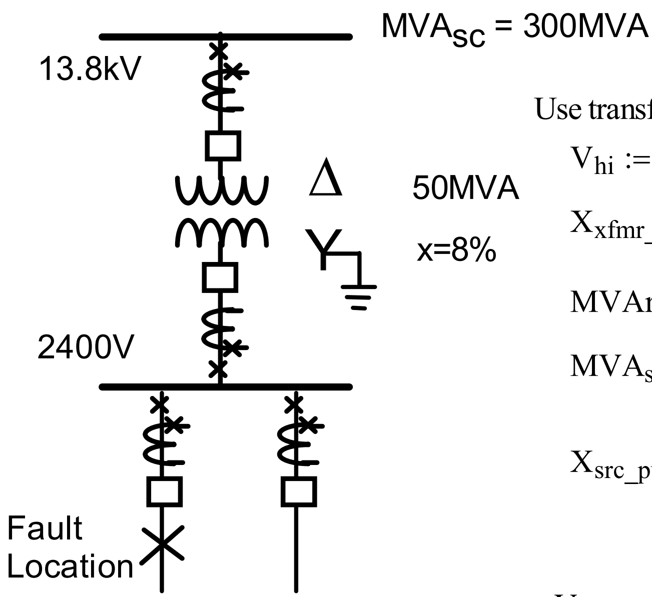
Unbalanced Fault Analysis Examples

$pu := 1$ $\qquad$ $MVA := 1000kW$

$a := 1e^{j \cdot 120deg}$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

Compare the per unit phase currents seen on the HV side of the transformer below with those seen by at the fault location for three phase, SLG and LL faults the left side feeder. Assume wye connected CT's.



Use transformer ratings as the per unit base.

$V_{hi} := 13.8kV$ $\qquad$ $V_{lo} := 2400V$

$X_{xfmr\_pu} := 0.08pu$

$MVArated := 50MVA$ $\qquad$ $S_B := 50MVA$

$MVA_{sc} := 300MVA$

$$X_{src\_pu} := \frac{(1.0pu)^2}{\left(\frac{MVA_{sc}}{S_B}\right)} \qquad X_{src\_pu} = 0.17 \cdot pu$$

$V_{src} := 1.0pu$

Sequence Impedances for Faults:

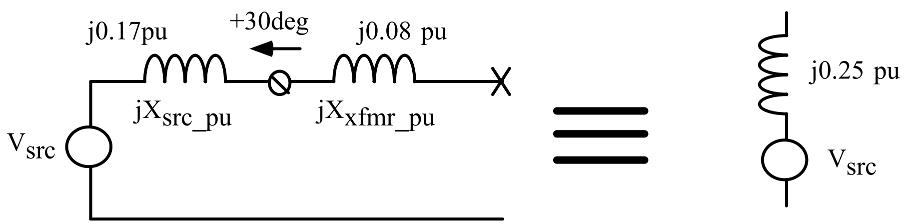
$X_1 := X_{src\_pu} + X_{xfmr\_pu}$ $\qquad$ $X_1 = 0.25 \cdot pu$ $\qquad$ $X_2 := X_1$

$$X_{ground} := 0pu$$

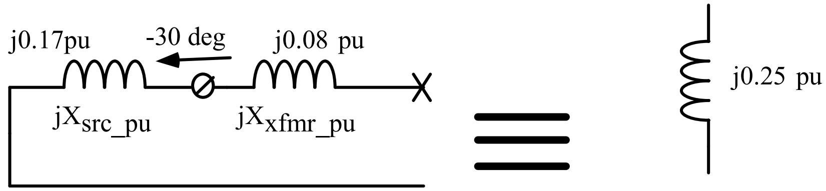
$$X_0 := X_{xfmr\_pu} + 3 \cdot X_{ground}$$

Sequence Networks:

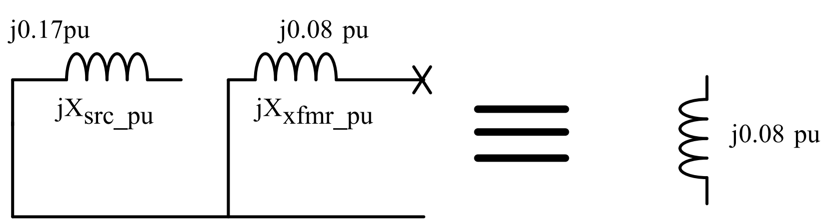
Positive Sequence



Negatve Sequence



Zero sequence



Three phase fault:

$$I_{f\_3\phi} := \frac{V_{src}}{jX_1} \qquad I_{f\_3\phi} = -4.05i \cdot pu$$

Per unit current magnitudes will be the same on the low side as they are on the high side. But there will be a plus 30 degree shift.

$$I_{f\_abc3ph} := I_{f\_3\phi} \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} \qquad \overrightarrow{\left|I_{f\_abc3ph}\right|} = \begin{pmatrix} 4.05 \\ 4.05 \\ 4.05 \end{pmatrix} \qquad \overrightarrow{\arg\left(I_{f\_abc3ph}\right)} = \begin{pmatrix} -90 \\ 150 \\ 30 \end{pmatrix} \cdot deg$$

$$I_{HV\_abc3ph} := I_{f\_3\phi} \cdot e^{j \cdot 30deg} \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} \qquad \overrightarrow{\left|I_{HV\_abc3ph}\right|} = \begin{pmatrix} 4.05 \\ 4.05 \\ 4.05 \end{pmatrix} \qquad \overrightarrow{\arg\left(I_{HV\_abc3ph}\right)} = \begin{pmatrix} -60 \\ 180 \\ 60 \end{pmatrix} \cdot deg$$

Voltage at the fault point:

$$V_{af} := V_{src} - j \cdot X_1 \cdot I_{f\_3\phi} \qquad V_{af} = 0 \cdot pu$$

Line to Line fault:

$$I_{0\_LLf} := 0pu \qquad R_f := 0pu$$

$$I_{1\_LLf} := \frac{V_{src}}{j \cdot X_1 + j \cdot X_2 + R_f} \qquad I_{1\_LLf} = -2.03i \cdot pu$$

$$I_{2\_LLf} := -I_{1\_LLf}$$

$$I_{abc\_LLf} := A_{012} \cdot \begin{pmatrix} I_{0\_LLf} \\ I_{1\_LLf} \\ I_{2\_LLf} \end{pmatrix} \qquad \overrightarrow{\left|I_{abc\_LLf}\right|} = \begin{pmatrix} 0 \\ 3.51 \\ 3.51 \end{pmatrix} \cdot pu$$

$$\arg\left(I_{abc\_LLf_1}\right) = 180 \cdot deg$$

$$\arg\left(I_{abc\_LLf_2}\right) = 0 \cdot deg$$

Now move the currents from the low side to the high side (note, you must transform the sequence currents):

$$I_{1\_LLH} := I_{1\_LLf} \cdot e^{j \cdot 30deg} \qquad I_{0\_LLH} := 0pu$$

$$I_{2\_LLH} := I_{2\_LLf} \cdot e^{-j \cdot 30deg}$$

$$I_{abc\_LLh} := A_{012} \cdot \begin{pmatrix} I_{0\_LLH} \\ I_{1\_LLH} \\ I_{2\_LLH} \end{pmatrix} \qquad \overrightarrow{\left|I_{abc\_LLh}\right|} = \begin{pmatrix} 2.03 \\ 4.05 \\ 2.03 \end{pmatrix} \cdot pu$$

$$\overrightarrow{\arg\left(I_{abc\_LLh}\right)} = \begin{pmatrix} 0 \\ 180 \\ 0 \end{pmatrix} \cdot deg$$

Phase B per unit current is larger on the high side of the transformer than on the low side.

Voltage at the fault point:

$$V_{1\_LL} := V_{src} - j \cdot X_1 \cdot I_{1\_LLf} \qquad \left|V_{1\_LL}\right| = 0.5 \cdot pu \qquad \arg\left(V_{1\_LL}\right) = 0 \cdot deg$$

$$V_{2\_LL} := 0 - j \cdot X_2 \cdot I_{2\_LLf} \qquad \left|V_{2\_LL}\right| = 0.5 \cdot pu \qquad \arg\left(V_{2\_LL}\right) = 0 \cdot deg$$

$$V_{0\_LL} := 0 - j \cdot X_0 \cdot I_{0\_LLf} \qquad \left|V_{0\_LL}\right| = 0 \cdot pu$$

$$V_{abc\_LL} := A_{012} \cdot \begin{pmatrix} V_{0\_LL} \\ V_{1\_LL} \\ V_{2\_LL} \end{pmatrix}$$

$$\overrightarrow{\left|V_{abc\_LL}\right|} = \begin{pmatrix} 1 \\ 0.5 \\ 0.5 \end{pmatrix} \cdot pu$$

$$\overrightarrow{\arg\left(V_{abc\_LL}\right)} = \begin{pmatrix} 0 \\ 180 \\ 180 \end{pmatrix} \cdot deg$$

Voltage at the 13.8kV Bus

$$V_{1\_LLHV} := V_{src} \cdot e^{j \cdot 30deg} - j \cdot X_{src\_pu} \cdot I_{1\_LLH} \qquad \left|V_{1\_LLHV}\right| = 0.66 \cdot pu \qquad \arg\left(V_{1\_LLHV}\right) = 30 \cdot deg$$

$$V_{2\_LLHV} := 0 - j \cdot X_{src\_pu} \cdot I_{2\_LLH} \qquad \left|V_{2\_LLHV}\right| = 0.34 \cdot pu \qquad \arg\left(V_{2\_LLHV}\right) = -30 \cdot deg$$

$$V_{0\_LLHV} := 0 \qquad \left|V_{0\_LLHV}\right| = 0 \cdot pu$$

$$V_{abc\_LLHV} := A_{012} \cdot \begin{pmatrix} V_{0\_LLHV} \\ V_{1\_LLHV} \\ V_{2\_LLHV} \end{pmatrix}$$

$$\overrightarrow{\left|V_{abc\_LLHV}\right|} = \begin{pmatrix} 0.88 \\ 0.32 \\ 0.88 \end{pmatrix} \cdot pu$$

$$\overrightarrow{\arg\left(V_{abc\_LLHV}\right)} = \begin{pmatrix} 10.61 \\ -90 \\ 169.39 \end{pmatrix} \cdot deg$$

Single Line to Ground fault:

$$I_{0\_SLGf} := \frac{V_{src}}{j \cdot X_1 + j \cdot X_2 + j \cdot X_0 + 3 \cdot R_f} \qquad I_{0\_SLGf} = -1.74i \cdot pu$$

$$I_{1\_SLGf} := I_{0\_SLGf} \qquad I_{2\_SLGf} := I_{0\_SLGf}$$

$$I_{abc\_SLGf} := A_{012} \cdot \begin{pmatrix} I_{0\_SLGf} \\ I_{1\_SLGf} \\ I_{2\_SLGf} \end{pmatrix}$$

$$\overrightarrow{\left|I_{abc\_SLGf}\right|} = \begin{pmatrix} 5.23 \\ 0 \\ 0 \end{pmatrix} \cdot pu$$

$$\overrightarrow{\arg\left(I_{abc\_SLGf}\right)} = \begin{pmatrix} -90 \\ 18.43 \\ 18.43 \end{pmatrix} \cdot deg$$

Now move the currents from the low side to the high side:

$$I_{1\_SLGH} := I_{1\_SLGf} \cdot e^{j \cdot 30deg} \qquad I_{0\_SLGH} := 0pu$$

$$I_{2\_SLGH} := I_{2\_SLGf} \cdot e^{-j \cdot 30deg}$$

$$I_{abc\_SLGh} := A_{012} \cdot \begin{pmatrix} I_{0\_SLGH} \\ I_{1\_SLGH} \\ I_{2\_SLGH} \end{pmatrix}$$

$$\overrightarrow{\left|I_{abc\_SLGh}\right|} = \begin{pmatrix} 3.02 \\ 0 \\ 3.02 \end{pmatrix} \cdot pu$$

Currents are lower on the high side and appear as a L-L fault not SLG

$$\overrightarrow{\arg\left(I_{abc\_SLGh}\right)} = \begin{pmatrix} -90 \\ 90 \\ 90 \end{pmatrix} \cdot deg$$

Voltage at the fault point:

$$V_{1\_SLG} := V_{src} - j \cdot X_1 \cdot I_{1\_SLGf} \qquad \left|V_{1\_SLG}\right| = 0.57 \cdot pu \qquad \arg\left(V_{1\_SLG}\right) = 0 \cdot deg$$

$$V_{2\_SLG} := 0 - j \cdot X_2 \cdot I_{2\_SLGf} \qquad \left|V_{2\_SLG}\right| = 0.43 \cdot pu \qquad \arg\left(V_{2\_SLG}\right) = 180 \cdot deg$$

$$V_{0\_SLG} := 0 - j \cdot X_0 \cdot I_{0\_SLGf} \qquad \left|V_{0\_SLG}\right| = 0.14 \cdot pu \qquad \arg\left(V_{0\_SLG}\right) = 180 \cdot deg$$

$$V_{abc\_SLG} := A_{012} \cdot \begin{pmatrix} V_{0\_SLG} \\ V_{1\_SLG} \\ V_{2\_SLG} \end{pmatrix} \qquad \overrightarrow{\left|V_{abc\_SLG}\right|} = \begin{pmatrix} 0 \\ 0.89 \\ 0.89 \end{pmatrix} \cdot pu$$

$$\arg\left(V_{abc\_SLG_1}\right) = -103.59 \cdot deg$$

$$\arg\left(V_{abc\_SLG_2}\right) = 103.59 \cdot deg$$

Voltage on 13.kV bus

$$V_{1\_SLGHV} := \left(V_{src} \cdot e^{j \cdot 30deg} - j \cdot X_{src\_pu} \cdot I_{1\_SLGH}\right) \qquad \left|V_{1\_SLGHV}\right| = 0.71 \cdot pu \quad \arg\left(V_{1\_SLGHV}\right) = 30 \cdot deg$$

$$V_{2\_SLGHV} := \left(0 - j \cdot X_{src\_pu} \cdot I_{2\_SLGH}\right) \qquad \left|V_{2\_SLGHV}\right| = 0.29 \cdot pu \quad \arg\left(V_{2\_SLGHV}\right) = 150 \cdot deg$$

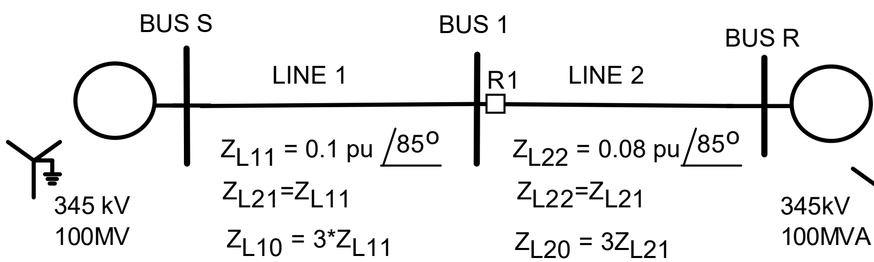
$$V_{0\_SLGHV} := 0 - j \cdot X_{src\_pu} \cdot I_{0\_SLGH} \qquad \left|V_{0\_SLGHV}\right| = 0 \cdot pu$$

$$V_{abc\_SLGHV} := A_{012} \cdot \begin{pmatrix} V_{0\_SLGHV} \\ V_{1\_SLGHV} \\ V_{2\_SLGHV} \end{pmatrix}$$

$$\overrightarrow{|V_{abc\_SLGHV}|} = \begin{pmatrix} 0.62 \\ 1 \\ 0.62 \end{pmatrix} \cdot pu$$

$$\overrightarrow{\arg(V_{abc\_SLGHV})} = \begin{pmatrix} 54.06 \\ -90 \\ 125.94 \end{pmatrix} \cdot deg$$

- ***Example with two sources:***



V_S = 1.0pu @ 0 deg

Z_{S1} = j0.03 pu

$Z_{S2}=Z_{S1}$

$Z_{S0} = 3*Z_{S1}$

Per unit line impedances calculated with SB=100MVA and VB=345kV LL

V_R = 1.0pu at 0 deg

Z_{R1} = j0.06 pu

$Z_{R2}=Z_{R1}$

$Z_{R0} = 3*Z_{R1}$

$Z_{S1} := j \cdot 0.03pu$

$Z_{S2} := Z_{S1}$

$Z_{L11} := 0.1pu \cdot e^{j \cdot 85deg}$

$Z_{L11} = 0.01 + 0.1i$

$Z_{L21} := 0.08pu \cdot e^{j \cdot 85deg}$

$Z_{L21} = 0.01 + 0.08i$

$Z_{S0} := 3Z_{S1}$

$Z_{R1} := j\cdot 0.06pu$

$Z_{R2} := Z_{R1}$

$Z_{R0} := 3Z_{R1}$

$Z_{L12} := Z_{L11}$

$Z_{L10} := 3\cdot Z_{L11}$

$Z_{L10} = 0.03 + 0.3i$

$Z_{L22} := Z_{L21}$

$Z_{L20} := 3\cdot Z_{L21}$

$Z_{L20} = 0.02 + 0.24i$

- For faults on Line 2:

$$Z_{L2\_1\_thev}(n) := \left[\frac{1}{Z_{S1} + Z_{L11} + n\cdot Z_{L21}} + \frac{1}{(1-n)\cdot Z_{L21} + Z_{R1}}\right]^{-1}$$

$$Z_{L2\_2\_thev}(n) := \left[\frac{1}{Z_{S2} + Z_{L12} + n\cdot Z_{L22}} + \frac{1}{(1-n)\cdot Z_{L22} + Z_{R2}}\right]^{-1}$$

$$Z_{L2\_0\_thev}(n) := \left[\frac{1}{Z_{S0} + Z_{L10} + n\cdot Z_{L20}} + \frac{1}{(1-n)\cdot Z_{L20} + Z_{R0}}\right]^{-1}$$

- ***3 phase faults:*** $V_f := 1$

$$I_{f1}(m) := \frac{V_f}{Z_{L2\_1\_thev}(m)}$$

$$I_{left\_1}(m) := \frac{I_{f1}(m)\cdot\left[(1-m)\cdot Z_{L21} + Z_{R1}\right]}{Z_{S1} + Z_{L11} + m\cdot Z_{L21} + \left[(1-m)\cdot Z_{L21} + Z_{R1}\right]}$$

$$V_{R1\_1}(m) := V_f - I_{left\_1}(m)\cdot\left(Z_{S1} + Z_{L11}\right)$$

$$\left|I_{left\_1}(0.5)\right| = 5.89\cdot pu \qquad \left|V_{R1\_1}(0.5)\right| = 0.24\cdot pu$$

$$\arg\left(I_{left\_1}(0.5)\right) = -85.88\cdot deg \qquad \arg\left(V_{R1\_1}(0.5)\right) = -0.88\cdot deg$$

- ***SLG faults:***

$$I_{f0}(m) := \frac{V_f}{Z_{L2\_1\_thev}(m) + Z_{L2\_2\_thev}(m) + Z_{L2\_0\_thev}(m)} \qquad I_{f1}(m) := I_{f0}(m) \qquad I_{f2}(m) := I_{f0}(m)$$

$$I_{left\_1}(m) := \frac{I_{f1}(m)\cdot\left[(1-m)\cdot Z_{L21} + Z_{R1}\right]}{Z_{S1} + Z_{L11} + m\cdot Z_{L21} + \left[(1-m)\cdot Z_{L21} + Z_{R1}\right]}$$

$$I_{left\_2}(m) := \frac{I_{f2}(m)\cdot\left[(1-m)\cdot Z_{L22} + Z_{R2}\right]}{Z_{S2} + Z_{L12} + m\cdot Z_{L22} + \left[(1-m)\cdot Z_{L22} + Z_{R2}\right]}$$

$$I_{left\_0}(m) := \frac{I_{f0}(m)\cdot\left[(1-m)\cdot Z_{L20} + Z_{R0}\right]}{Z_{S0} + Z_{L10} + m\cdot Z_{L20} + \left[(1-m)\cdot Z_{L20} + Z_{R0}\right]}$$

$$V_{R1\_1}(m) := V_f - I_{left\_1}(m)\cdot\left(Z_{S1} + Z_{L11}\right)$$

$$V_{R1\_2}(m) := 0 - I_{left\_2}(m)\cdot\left(Z_{S2} + Z_{L12}\right)$$

$$V_{R1\_0}(m) := 0 - I_{left\_0}(m)\cdot\left(Z_{S0} + Z_{L10}\right)$$

$$V_{ABC\_R1}(m) := A_{012}\cdot\begin{pmatrix} V_{R1\_0}(m) \\ V_{R1\_1}(m) \\ V_{R1\_2}(m) \end{pmatrix} \qquad \overrightarrow{\left|V_{ABC\_R1}(0.5)\right|} = \begin{pmatrix} 0.24 \\ 1.18 \\ 1.18 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(V_{ABC\_R1}(0.5)\right)} = \begin{pmatrix} -0.88 \\ -132.89 \\ 132.99 \end{pmatrix}\cdot deg$$

$$I_{ABC\_R1}(m) := A_{012}\cdot\begin{pmatrix} I_{left\_0}(m) \\ I_{left\_1}(m) \\ I_{left\_2}(m) \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_R1}(0.5)\right|} = \begin{pmatrix} 3.53 \\ 0 \\ 0 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_R1}(0.5)\right)} = \begin{pmatrix} -85.88 \\ 49.05 \\ 49.05 \end{pmatrix}\cdot deg$$

- ***DLG faults:***

$$I_{f1}(m) := \frac{V_f}{Z_{L2\_1\_thev}(m) + \left(\frac{1}{Z_{L2\_2\_thev}(m)} + \frac{1}{Z_{L2\_0\_thev}(m)}\right)^{-1}}$$

$$I_{f2}(m) := -I_{f1}(m)\cdot\left(\frac{Z_{L2\_0\_thev}(m)}{Z_{L2\_2\_thev}(m) + Z_{L2\_0\_thev}(m)}\right)$$

$$I_{f0}(m) := -I_{f1}(m)\left(\frac{Z_{L2\_2\_thev}(m)}{Z_{L2\_2\_thev}(m) + Z_{L2\_0\_thev}(m)}\right)$$

$$I_{left\_1}(m) := \frac{I_{f1}(m)\cdot\left[(1-m)\cdot Z_{L21} + Z_{R1}\right]}{Z_{S1} + Z_{L11} + m\cdot Z_{L21} + \left[(1-m)\cdot Z_{L21} + Z_{R1}\right]}$$

$$I_{left\_2}(m) := \frac{I_{f2}(m)\cdot\left[(1-m)\cdot Z_{L22} + Z_{R2}\right]}{Z_{S2} + Z_{L12} + m\cdot Z_{L22} + \left[(1-m)\cdot Z_{L22} + Z_{R2}\right]}$$

$$I_{left\_0}(m) := \frac{I_{f0}(m)\cdot\left[(1-m)\cdot Z_{L20} + Z_{R0}\right]}{Z_{S0} + Z_{L10} + m\cdot Z_{L20} + \left[(1-m)\cdot Z_{L20} + Z_{R0}\right]}$$

$$V_{R1\_1}(m) := V_f - I_{left\_1}(m)\cdot\left(Z_{S1} + Z_{L11}\right)$$

$$V_{R1\_2}(m) := 0 - I_{left\_2}(m)\cdot\left(Z_{S2} + Z_{L12}\right)$$

$$V_{R1\_0}(m) := 0 - I_{left\_0}(m)\cdot\left(Z_{S0} + Z_{L10}\right)$$

$$V_{ABC\_R1}(m) := A_{012}\cdot\begin{pmatrix} V_{R1\_0}(m) \\ V_{R1\_1}(m) \\ V_{R1\_2}(m) \end{pmatrix} \qquad \overrightarrow{\left|V_{ABC\_R1}(0.5)\right|} = \begin{pmatrix} 1.22 \\ 0.24 \\ 0.24 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(V_{ABC\_R1}(0.5)\right)} = \begin{pmatrix} 0.05 \\ -120.88 \\ 119.12 \end{pmatrix}\cdot deg$$

$$I_{ABC\_R1}(m) := A_{012}\cdot\begin{pmatrix} I_{left\_0}(m) \\ I_{left\_1}(m) \\ I_{left\_2}(m) \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_R1}(0.5)\right|} = \begin{pmatrix} 0 \\ 5.25 \\ 5.25 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_R1}\left(0.5 + 10^{-15}\right)\right)} = \begin{pmatrix} 93.58 \\ 170.22 \\ 18.02 \end{pmatrix}\cdot deg$$

- Adding the 10^{-15} avoided a divide by 0 problem in angle calculation