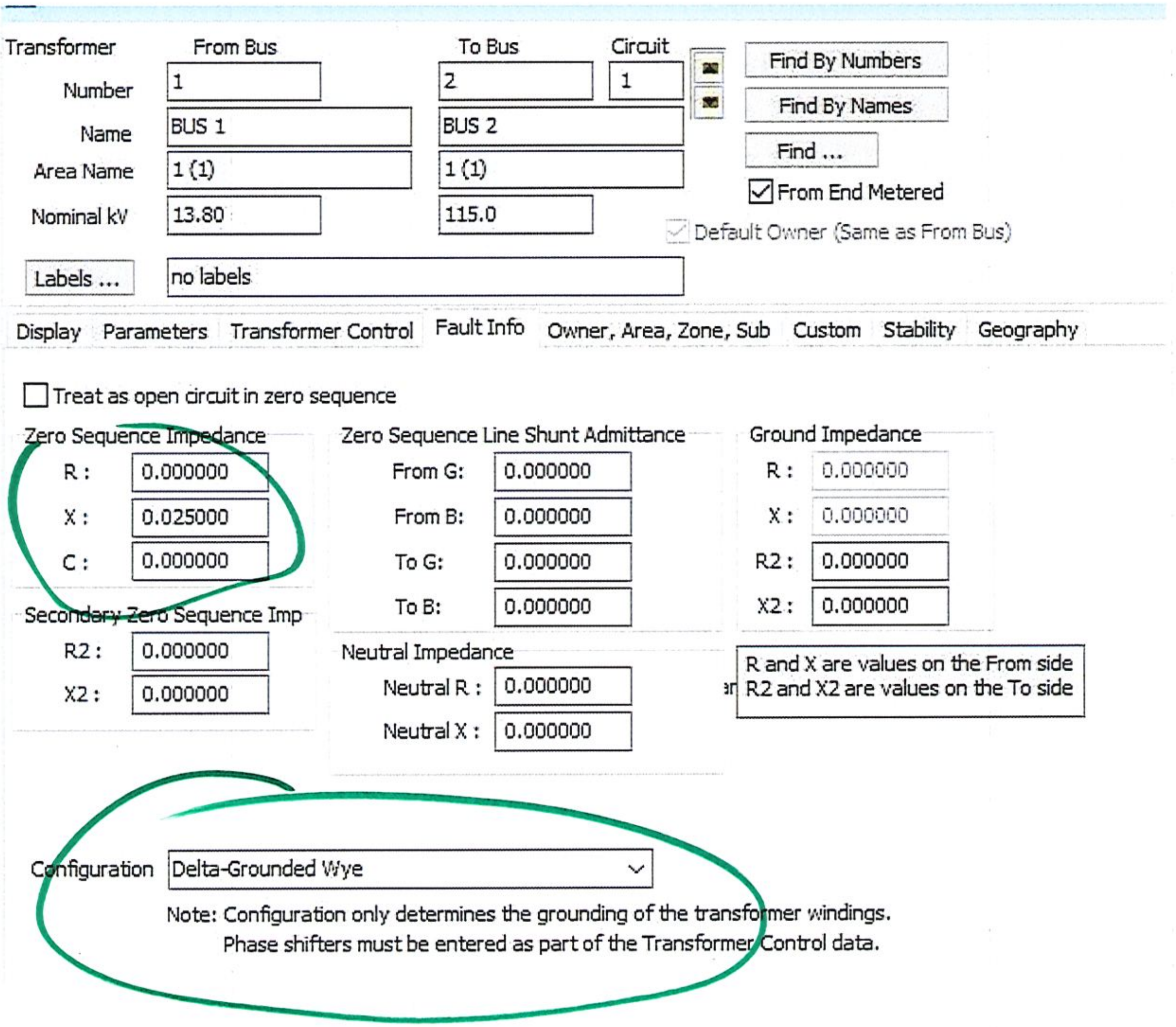

Transformer	From Bus	To Bus	Circuit
Number	1	2	1
Name	BUS 1	BUS 2	
Area Name	1 (1)	1 (1)	
Nominal kV	13.80	115.0	

☑ From End Metered
☑ Default Owner (Same as From Bus)

Labels: no labels

☐ Treat as open circuit in zero sequence

Zero Sequence Impedance
R : 0.000000
X : 0.025000
C : 0.000000

Secondary Zero Sequence Imp
R2 : 0.000000
X2 : 0.000000

Zero Sequence Line Shunt Admittance
From G: 0.000000
From B: 0.000000
To G: 0.000000
To B: 0.000000

Neutral Impedance
Neutral R : 0.000000
Neutral X : 0.000000

Ground Impedance
R : 0.000000
X : 0.000000
R2 : 0.000000
X2 : 0.000000

R and X are values on the From side
R2 and X2 are values on the To side

Configuration: Delta-Grounded Wye

Note: Configuration only determines the grounding of the transformer windings.
Phase shifters must be entered as part of the Transformer Control data.

- Getting fault Ybus: (fault at 50% of the line)

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Name	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5
BUS 1	0.00 - j60.00	20.00 + j34.64			
BUS 2	-20.00 + j34.64	0.00 - j44.00	0.00 + j0.00		-0.00 + j4.00
BUS 3		0.00 + j0.00	0.00 - j11.50	-3.75 + j6.50	-0.00 + j4.00
BUS 4			3.75 + j6.50	0.00 - j11.25	
FaultPt		-0.00 + j4.00	-0.00 + j4.00		0.00 - j8.00

pos

neg seq

zero seq volt

Bus voltages

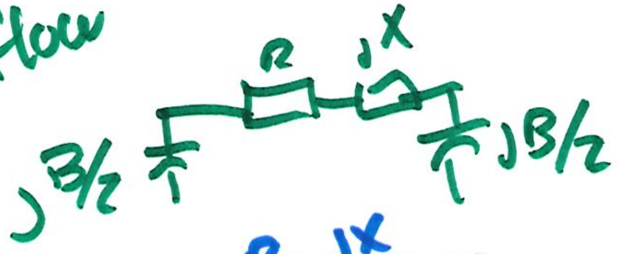
Name	Phase Volt A	Phase Ang A	Phase Volt B	Phase Ang B	Phase Volt C	Phase Ang C	Seq. Volt +	Seq. Angle +	Seq. Volt -	Seq. Angle -	Seq. Volt 0	Seq. Angle 0
BUS 1	0.84615	-30	0.84615	-150	0.84615	90	0.8462	-30	0	90	0	90
BUS 2	0.76923	0	0.76923	-120	0.76923	120	0.7692	0	0	90	0	90
BUS 3	0.38462	0	0.38462	-120	0.38462	120	0.3846	0	0	90	0	90
BUS 4	0.58974	-30	0.58974	-150	0.58974	90	0.5897	-30	0	90	0	90
FaultPt	0	0	0	-120	0	120	0	0	0	90	0	90

faulted branch - replace

Branch currents:

From Name	To Name	Phase Cur A From	Phase Ang A From	Phase Cur B From	Phase Ang B From	Phase Cur C From	Phase Ang C From	Seq. Cur + From	Seq. Ang + From
BUS 1	BUS 2	3.07692	-120	3.07692	120	3.07692	0	3.07692	-119.99998
BUS 2	BUS 3	0	0	0	0	0	0	0	0
BUS 2	FaultPt	3.07692	-90	3.07692	150	3.07692	30	3.07692	-89.99996
BUS 4	BUS 3	1.53846	-120	1.53846	120	1.53846	0	1.53846	-119.99995
FaultPt	BUS 3	1.53846	90	1.53846	-30	1.53846	-150	1.53846	90.00004

Power flow



fault

Analysis of unbalanced faults

1. Shunt faults
   - with or w/o fault resistance
   - with and w/o load flow
2. Series faults
3. Simultaneous faults

Conducting the Analysis

- Pos, neg, zero sequence equiv circuits

$$\rightarrow \underbrace{Y_{BUS1},\ Y_{BUS2},\ Y_{BUS0}}_{\Downarrow} \quad [Y_{BUS2}]^{-1} = Z_{BUS}$$

$$Z_{BUS}$$

- Perform all steps of fault analysis in sequence domain → results back to ABC

Each fault type

- → Boundary conditions for that fault
- → convert to sequence domain
- → find formula/equiv ckt for fault current calculation in sequence domain
- → once we have fault currents do circuit analysis for rest of ckt in sequence domain