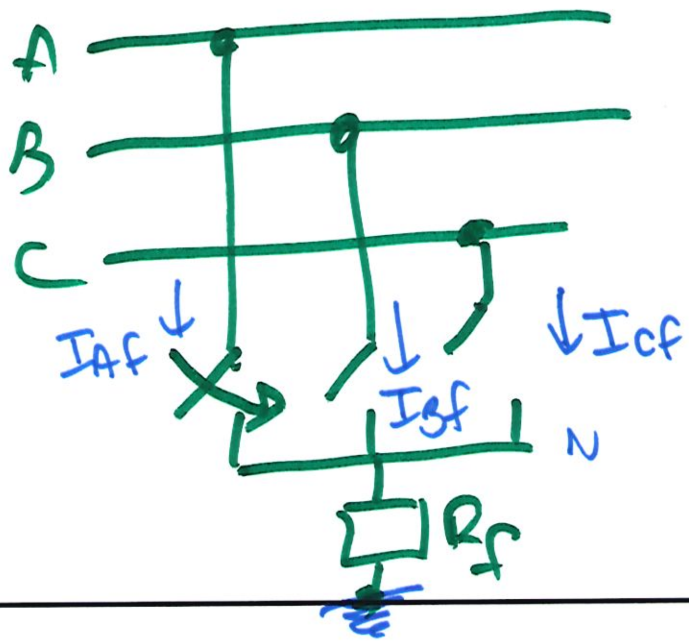
ECE 523
Symmetrical Components

Session 11

Single Line to Ground Fault (SLG)

① Boundary conditions

→ fault currents at fault location (not same as line currents)



SLG on phase A

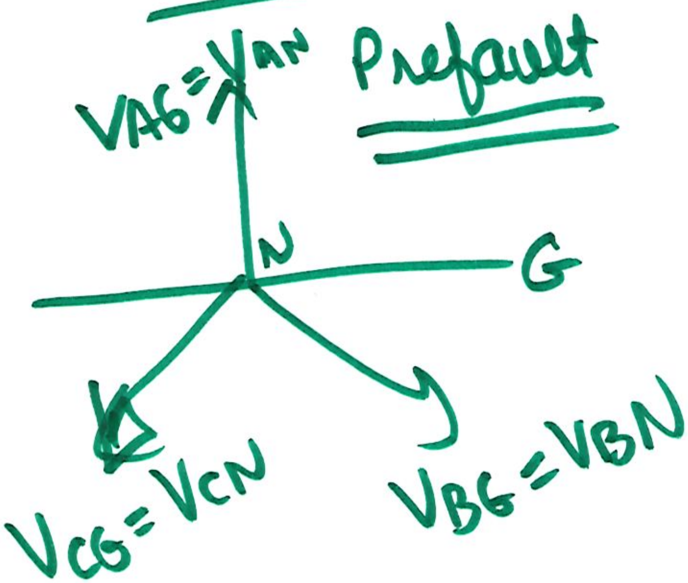
$I_{Af} \neq 0$

$I_{Bf} = 0$

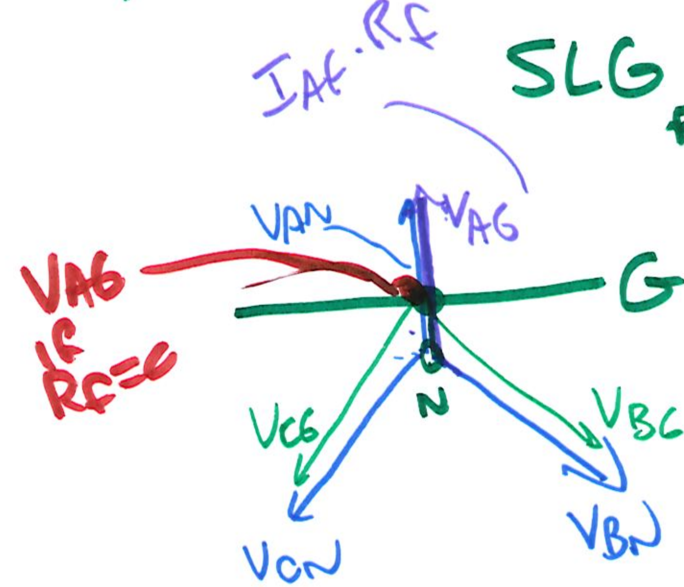
$I_{Cf} = 0$

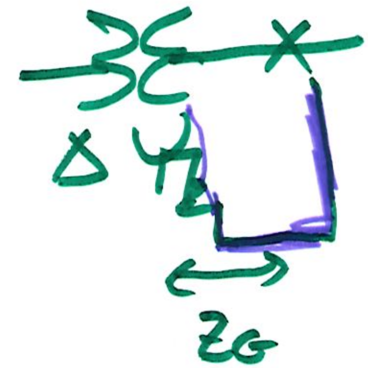
$V_{AG} = I_{Af} R_f$

Grounded System - closed to ground point



SLG Fault on A





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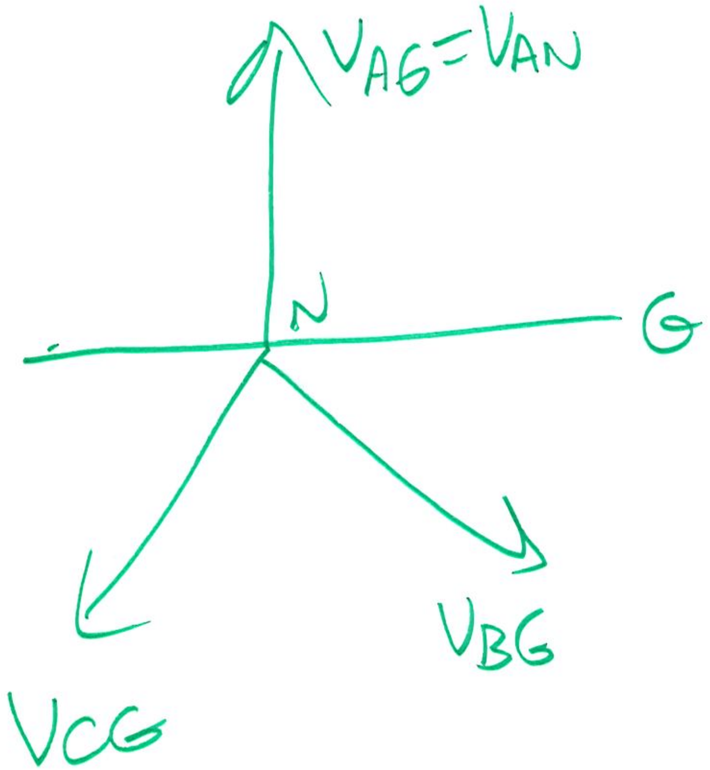
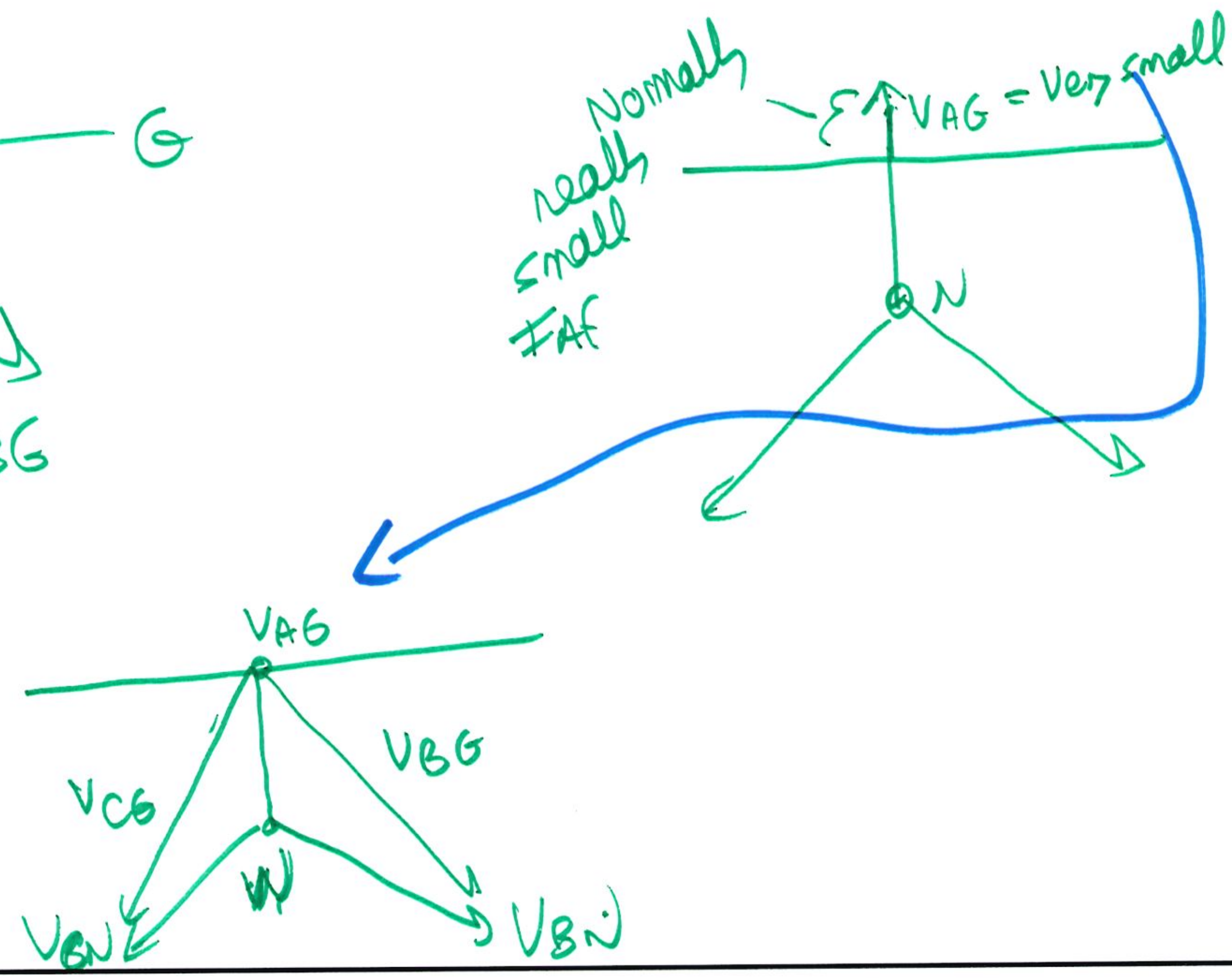
ungrounded system

Prefault



SLG



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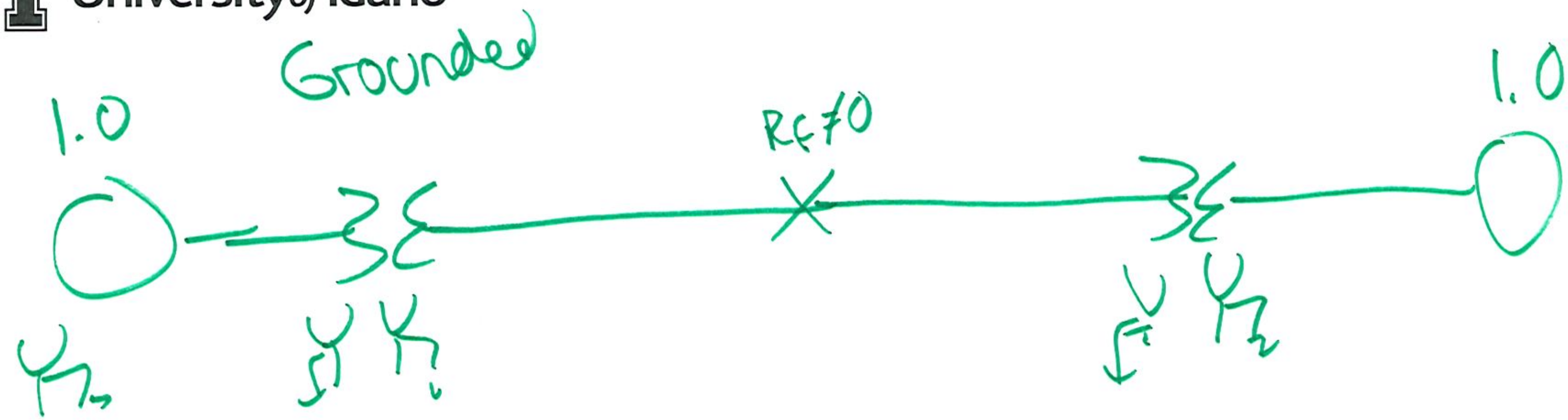
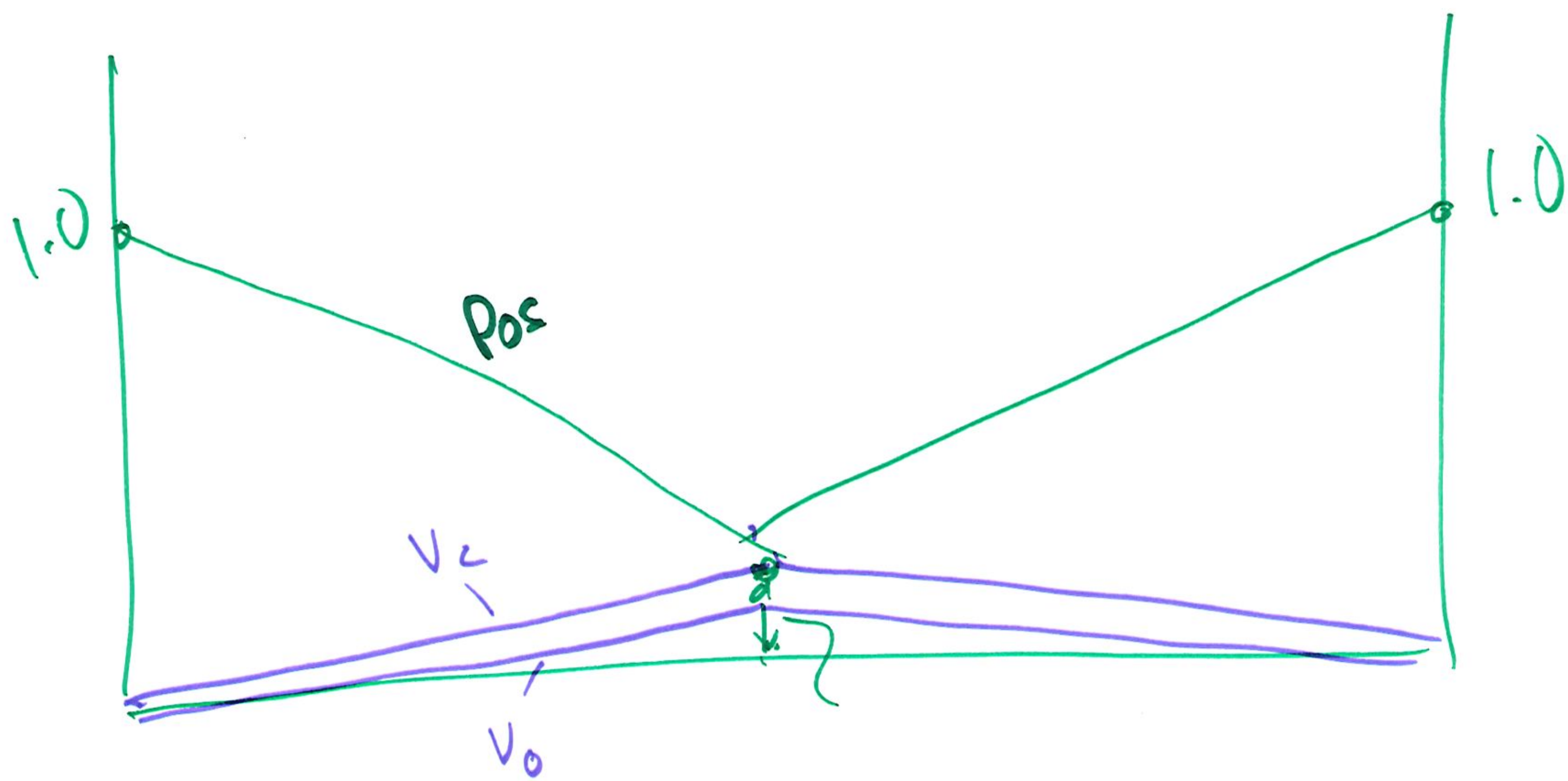
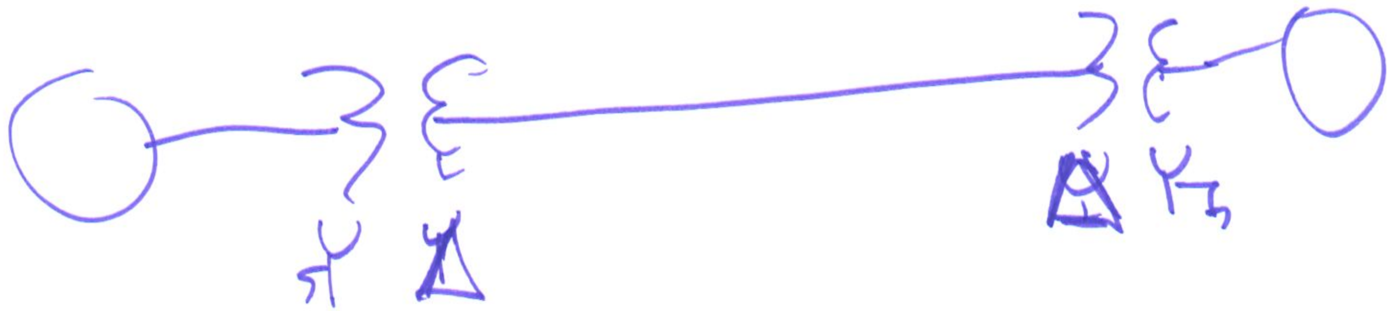
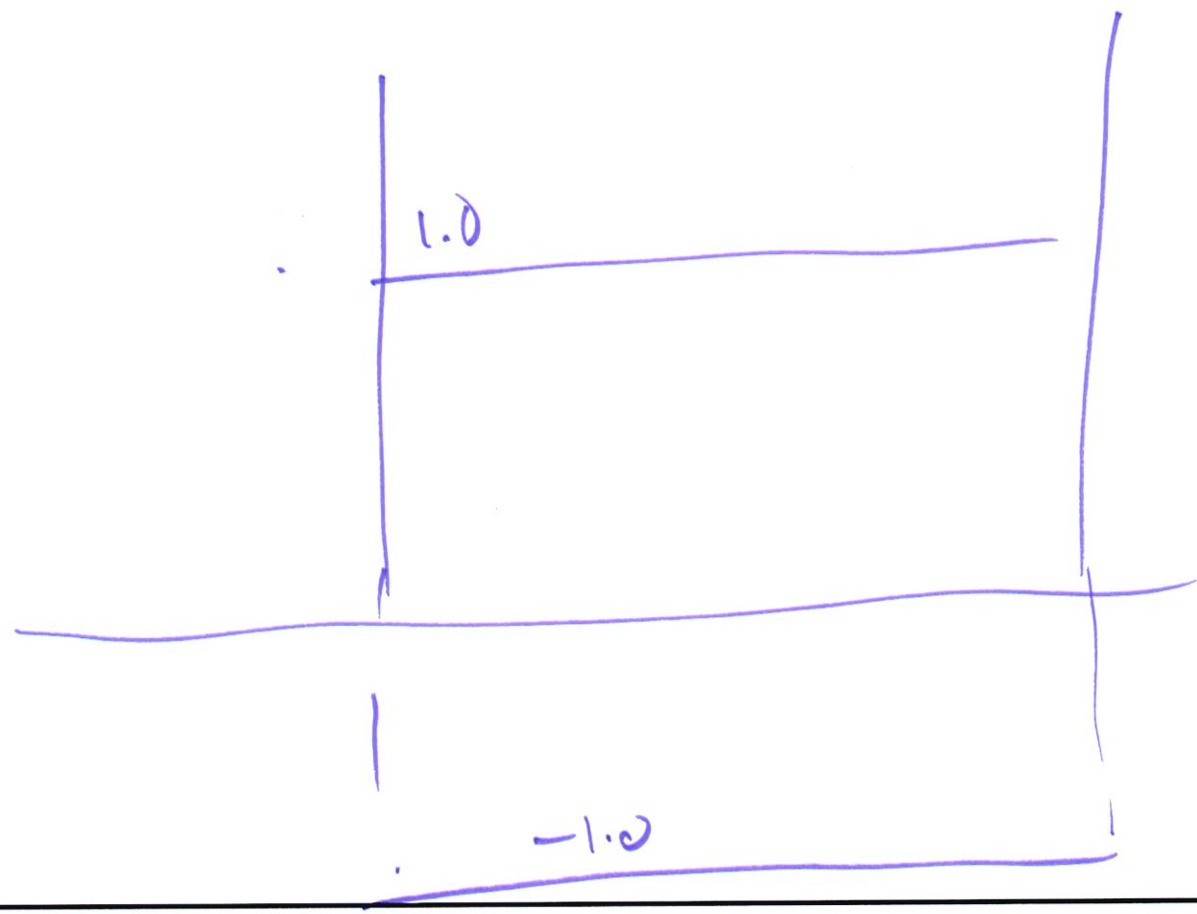




Ungrounded





SLG - general case

$I_{Bf} = I_{Cf} = 0$

$I_{Af} \neq 0$

$V_{AG} = I_{Af} \cdot Z_f$

⇒ symmetrical components domain

voltage equation

$$(V_{A0} + V_{A1} + V_{A2}) = (I_{A0} + I_{A1} + I_{A2}) \cdot R_f$$

current contraints

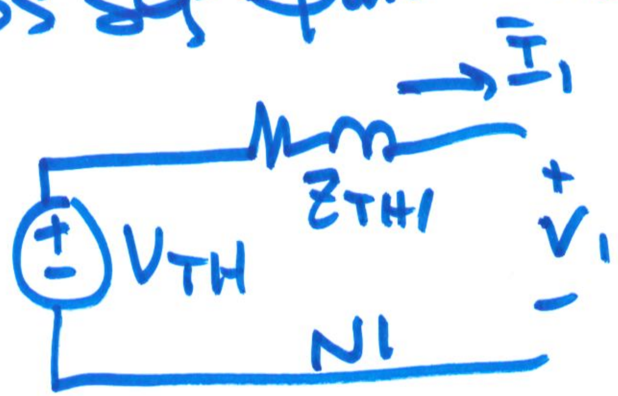
$$\begin{bmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \cdot \begin{bmatrix} I_{Af} \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{I}_{A0} = \frac{\bar{I}_{AF}}{3} = \bar{I}_{A1} = \bar{I}_{A2}$$

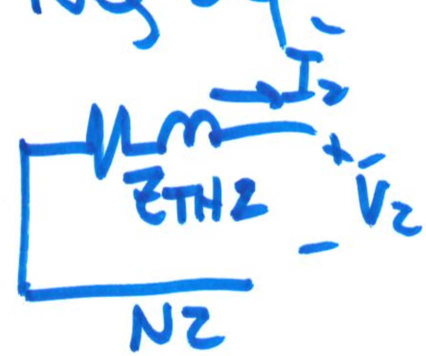
$$(V_{A0} + V_{A1} + V_{A2}) = 3 I_{A0} R_F$$

connect sequence networks based on this

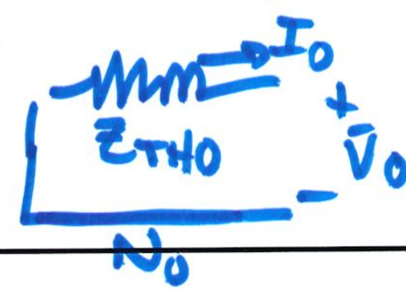
Pos seq equiv - Thevenin Equiv



Neg Seq



Zero seq



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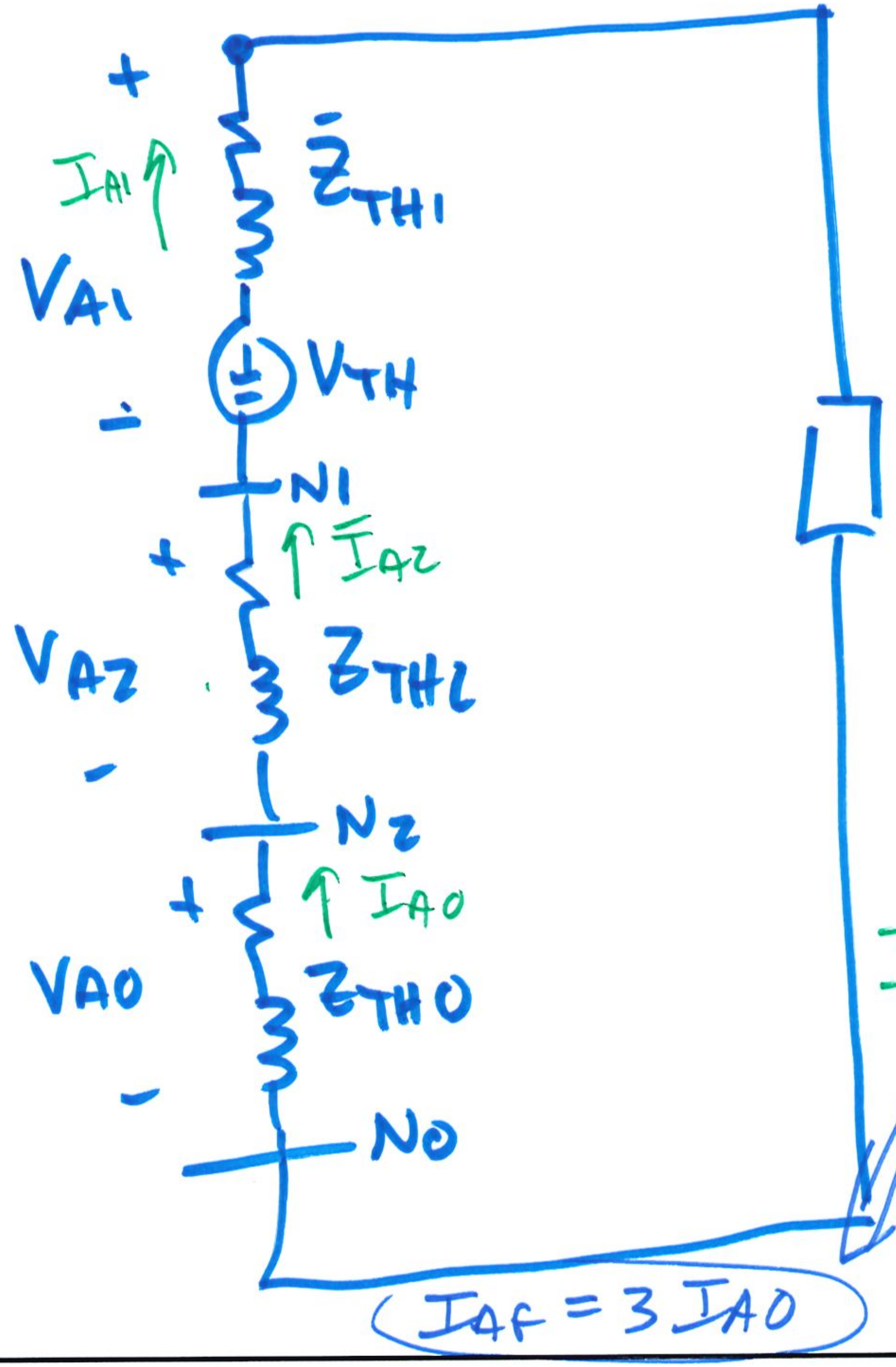

$$I_{A0} = \frac{\bar{V}_{TH}}{Z_{TH1} + Z_{TH2} + Z_{TH0} + 3R_f}$$

↑ solving the circuit

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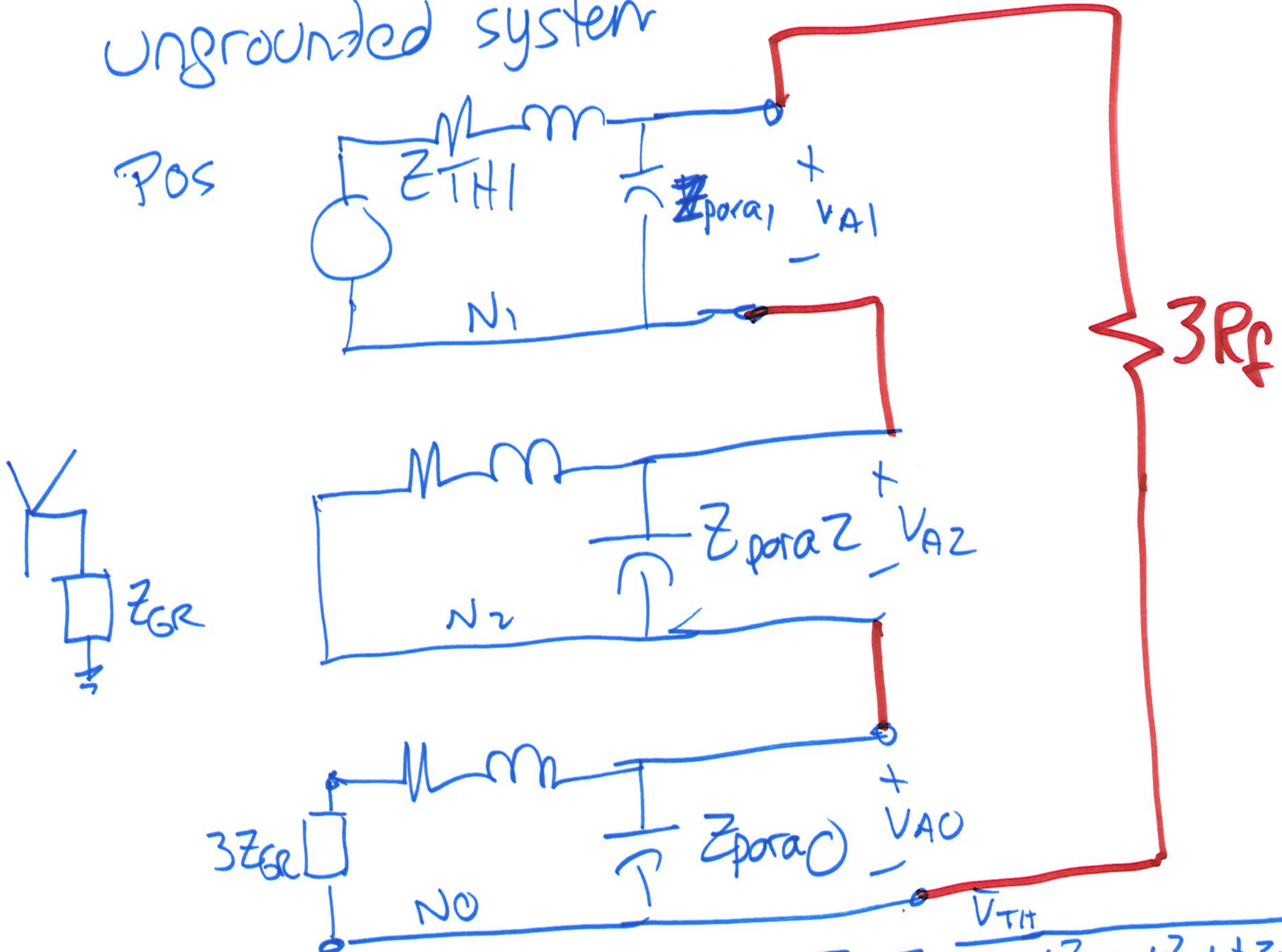
ungrounded system



$$I_{A0} = \frac{V_{TH}}{Z_{TH1} + Z_{TH2} + Z_{TH0} + 3Z_{gr} + 3R_f}$$

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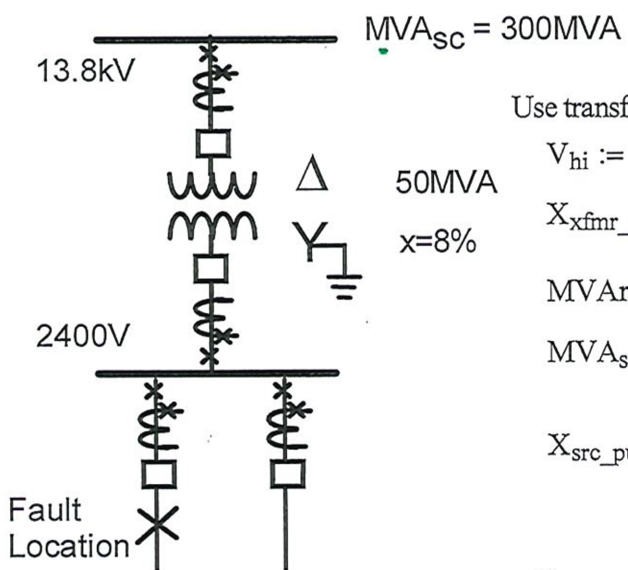
Unbalanced Fault Analysis Examples

$pu := 1$ $\qquad MVA := 1000kW$

$a := 1e^{j \cdot 120deg}$ $\qquad A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$

Compare the per unit phase currents seen on the HV side of the transformer below with those seen by at the fault location for three phase, SLG and LL faults the left side feeder. Assume wye connected CT's.



Use transformer ratings as the per unit base.

$V_{hi} := 13.8kV$ $\qquad V_{lo} := 2400V$

$X_{xfmr\_pu} := 0.08pu$

$MVArated := 50MVA$ $\qquad S_B := 50MVA$

$MVA_{sc} := 300MVA$

$X_{src\_pu} := \dfrac{(1.0pu)^2}{\left(\dfrac{MVA_{sc}}{S_B}\right)}$ $\qquad X_{src\_pu} = 0.17 \cdot pu$

convert to per unit

$V_{src} := 1.0pu$

$|\bar{I}_{3\phi}| = \dfrac{1pu}{|jX_{src}|}$

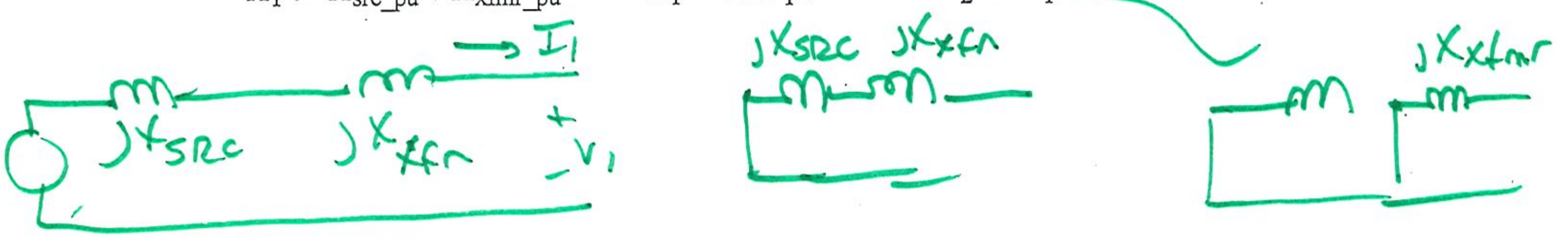
$MVA_{sc} = V \cdot I_{sc}$

$= 1 \cdot pu \cdot I_{sc}$

$= \dfrac{(1pu)^2}{|jX_{src}|}$

Sequence Impedances for Faults:

$X_1 := X_{src\_pu} + X_{xfmr\_pu}$ $\qquad X_1 = 0.25 \cdot pu$ $\qquad X_2 := X_1$



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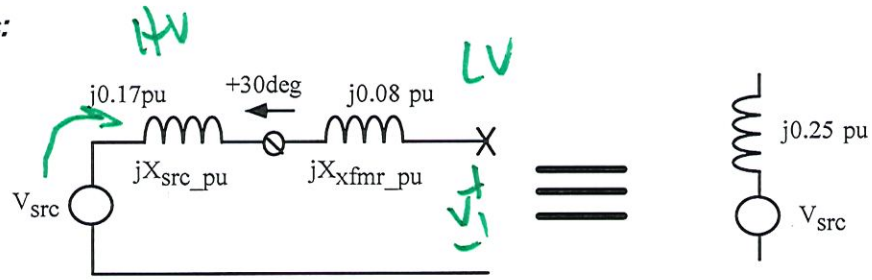
$X_{ground} := 0pu$

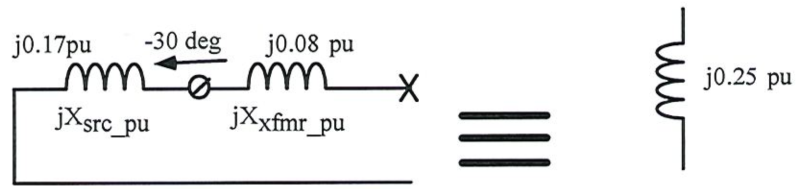
$X_0 := X_{xfmr\_pu} + 3 \cdot X_{ground}$

Sequence Networks:

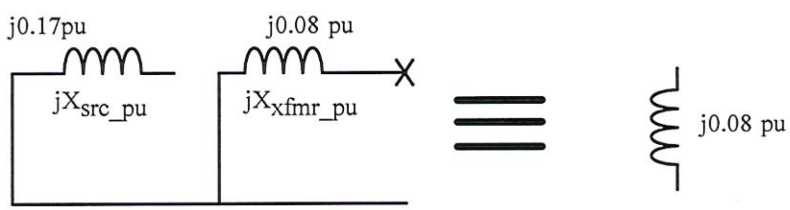
Positive Sequence



Negatve Sequence



Zero sequence



Three phase fault:

$I_{f\_3\phi} := \frac{V_{src}}{jX_1}$ $\qquad I_{f\_3\phi} = -4.05i \cdot pu$

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Per unit current magnitudes will be the same on the low side as they are on the high side. But there will be a plus 30 degree shift.

$$I_{f\_abc3ph} := I_{f\_3\phi} \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} \qquad \overrightarrow{\left|I_{f\_abc3ph}\right|} = \begin{pmatrix} 4.05 \\ 4.05 \\ 4.05 \end{pmatrix} \qquad \overrightarrow{\arg\left(I_{f\_abc3ph}\right)} = \begin{pmatrix} -90 \\ 150 \\ 30 \end{pmatrix} \cdot deg$$

$$I_{HV\_abc3ph} := I_{f\_3\phi} \cdot e^{j \cdot 30deg} \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} \qquad \overrightarrow{\left|I_{HV\_abc3ph}\right|} = \begin{pmatrix} 4.05 \\ 4.05 \\ 4.05 \end{pmatrix} \qquad \overrightarrow{\arg\left(I_{HV\_abc3ph}\right)} = \begin{pmatrix} -60 \\ 180 \\ 60 \end{pmatrix} \cdot deg$$

Voltage at the fault point:

$$V_{af} := V_{src} - j \cdot X_1 \cdot I_{f\_3\phi} \qquad V_{af} = 0 \cdot pu = I_f \cdot R_f = 0$$

Line to Line fault:

$$I_{0\_LLf} := 0pu \qquad R_f := 0pu$$

$$I_{1\_LLf} := \frac{V_{src}}{j \cdot X_1 + j \cdot X_2 + R_f} \qquad I_{1\_LLf} = -2.03i \cdot pu$$

$$I_{2\_LLf} := -I_{1\_LLf}$$

$$I_{abc\_LLf} := A_{012} \cdot \begin{pmatrix} I_{0\_LLf} \\ I_{1\_LLf} \\ I_{2\_LLf} \end{pmatrix} \qquad \overrightarrow{\left|I_{abc\_LLf}\right|} = \begin{pmatrix} 0 \\ 3.51 \\ 3.51 \end{pmatrix} \cdot pu$$

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Single Line to Ground fault:

$R_f = 0$

$$I_{0\_SLGf} := \frac{V_{src}}{j \cdot X_1 + j \cdot X_2 + j \cdot X_0 + 3 \cdot R_f} \qquad I_{0\_SLGf} = -1.74i \cdot pu$$

$$I_{1\_SLGf} := I_{0\_SLGf} \qquad I_{2\_SLGf} := I_{0\_SLGf}$$

$$I_{abc\_SLGf} := A_{012} \cdot \begin{pmatrix} I_{0\_SLGf} \\ I_{1\_SLGf} \\ I_{2\_SLGf} \end{pmatrix} \qquad \overrightarrow{\left|I_{abc\_SLGf}\right|} = \begin{pmatrix} 5.23 \\ 0 \\ 0 \end{pmatrix} \cdot pu$$

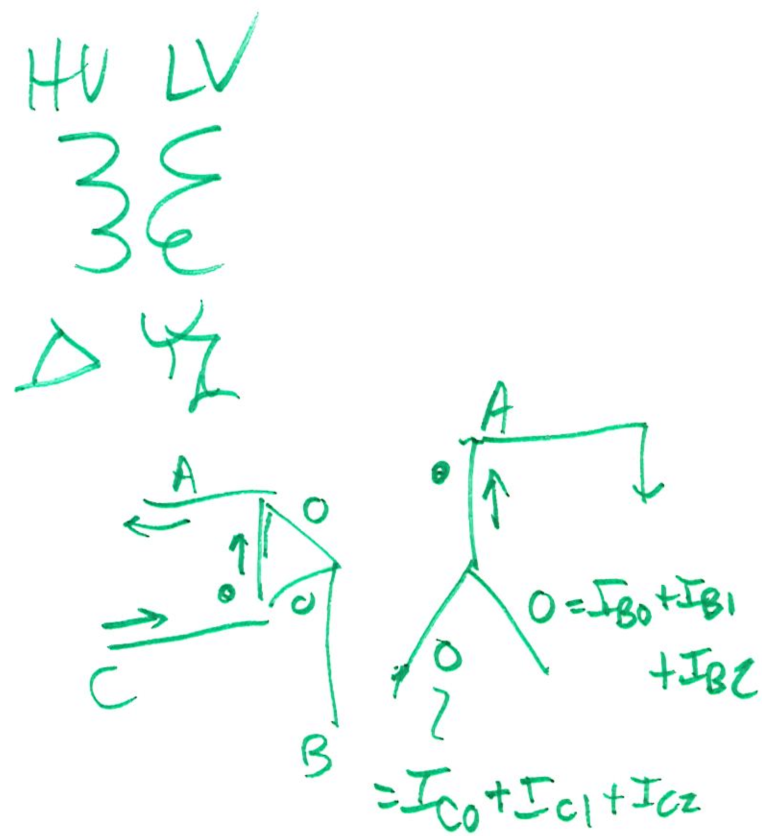
$$\overrightarrow{\arg\left(I_{abc\_SLGf}\right)} = \begin{pmatrix} -90 \\ 18.43 \\ 18.43 \end{pmatrix} \cdot deg$$

Now move the currents from the low side to the high side:

$$I_{1\_SLGH} := I_{1\_SLGf} \cdot e^{j \cdot 30deg} \qquad I_{0\_SLGH} := 0pu$$

$$I_{2\_SLGH} := I_{2\_SLGf} \cdot e^{-j \cdot 30deg}$$

$$I_{abc\_SLGh} := A_{012} \cdot \begin{pmatrix} I_{0\_SLGH} \\ I_{1\_SLGH} \\ I_{2\_SLGH} \end{pmatrix} \qquad \overrightarrow{\left|I_{abc\_SLGh}\right|} = \begin{pmatrix} 3.02 \\ 0 \\ 3.02 \end{pmatrix} \cdot pu$$



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Currents are lower on the high side and appear as a L-L fault not SLG

$$\overrightarrow{\arg(I_{abc\_SLGh})} = \begin{pmatrix} -90 \\ 90 \\ 90 \end{pmatrix} \cdot deg$$

Voltage at the fault point: $-Z_{TH1}$

$$V_{1\_SLG} := V_{src} - j \cdot X_1 \cdot I_{1\_SLGf} \qquad |V_{1\_SLG}| = 0.57 \cdot pu \qquad \arg(V_{1\_SLG}) = 0 \cdot deg$$

$$V_{2\_SLG} := 0 - j \cdot X_2 \cdot I_{2\_SLGf} \qquad |V_{2\_SLG}| = 0.43 \cdot pu \qquad \arg(V_{2\_SLG}) = 180 \cdot deg$$

$$V_{0\_SLG} := 0 - j \cdot X_0 \cdot I_{0\_SLGf} \qquad |V_{0\_SLG}| = 0.14 \cdot pu \qquad \arg(V_{0\_SLG}) = 180 \cdot deg$$

$\bar{V}_1 + \bar{V}_2 + \bar{V}_0 = 3R_f I_{A0}$

$$V_{abc\_SLG} := A_{012} \cdot \begin{pmatrix} V_{0\_SLG} \\ V_{1\_SLG} \\ V_{2\_SLG} \end{pmatrix} \qquad \overrightarrow{|V_{abc\_SLG}|} = \begin{pmatrix} 0 \\ 0.89 \\ 0.89 \end{pmatrix} \cdot pu$$

$$\arg(V_{abc\_SLG_1}) = -103.59 \cdot deg$$

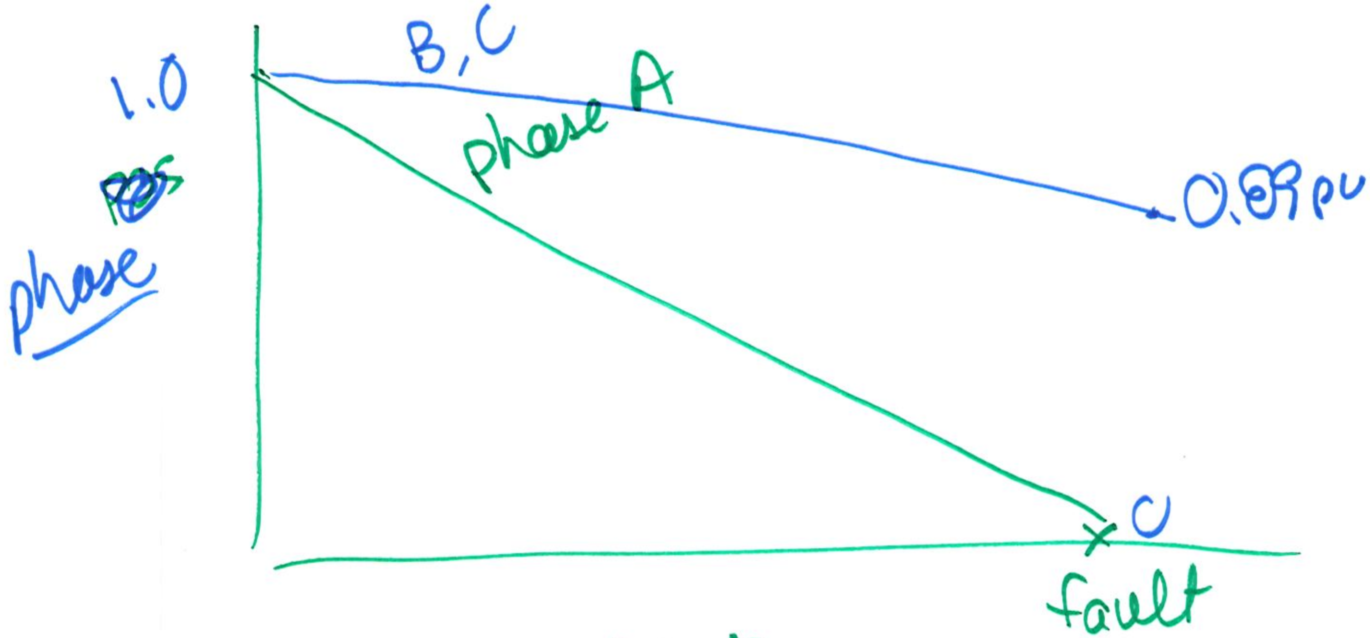
$$\arg(V_{abc\_SLG_2}) = 103.59 \cdot deg$$

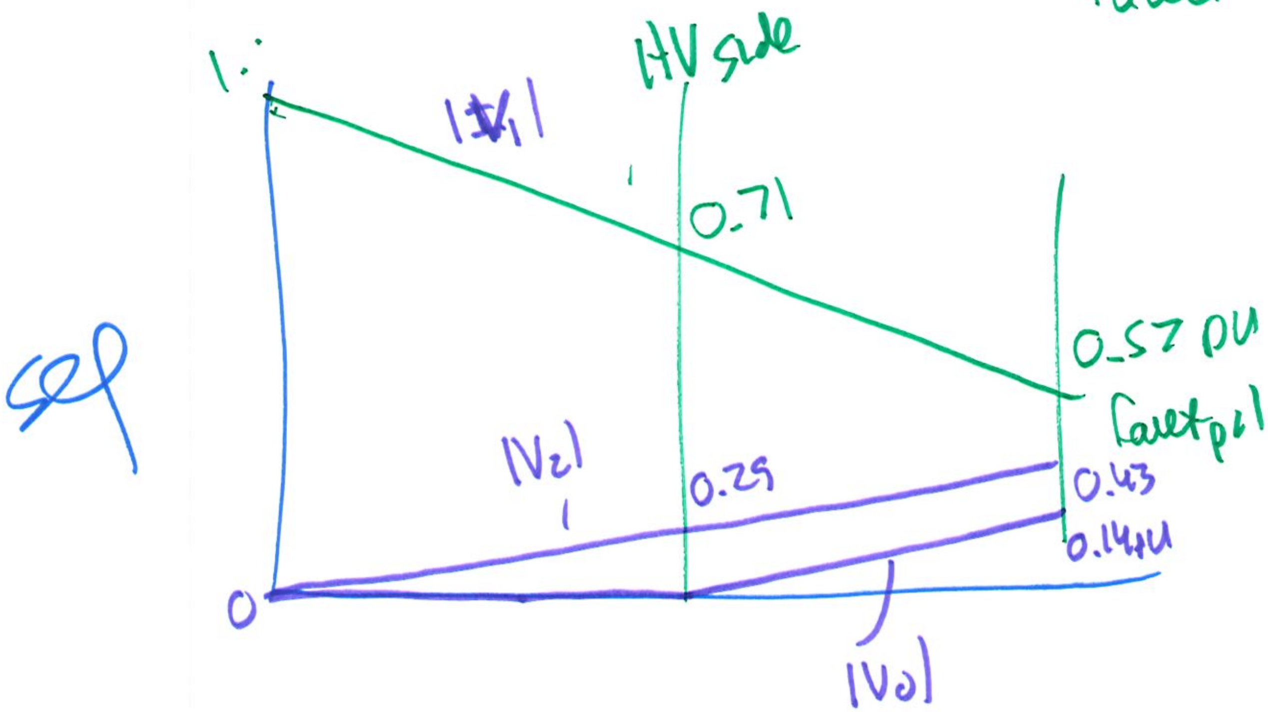
Voltage on 13.kV bus (refer to HV side)

$$V_{1\_SLGHV} := \left(V_{src} \cdot e^{j \cdot 30deg} - j \cdot X_{src\_pu} \cdot I_{1\_SLGH}\right) \qquad |V_{1\_SLGHV}| = 0.71 \cdot pu \qquad \arg(V_{1\_SLGHV}) = 30 \cdot deg$$

$$V_{2\_SLGHV} := \left(0 - j \cdot X_{src\_pu} \cdot I_{2\_SLGH}\right) \qquad |V_{2\_SLGHV}| = 0.29 \cdot pu \qquad \arg(V_{2\_SLGHV}) = 150 \cdot deg$$

$$V_{0\_SLGHV} := 0 - j \cdot X_{src\_pu} \cdot I_{0\_SLGH} \qquad |V_{0\_SLGHV}| = 0 \cdot pu$$


1.0
phase
B, C
phase A
0.89 pu
0
fault


1
HV side
$|V_1|$
0.71
0.57 pu
$fault_{p.l}$
seq
$|V_2|$
0.29
0.43
0.14 pu
0
$|V_0|$