Connect circuits

R_f, R_g, R_h, R_r

3

1

V_0

1

V_1

V_2
Powerworld Results

\[ I_{B,LL} := 3.997 \angle (180\text{deg}) \]

<table>
<thead>
<tr>
<th>Fault Data - Buses</th>
<th>Name</th>
<th>Phase Volt A</th>
<th>Phase Ang A</th>
<th>Phase Volt B</th>
<th>Phase Ang B</th>
<th>Phase Volt C</th>
<th>Phase Ang C</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUS S</td>
<td>0.96384</td>
<td>-26.04</td>
<td>0.96384</td>
<td>-153.96</td>
<td>0.84615</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>BUS 1</td>
<td>0.83294</td>
<td>0</td>
<td>0.83294</td>
<td>-126.89</td>
<td>0.83294</td>
<td>126.89</td>
<td></td>
</tr>
<tr>
<td>BUS 2</td>
<td>0.60079</td>
<td>0</td>
<td>0.60079</td>
<td>-146.33</td>
<td>0.60079</td>
<td>146.33</td>
<td></td>
</tr>
<tr>
<td>BUS R</td>
<td>0.91485</td>
<td>-18.8</td>
<td>0.91485</td>
<td>-161.2</td>
<td>0.58974</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>FaultPt</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
<td>180</td>
<td>0.5</td>
<td>-180</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fault Data - Lines</th>
<th>From Name</th>
<th>To Name</th>
<th>Phase Cur A From</th>
<th>Phase Ang A From</th>
<th>Phase Cur B From</th>
<th>Phase Ang B From</th>
<th>Phase Cur C From</th>
<th>Phase Ang C From</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUS S</td>
<td>BUS 1</td>
<td></td>
<td>1.53846</td>
<td>-180</td>
<td>1.53846</td>
<td>-180</td>
<td>3.07692</td>
<td>0</td>
</tr>
<tr>
<td>BUS 1</td>
<td>BUS 2</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>BUS 1</td>
<td>FaultPt</td>
<td></td>
<td>0</td>
<td>0</td>
<td>2.66469</td>
<td>-180</td>
<td>2.66469</td>
<td>0</td>
</tr>
<tr>
<td>BUS R</td>
<td>BUS 2</td>
<td></td>
<td>0.76923</td>
<td>-180</td>
<td>0.76923</td>
<td>-180</td>
<td>1.53846</td>
<td>0</td>
</tr>
<tr>
<td>FaultPt</td>
<td>BUS 2</td>
<td></td>
<td>0</td>
<td>0</td>
<td>1.33235</td>
<td>0</td>
<td>1.33235</td>
<td>-180</td>
</tr>
</tbody>
</table>

**DLG Fault:**

\[
I_{1,DLG}(M, R_f) := \frac{V_f}{\left(Z_1(M, 4, 4) + \left[\frac{1}{Z_2(M, 4, 4)} + \frac{1}{Z_0(M, 4, 4) + 3 \cdot R_f}\right]\right)^{-1}}
\]

\[
|I_{1,DLG}(0.5, 0)| = 2.7269 \quad \text{arg}(I_{1,DLG}(0.5, 0)) = -90\text{-deg}
\]

\[ I_1(M, R_f, R_G) = (Z_{1th} + R_f) + \left[\frac{1}{Z_{CTH} + R_f} + \frac{1}{Z_{0th} + R_f + \frac{Z_{R}}{3}}\right] \]
Current dividers for $I_0$ and $I_2$

$$I_{0\_DLG}(M, R_f) := -I_{1\_DLG}(M, R_f) \cdot \frac{Z_2(M)_{4,4}}{Z_2(M)_{4,4} + (Z_0(M)_{4,4} + 3\cdot R_f)}$$

$$I_{2\_DLG}(M, R_f) := -I_{1\_DLG}(M, R_f) \cdot \frac{Z_0(M)_{4,4} + 3\cdot R_f}{Z_2(M)_{4,4} + (Z_0(M)_{4,4} + 3\cdot R_f)}$$

$$I_{ABC\_DLG}(M, R_f) := A_{012} \cdot \begin{bmatrix} I_{0\_DLG}(M, R_f) \\ I_{1\_DLG}(M, R_f) \\ I_{2\_DLG}(M, R_f) \end{bmatrix}$$

$$|I_{ABC\_DLG}(0.5, 0)| = \begin{bmatrix} 0 \\ 4.1902 \\ 4.1902 \end{bmatrix} \cdot \text{pu}$$

$$\arg(I_{ABC\_DLG}(0.5, 0)) = \begin{bmatrix} -90 \\ 162.5359 \\ 17.4641 \end{bmatrix} \cdot \text{deg}$$

- Angles meaningless when magnitude is 0

Now find voltages in each sequence component

$$\Delta V_{1\_DLG}(M, R_f) := Z_1(M) \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_{1\_DLG}(M, R_f) \end{bmatrix}$$

$$|\Delta V_{1\_DLG}(0.5, 0)| = \begin{bmatrix} 0.0909 \\ 0.1363 \\ 0.3636 \\ 0.2424 \\ 0.5908 \end{bmatrix}$$

$$\arg(\Delta V_{1\_DLG}(0.5, 0)) = \begin{bmatrix} 150 \\ 180 \\ 180 \end{bmatrix} \cdot \text{deg}$$
\[ I_{1\_B1\_DLG}(M, R_f) := \frac{V_{1\_DLG}(M, R_f) - V_{1\_DLG}(M, R_f)_0}{M \cdot Z_{L11}} \]

\[ |I_{1\_B1\_DLG}(0.5, 0)| = 1.8179 \quad \text{arg}(I_{1\_B1\_DLG}(0.5, 0)) = -90\text{-deg} \]

\[ I_{2\_B1\_DLG}(M, R_f) := \frac{V_{2\_DLG}(M, R_f) - V_{2\_DLG}(M, R_f)_0}{M \cdot Z_{L12}} \]

\[ |I_{2\_B1\_DLG}(0.5, 0)| = 1.259 \quad \text{arg}(I_{2\_B1\_DLG}(0.5, 0)) = 90\text{-deg} \]

\[ I_{0\_B1\_DLG}(M, R_f) := \frac{V_{0\_DLG}(M, R_f) - V_{0\_DLG}(M, R_f)_0}{M \cdot Z_{L10}} \]

\[ |I_{0\_B1\_DLG}(0.5, 0)| = 0.4424 \quad \text{arg}(I_{0\_B1\_DLG}(0.5, 0)) = 90\text{-deg} \]

\[ I_{ABC\_B1\_DLG}(M, R_f) := A_{012} \begin{pmatrix} I_{c\_B1\_DLG}(M, R_f) \\ I_{1\_B1\_DLG}(M, R_f) \\ I_{2\_B1\_DLG}(M, R_f) \end{pmatrix} \]

\[ |I_{ABC\_B1\_DLG}(0.5, 0)| = \begin{pmatrix} 0.1165 \\ 2.7607 \\ 2.7607 \end{pmatrix} \cdot \text{pu} \quad \text{arg}(I_{ABC\_B1\_DLG}(0.5, 0)) = \begin{pmatrix} -90 \\ 164.8436 \\ 15.1564 \end{pmatrix} \text{-deg} \]

- LV side of transformer (Bus S)

\[ I_{1\_BS\_DLG}(M, R_f) := \frac{V_S \cdot e^{-j \cdot 30\text{deg}} - V_{1\_DLG}(M, R_f)_0}{jX_{G11}} \]

\[ |I_{1\_BS\_DLG}(0.5, 0)| = 1.8179 \cdot \text{pu} \quad \text{arg}(I_{1\_BS\_DLG}(0.5, 0)) = -120\text{-deg} \]

\[ I_{2\_BS\_DLG}(M, R_f) := \frac{0 - V_{2\_DLG}(M, R_f)_0}{jX_{G12}} \]

\[ |I_{2\_BS\_DLG}(0.5, 0)| = 1.259 \cdot \text{pu} \quad \text{arg}(I_{2\_BS\_DLG}(0.5, 0)) = 120\text{-deg} \]

\[ I_{0\_BS\_DLG} := 0 \]
- DLG sequence volt & current signatures

- If no Rg or Rc - at fault location

  \[ V_1 \quad V_2 \quad V_0 \]

  \[ \uparrow \quad \uparrow \quad \uparrow \rightarrow A \text{ rel comp} \]

  for BCG

Suppose ACG fault with Anel

\[ V_0 \]

\[ \uparrow \]
with $R_0 \neq 0$, BCG, ANL

$V_1 \uparrow \uparrow \uparrow V_2 \uparrow \uparrow \uparrow V_0$

If $R_C \neq 0$ and $R_0 = 0$

If $Z_{24TH} = Z_{OTH}$

$V_1 \uparrow \uparrow \uparrow V_2 \uparrow \uparrow \uparrow V_0$

If both $Z_{OTH} > Z_{24TH}$
Sequence currents

DLG at fault location

\[ \begin{align*}
I_2 & \quad I_1 \\
I_0 & \\
\end{align*} \]

\{ sum to 0

\[ I_{AF} = 0 \]

LL

\[ \begin{align*}
I_2 & \quad I_1 \\
\end{align*} \]

SLG

\[ \begin{align*}
I_0 & \\
I_1 & \\
I_2 & \\
\end{align*} \]
Add Power Flow

For fault analysis, it matters more when:

1. High fault resistance
2. Heavily loaded cases
   - Weak system → high source impedances
      → High source impedance ratio
         \[
         \frac{Z_{S}}{Z_{L.m}}
         \]
3. Reproduce a field event
Ways to model 1000 conditions

1. Add a load impedance at Bus where it is located

   (Solve pre-fault power flow - maybe)

2. Simply use pre-fault power flow results at each bus
Set line impedance parameters (set zero sequence line impedances to 3 times the positive sequence values):

\[ Z_{L,23} := j \cdot 0.05 \text{pu} \quad B_{c,23} := 0.005 \cdot \text{pu} \]
\[ Z_{L,25} := j \cdot 0.35 \text{pu} \quad B_{c,25} := 0.035 \cdot \text{pu} \]
\[ Z_{L,35} := j \cdot 0.35 \text{pu} \quad B_{c,35} := 0.035 \cdot \text{pu} \]

Transformer 1 Change of base (zero sequence impedances match positive sequence):

\[ X_{t,11} := 0.1 \cdot \left( \frac{S_B}{200 \text{MVA}} \right) \quad X_{t,11} = 0.05 \cdot \text{pu} \quad X_{t,10} := X_{t,11} \]

Transformer 2 Change of base (zero sequence impedances match positive sequence):

\[ X_{t,21} := 0.1 \cdot \left( \frac{S_B}{200 \text{MVA}} \right) \quad X_{t,21} = 0.05 \cdot \text{pu} \quad X_{t,20} := X_{t,11} \]

A. Assuming the load at Bus 5 is 100MW at 0.9 lagging power factor, perform a power flow solution. Use the voltage at Bus 3 as your angle reference.

Options,

1. Solve the power flow equations for the entire system using Mathcad solve blocks
2. Use Powerworld or a similar load flow problem (at least to check results)

- In order to use V3 as the reference angle, use the angle for V3 from the power flow solution and shift the slack bus angle such that the new angle at Bus 3 is 0.

**1) Solve Full Powerflow Solution Using MathCAD Solve Block**

- Positive sequence Y bus for power flow calculations (ignore phase shifts for the moment):

\[ Y_{11} := \frac{1}{j \cdot X_{t,11}} \quad Y_{12} := -\frac{1}{j \cdot X_{t,11}} \quad \text{Symmetry assumed} \]
\[
Y_{22} := \frac{1}{j\cdot X_{t11}} + \frac{1}{Z_{L23}} + \frac{1}{j\cdot B_{c23}} \frac{1}{2} + \frac{j\cdot B_{c25}}{2} \quad Y_{23} := \frac{-1}{Z_{L23}} \quad Y_{25} := \frac{-1}{Z_{L25}}
\]
\[
Y_{33} := \frac{1}{j\cdot X_{t11}} + \frac{1}{Z_{L23}} + \frac{1}{j\cdot B_{c23}} \frac{1}{2} + \frac{j\cdot B_{c35}}{2} \quad Y_{34} := \frac{-1}{j\cdot X_{t11}} \quad Y_{35} := \frac{-1}{Z_{L35}}
\]
\[
Y_{44} := \frac{1}{j\cdot X_{t11}} \quad Y_{55} := \frac{1}{Z_{L25}} + \frac{1}{Z_{L35}} + \left( \frac{j\cdot B_{c25}}{2} + \frac{j\cdot B_{c35}}{2} \right)
\]

\[
Y_{\text{busPF}} := \begin{pmatrix}
Y_{11} & Y_{12} & 0 & 0 & 0 \\
Y_{12} & Y_{22} & Y_{23} & 0 & Y_{25} \\
0 & Y_{23} & Y_{33} & Y_{34} & Y_{35} \\
0 & 0 & Y_{34} & Y_{44} & 0 \\
0 & 0 & Y_{25} & Y_{35} & Y_{55}
\end{pmatrix}
\]

\[
Y_{\text{busPF}} = \begin{pmatrix}
-20i & 20i & 0 & 0 & 0 \\
20i & -42.8371i & 20i & 0 & 2.8571i \\
0 & 20i & -42.8371i & 20i & 2.8571i \\
0 & 0 & 20i & -20i & 0 \\
0 & 2.8571i & 2.8571i & 0 & -5.6793i
\end{pmatrix}
\]

\[
P2 := 0 \quad P3 := 0 \quad P4 := 0.5\text{pu} \quad P5 := -1.0\text{pu}
\]

\[
Q2 := 0 \quad Q3 := 0 \quad Q5 := P5 \cdot \tan(\cos(0.9)) \quad Q5 = -0.4843\text{pu}
\]

- P5 and Q5 are negative injections since they represent a load

- Initial guesses

\[
V1 := 1 \quad V4 := 1 \quad V1 := 1 \quad V3 := 1 \quad a1 := 0 \quad a2 := 0\text{deg} \quad a3 := 0\text{deg}
\]
• Note, that if we include the transformer phase shift, we need to subtract 30 degrees from the voltages at Bus 1 and Bus 4:

\[
\begin{align*}
\theta_{1\,LV} &= a1 - 30\text{deg} & \theta_{1\,LV} &= -30\text{-deg} \\
\theta_{4\,LV} &= \theta_4 - 30\text{deg} & \theta_{4\,LV} &= -30\text{-deg}
\end{align*}
\]

• Finally, we need to shift the angle, by subtracting the angle of \( \theta_3 \) to each of the bus angles such that angle at Bus 3 = 0:

\[
\begin{align*}
\theta_{1\,p} &= \theta_{1\,LV} - \theta_3 & \theta_{1\,p} &= -28.541\text{-deg} \\
\theta_{2\,p} &= \theta_2 - \theta_3 & \theta_{2\,p} &= 0\text{-deg} \\
\theta_{3\,p} &= \theta_3 - \theta_3 & \theta_{3\,p} &= 0\text{-deg} \\
\theta_{4\,p} &= \theta_{4\,LV} - \theta_3 & \theta_{4\,p} &= -28.541\text{-deg} \\
\theta_{5\,p} &= \theta_5 - \theta_3 & \theta_{5\,p} &= -11.8393\text{-deg}
\end{align*}
\]

• Calculate generator currents (note that you need to account for transformer phase shift)

\[
I_{gen1} := \frac{V_1 \cdot e^{j\theta_{1\,p}} - mV_2 \cdot e^{j\theta_{2\,p}} \cdot (e^{-j \cdot 30\text{deg}})}{(j \cdot X_{t11})} \\
|I_{gen1}| = 0.6216\cdot\text{pu} \\
\text{arg}(I_{gen1}) = -64.9864\text{-deg}
\]

\[
S_{gen1} := V_1 \cdot e^{j\theta_{1\,p}} \cdot I_{gen1} \\
S_{gen1} = (0.5 + 0.3692i)\cdot\text{pu}
\]

\[
I_{gen2} := \frac{V_4 \cdot e^{j\theta_{4\,p}} - mV_3 \cdot e^{j\theta_{3\,p}} \cdot (e^{-j \cdot 30\text{deg}})}{(j \cdot X_{t21})} \\
|I_{gen2}| = 0.6216\cdot\text{pu} \\
\text{arg}(I_{gen2}) = -64.9864\text{-deg}
\]

\[
S_{gen2} := V_4 \cdot e^{j\theta_{4\,p}} \cdot I_{gen2} \\
S_{gen2} = (0.5 + 0.3692i)\cdot\text{pu}
\]
Fault Calculations with Power Flow

- This is much easier to do with Zbus methods. This approach only works for simple systems.
- The system below has a pre-fault power flow condition due to the angle and magnitude differences between the sources.
- The fault calculations need to change a little to ensure that the positive sequence current reflects this power flow in the case of a fault where power flow can continue to flow.
- Let's look at a SLG fault case.

![Diagram showing fault calculations](image)

- The negative and zero sequence circuits will be the same as one would in a case where the sources have equal angles and magnitudes, so they will not be described here.
- Positive sequence equivalent circuit:

![Positive sequence equivalent circuit](image)

- There are effectively two components to the current seen at each relay, and they can be determined using superposition.
  1. The fault current that flows due to the fault and leave this network at point F and reenters from the neutral plane.
  2. The current that flows between the two sources, the load current.

1. Determining fault current

- We need to find a Thevenin equivalent circuit.
- The process is actually a standard circuit analysis approach (Millman's Theorem), but is typically avoided if the voltage sources are all assumed to have the same magnitude and angle.

1. Convert the two sources to their Norton equivalents, using the impedance between the source and the fault point. Note that these are phasor calculations.

\[
Z_{\text{left}} = Z_{S1} + m \cdot Z_{L1} \\
I_{\text{Norton \_ left}} = \frac{V_{S1}}{Z_{\text{left}}} \\
Z_{\text{right}} = Z_{R1} + (1 - m) \cdot Z_{L1} \\
I_{\text{Norton \_ right}} = \frac{V_{S1}}{Z_{\text{right}}} 
\]
2. Note that the impedances are in parallel and the current sources are effectively in parallel
   a. Combine the impedances in parallel
   b. Combine the two current sources (note that this is not limited to two sources)

\[
Z_{\text{equiv1}} = \left( \frac{1}{Z_{\text{left1}}} + \frac{1}{Z_{\text{right1}}} \right)^{-1}
\]

\[
I_{\text{equiv1}} = I_{\text{Norton_left}} + I_{\text{Norton_right}}
\]

   c. Then convert back to a Thevenin equivalent

\[
Z_{\text{thev1}} = Z_{\text{equiv1}}
\]

\[
V_{\text{thev1}} = I_{\text{equiv1}} \cdot Z_{\text{equiv1}}
\]

- This Thevenin equivalent source is used for the fault calculations. *But not for the power flow calculation*
  1. Note that the Thevenin impedance is the same as we always do.
  2. Now the voltage source has a magnitude and angle that reflects the difference between the two sources.
  3. If the sources both have the same magnitude and angle, the resulting Thevenin voltage source will match that.
2. Determining power flow current

- This is just like any other power flow calculation. In this case you can look between the two known source voltages and the total impedance between them. In other cases you might need to find V1 and V2 and just use the line impedance.

\[
I_{12} = \frac{V_{S1} - V_{R1}}{Z_{S1} + Z_{L1} + Z_{R1}}
\]

\[
I_{21} = \frac{V_{R1} - V_{S1}}{(Z_{S1} + Z_{L1} + Z_{R1})}
\]

- Notes:
  1. The fault location doesn't matter in this calculation
  2. The Thevenin equivalent source from above is not used
  3. \( I_{12} \) flows in the opposite direction \( I_{21} \)

3. Total sequence currents

- The positive sequence current for the relay at bus 1 (phasor sums):

\[
I_{\text{relay1}} = I_{F_{\text{relay1}}} + I_{12_{\text{relay1}}}
\]

\[
I_{\text{relay2}} = I_{F_{\text{relay2}}} - I_{12_{\text{relay1}}}
\]

- \( I_{F_{\text{relay1}}} \) and \( I_{F_{\text{relay2}}} \) come from current dividers as usual

- The negative and zero sequence currents do not include an load flow current and are simply from current dividers from the fault calculation.
• For faults on Line 2:

\[ Z_{L2_1\_thev}(n) := \left[ \frac{1}{Z_{S1} + Z_{L11} + n \cdot Z_{L21}} + \frac{1}{(1 - n) \cdot Z_{L21} + Z_{R1}} \right]^{-1} \]

\[ Z_{L2_2\_thev}(n) := \left[ \frac{1}{Z_{S2} + Z_{L12} + n \cdot Z_{L22}} + \frac{1}{(1 - n) \cdot Z_{L22} + Z_{R2}} \right]^{-1} \]

\[ Z_{L2_0\_thev}(n) := \left[ \frac{1}{Z_{S0} + Z_{L10} + n \cdot Z_{L20}} + \frac{1}{(1 - n) \cdot Z_{L20} + Z_{R0}} \right]^{-1} \]

**Impedance Matrix Approach**

• Need positive, negative and zero sequence matrices

\[ Y_{bus1}(m) := \begin{bmatrix}
\frac{1}{Z_{S1}} + \frac{1}{Z_{L11}} & -\frac{1}{Z_{L11}} & 0 & 0 \\
-\frac{1}{Z_{L11}} & \frac{1}{Z_{L11}} + \frac{1}{m \cdot Z_{L21}} & 0 & -\frac{1}{m \cdot Z_{L21}} \\
0 & 0 & \frac{1}{(1 - m) \cdot Z_{L21}} + \frac{1}{Z_{R1}} & -\frac{1}{(1 - m) \cdot Z_{L21}} \\
0 & \frac{-1}{m \cdot Z_{L21}} & -\frac{1}{(1 - m) \cdot Z_{L21}} & \frac{1}{m \cdot Z_{L21}} + \frac{1}{(1 - m) \cdot Z_{L21}}
\end{bmatrix} \]
Fault Analysis with Power Flow on the System

\[ pu := 1 \quad MVA := 1000\text{kW} \]

\[ a := 1e^{j \cdot 120\text{deg}} \]

\[ A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \]

- **Example with two sources:**

![Diagram of electrical system](image)

- **Per unit line impedances calculated with \( S_B=100\text{MVA} \) and \( V_B=345\text{kV LL} \):**

- **Data:**
  - \( V_S = 1.0\text{pu} @ 0\text{deg} \)
  - \( Z_{S1} = j0.03\text{ pu} \)
  - \( Z_{S2} = Z_{S1} \)
  - \( Z_{S0} = 3\cdot Z_{S1} \)
  - \( Z_{L11} := 0.1\text{pu} \cdot e^{j \cdot 85\text{deg}} \)
  - \( Z_{L11} = 0.01 + 0.1i \)
  - \( Z_{L10} := 3\cdot Z_{L11} \)
  - \( Z_{L10} = 0.03 + 0.3i \)
  - \( Z_{L21} := 0.08\text{pu} \cdot e^{j \cdot 85\text{deg}} \)
  - \( Z_{L21} = 0.01 + 0.08i \)
  - \( Z_{L20} := 3\cdot Z_{L21} \)
  - \( Z_{L20} = 0.02 + 0.24i \)

- **Data at BUS R:**
  - \( Z_{S1} := j0.03\text{pu} \)
  - \( Z_{S2} := Z_{S1} \)
  - \( Z_{S0} := 3Z_{S1} \)
  - \( Z_{R1} := j0.06\text{pu} \)
  - \( Z_{R2} := Z_{R1} \)
  - \( Z_{R0} := 3Z_{R1} \)
Now add power flow based on phase angle differences

- Make Bus S the slack bus at 1.0pu
- Set Bus R magnitude and angle

\[ V_{S1} := 1.0\text{pu} e^{-j0\text{deg}} \]
\[ V_{R1} := 1.02\text{pu} e^{-j10.074\text{deg}} \]

- This case is simple enough that we don't need to do a normal power flow solution.

\[ I_{SR1} := \frac{V_{S1} - V_{R1}}{Z_{L11} + Z_{L21}} \quad |I_{SR1}| = 0.99\text{-pu} \quad \arg(I_{SR1}) = 6.37\text{-deg} \]
\[ V_{B1} := V_{S1} - I_{SR1} \cdot Z_{L11} \quad |V_{B1}| = 1.01\text{-pu} \quad \arg(V_{B1}) = -5.65\text{-deg} \]

- But for fault analysis we need the voltages behind the source impedances

\[ V_{\text{src}_S} := V_{S1} + I_{SR1} \cdot Z_{S1} \quad |V_{\text{src}_S}| = 0.997 \quad \arg(V_{\text{src}_S}) = 1.7\text{-deg} \]
\[ V_{\text{src}_R} := V_{R1} - I_{SR1} \cdot Z_{R1} \quad |V_{\text{src}_R}| = 1.038 \quad \arg(V_{\text{src}_R}) = -13.22\text{-deg} \]

\[ I_{s1\text{Nor}(m)} := \frac{V_{\text{src}_S}}{Z_{S1} + Z_{L11} + m \cdot Z_{L21}} \quad I_{r1\text{Nor}(m)} := \frac{V_{\text{src}_R}}{Z_{R1} + (1 - m) \cdot Z_{L21}} \]
\[ I_{1\text{-Nor}(m)} := I_{s1\text{Nor}(m)} + I_{r1\text{Nor}(m)} \]
\[ V_{1\text{-Thev}(m)} := I_{1\text{-Nor}(m)} \cdot Z_{L2 \_1\text{-thev}(m)} \quad |V_{1\text{-Thev}(0.5)}| = 1.01\text{-pu} \quad \arg(V_{1\text{-Thev}(0.5)}) = -7.87\text{-deg} \]
- SLG Fault

\[ I_{f0}(m) := \frac{V_{1,Thev}(m)}{Z_{L2,1,Thev}(m) + Z_{L2,2,Thev}(m) + Z_{L2,0,Thev}(m)} \]

\[ I_{f1}(m) := I_{f0}(m) \]

\[ I_{f2}(m) := I_{f0}(m) \]

\[ |I_{f0}(0.5)| = 3.22 \text{ pu} \]

\[ \text{arg}(I_{f0}(0.5)) = -95.09 \text{ deg} \]

- Fault Currents at Relay 1:

\[ I_{fa1}(m) := I_{f1}(m) \frac{Z_1 + (1-m)Z_{L21}}{(Z_{S1} + Z_{L11} + mZ_{L21}) + [Z_{R1} + (1-m)Z_{L21}]} \]

\[ |I_{fa1}(0.5)| = 1.19 \]

\[ \text{arg}(I_{fa1}(0.5)) = -93.76 \text{ deg} \]

However, the positive sequence current seen by the relay will include the load current.

\[ I_{\text{relayA1}}(m) := I_{fa1}(m) + I_{SR1} \]

\[ I_{\text{relayA1}}(0.5) = 0.91 - 1.08i \]

\[ |I_{\text{relayA1}(0.5)}| = 1.41 \]

\[ \text{arg}(I_{\text{relayA1}(0.5)}) = -49.96 \text{ deg} \]

Negative sequence and zero sequence currents don't see load current.

\[ I_{\text{relayA2}}(m) := I_{f2}(m) \frac{(1-m)Z_{L22} + Z_{R2}}{(Z_{S2} + Z_{L12} + mZ_{L22}) + [Z_{R2} + (1-m)Z_{L22}]} \]

\[ |I_{\text{relayA2}}(0.5)| = 1.19 \]

\[ \text{arg}(I_{\text{relayA2}(0.5)}) = -93.76 \text{ deg} \]