

ECE 523
Symmetrical Components

Session 12

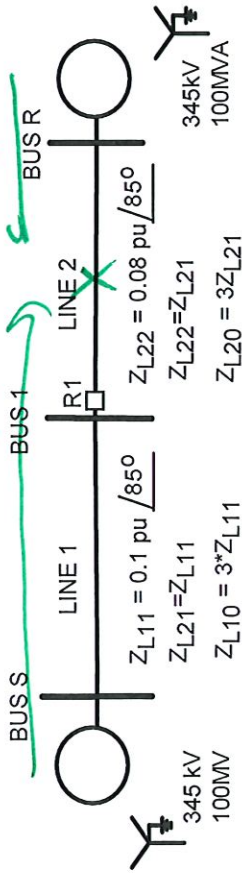
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$$V_{abc_SLGHV} := A_{012} \cdot \begin{pmatrix} V_{0_SLGHV} \\ V_{1_SLGHV} \\ V_{2_SLGHV} \end{pmatrix}$$

$$\left| V_{abc_SLGHV} \right| = \begin{pmatrix} 0.62 \\ 1 \\ 0.62 \end{pmatrix} \cdot pu$$

$$\arg(V_{abc_SLGHV}) = \begin{pmatrix} 54.06 \\ -90 \\ 125.94 \end{pmatrix} \cdot deg$$

• Example with two sources:



$V_S = 1.0 pu @ 0 deg$ Per unit line impedances calculated with $S_B = 100 MVA$ and $V_B = 345 KV LL$

$Z_{S1} = j0.03 pu$

$Z_{S2} = Z_{S1}$

$Z_{S0} = 3 * Z_{S1}$

$V_R = 1.0 pu$ at 0 deg

$Z_{R1} = j0.06 pu$

$Z_{R2} = Z_{R1}$

$Z_{R0} = 3 * Z_{R1}$

$$Z_{S1} := j \cdot 0.03 pu$$

$$Z_{S2} := Z_{S1}$$

$$Z_{L11} := 0.1 pu \cdot e^{j \cdot 85 deg}$$

$$Z_{L21} := 0.08 pu \cdot e^{j \cdot 85 deg}$$

$$Z_{L10} = 3 * Z_{L11}$$

$$Z_{L20} = 3 * Z_{L21}$$

$$Z_{L11} = 0.01 + 0.1i$$

$$Z_{L21} = 0.01 + 0.08i$$

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$$\begin{aligned}
 Z_{S0} &:= 3Z_{S1} & Z_{L12} &:= Z_{L11} & Z_{L22} &:= Z_{L21} \\
 Z_{R1} &:= j \cdot 0.06 \text{ pu} & Z_{L10} &:= 3 \cdot Z_{L11} & Z_{L20} &:= 3 \cdot Z_{L21} \\
 Z_{R2} &:= Z_{R1} & Z_{L10} &= 0.03 + 0.3i & Z_{L20} &= 0.02 + 0.24i \\
 Z_{R0} &:= 3Z_{R1} & & & &
 \end{aligned}$$

left of fault for right of fault

- For faults on Line 2:

$$Z_{L2_1_thev}(n) := \left[\frac{1}{Z_{S1} + Z_{L11} + n \cdot Z_{L21}} + \frac{1}{(1-n) \cdot Z_{L21} + Z_{R1}} \right]^{-1}$$

$$Z_{L2_2_thev}(n) := \left[\frac{1}{Z_{S2} + Z_{L12} + n \cdot Z_{L22}} + \frac{1}{(1-n) \cdot Z_{L22} + Z_{R2}} \right]^{-1}$$

$$Z_{L2_0_thev}(n) := \left[\frac{1}{Z_{S0} + Z_{L10} + n \cdot Z_{L20}} + \frac{1}{(1-n) \cdot Z_{L20} + Z_{R0}} \right]^{-1}$$

total fault current

- 3 phase faults: $V_f := 1$

$$I_{f1}(m) := \frac{V_f}{Z_{L2_1_thev}(m)}$$

$$I_{left_1}(m) := \frac{I_{f1}(m) \cdot [(1-m) \cdot Z_{L21} + Z_{R1}]}{Z_{S1} + Z_{L11} + m \cdot Z_{L21} + [(1-m) \cdot Z_{L21} + Z_{R1}]}$$

$$I_{fault_1}(m) = \frac{I_A(m) (Z_{S1} + Z_{L11} + m Z_{L21})}{Z_{TOT}}$$

divided current

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$$V_{R1_1}(m) := V_f - I_{left_1}(m) \cdot (Z_{S1} + Z_{L11})$$

$$|I_{left_1}(0.5)| = 5.89 \cdot pu$$

$$\arg(I_{left_1}(0.5)) = -85.88 \cdot deg$$

$$|V_{R1_1}(0.5)| = 0.24 \cdot pu$$

$$\arg(V_{R1_1}(0.5)) = -0.88 \cdot deg$$

• **SLG faults:**

$$I_{f0}(m) := \frac{V_f}{Z_{L2_1_thev}(m) + Z_{L2_2_thev}(m) + Z_{L2_0_thev}(m)}$$

$$I_{f1}(m) := I_{f0}(m) \quad I_{f2}(m) := I_{f0}(m)$$

$$I_{left_1}(m) := \frac{I_{f1}(m) \cdot [(1-m) \cdot Z_{L21} + Z_{R1}]}{Z_{S1} + Z_{L11} + m \cdot Z_{L21} + [(1-m) \cdot Z_{L21} + Z_{R1}]}$$

$$I_{left_2}(m) := \frac{I_{f2}(m) \cdot [(1-m) \cdot Z_{L22} + Z_{R2}]}{Z_{S2} + Z_{L12} + m \cdot Z_{L22} + [(1-m) \cdot Z_{L22} + Z_{R2}]}$$

$$I_{left_0}(m) := \frac{I_{f0}(m) \cdot [(1-m) \cdot Z_{L20} + Z_{R0}]}{Z_{S0} + Z_{L10} + m \cdot Z_{L20} + [(1-m) \cdot Z_{L20} + Z_{R0}]}$$

$$V_{R1_1}(m) := V_f - I_{left_1}(m) \cdot (Z_{S1} + Z_{L11})$$

$$V_{R1_2}(m) := 0 - I_{left_2}(m) \cdot (Z_{S2} + Z_{L12})$$

$$V_{R1_0}(m) := 0 - I_{left_0}(m) \cdot (Z_{S0} + Z_{L10})$$

Total Fault Current
 separate
 circuit from
 other faults

calculate
 I_{f0} first
 then I_{left}
 then V_{R1}

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$$V_{ABC_R1}(m) := A_{012} \cdot \begin{pmatrix} V_{R1_0}(m) \\ V_{R1_1}(m) \\ V_{R1_2}(m) \end{pmatrix} \quad \overrightarrow{|V_{ABC_R1}(0.5)|} = \begin{pmatrix} 0.24 \\ 1.18 \\ 1.18 \end{pmatrix} \cdot pu \quad \overrightarrow{\arg(V_{ABC_R1}(0.5))} = \begin{pmatrix} -0.88 \\ -132.89 \\ 132.99 \end{pmatrix} \cdot deg$$

$$I_{ABC_R1}(m) := A_{012} \cdot \begin{pmatrix} I_{left_0}(m) \\ I_{left_1}(m) \\ I_{left_2}(m) \end{pmatrix} \quad \overrightarrow{|I_{ABC_R1}(0.5)|} = \begin{pmatrix} 3.53 \\ 0 \\ 0 \end{pmatrix} \cdot pu \quad \overrightarrow{\arg(I_{ABC_R1}(0.5))} = \begin{pmatrix} -85.88 \\ 49.05 \\ 49.05 \end{pmatrix} \cdot deg$$

meanly less
magnitude = 0
C10
-13

• **DLG faults:**

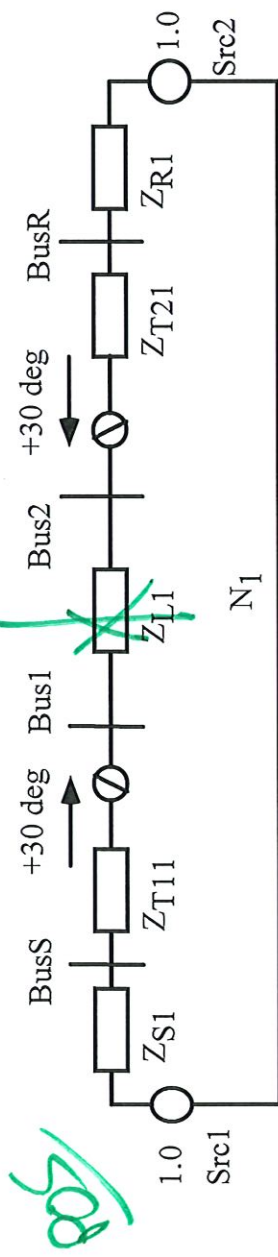
$$I_{f1}(m) := \frac{V_f}{Z_{L2_1_thev}(m) + \left(\frac{1}{Z_{L2_2_thev}(m)} + \frac{1}{Z_{L2_0_thev}(m)} \right)^{-1}}$$

$$I_{f2}(m) := -I_{f1}(m) \cdot \left(\frac{Z_{L2_0_thev}(m)}{Z_{L2_2_thev}(m) + Z_{L2_0_thev}(m)} \right)$$

$$I_{f0}(m) := -I_{f1}(m) \cdot \left(\frac{Z_{L2_2_thev}(m)}{Z_{L2_2_thev}(m) + Z_{L2_0_thev}(m)} \right)$$

$$I_{left_1}(m) := \frac{I_{f1}(m) \cdot [(1-m) \cdot Z_{L21} + Z_{R1}]}{Z_{S1} + Z_{L11} + m \cdot Z_{L21} + [(1-m) \cdot Z_{L21} + Z_{R1}]}$$

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• Positive sequence Ybus

$$Y_1(M) := \begin{bmatrix} \frac{1}{jX_{G11}} + \frac{1}{Z_{T11}} & \frac{-1 \cdot e^{-j \cdot 30 \text{deg}}}{Z_{T11}} & 0 & 0 & 0 \\ \frac{-1 \cdot e^{j \cdot 30 \text{deg}}}{Z_{T11}} & \frac{1}{Z_{T11}} + \frac{1}{M \cdot Z_{L11}} & 0 & 0 & \frac{-1}{M \cdot Z_{L11}} \\ 0 & 0 & \frac{1}{Z_{T21}} + \frac{1}{(1-M) \cdot Z_{L11}} & \frac{-1 \cdot e^{j \cdot 30 \text{deg}}}{Z_{T21}} & \frac{-1}{(1-M) \cdot Z_{L11}} \\ 0 & 0 & \frac{-1 \cdot e^{-j \cdot 30 \text{deg}}}{Z_{T21}} & \frac{1}{Z_{T21}} + \frac{1}{jX_{G21}} & 0 \\ 0 & \frac{-1}{M \cdot Z_{L11}} & \frac{-1}{(1-M) \cdot Z_{L11}} & 0 & \frac{1}{M \cdot Z_{L11}} + \frac{1}{(1-M) \cdot Z_{L11}} \end{bmatrix}$$

Bus S Bus 1 Bus 2 Bus R Bus F
Bus S Bus 1 Bus 2 Bus R Bus F

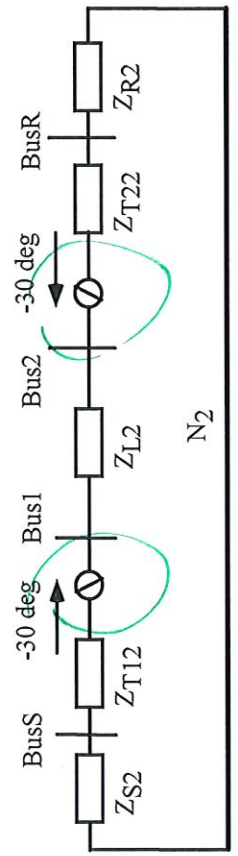
$$Z_1(M) := Y_1(M)^{-1}$$

$V_S := 1.0 \text{pu}$ $V_R := 1.0 \text{pu}$ $V_f := 1.0 \text{pu}$

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Negative Sequence Ybus

- All of the impedance values are the same in this case.
- Only change is in the Δ -Y phase shifts.
- Equivalent circuit looks the same, but no sources.

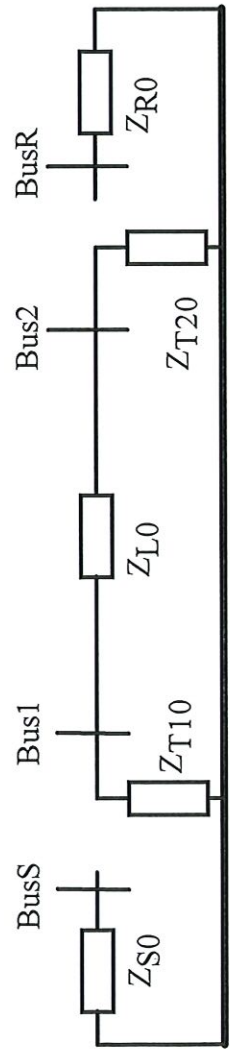


$$Y_2(M) := \begin{bmatrix} \frac{1}{jX_{G12}} + \frac{1}{Z_{T12}} & \frac{-1 \cdot e^{j \cdot 30 \text{deg}}}{Z_{T12}} & 0 & 0 & 0 \\ \frac{-1 \cdot e^{-j \cdot 30 \text{deg}}}{Z_{T12}} & \frac{1}{Z_{T12}} + \frac{1}{M \cdot Z_{L12}} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{Z_{T22}} + \frac{1}{(1-M) \cdot Z_{L12}} & \frac{-1 \cdot e^{-j \cdot 30 \text{deg}}}{Z_{T22}} & 0 \\ 0 & 0 & \frac{-1 \cdot e^{j \cdot 30 \text{deg}}}{Z_{T22}} & \frac{1}{Z_{T22}} + \frac{1}{jX_{G22}} & 0 \\ 0 & 0 & \frac{-1}{M \cdot Z_{L12}} & \frac{-1}{(1-M) \cdot Z_{L12}} & \frac{1}{M \cdot Z_{L12}} + \frac{1}{(1-M) \cdot Z_{L12}} \end{bmatrix}$$

$$Z_2(M) := Y_2(M)^{-1}$$

Zero Sequence Ybus

- Impedance values differ
- Model the transformer zero sequence connections
- Different looking equivalent circuit.



$$Y_0(M) := \begin{bmatrix} \frac{1}{Z_{G10}} & 0 & 0 & 0 \\ 0 & \frac{1}{Z_{T10}} + \frac{1}{M \cdot Z_{L10}} & 0 & -\frac{1}{M \cdot Z_{L10}} \\ 0 & 0 & \frac{1}{Z_{T20}} + \frac{1}{(1-M) \cdot Z_{L10}} & \frac{-1}{(1-M) \cdot Z_{L10}} \\ 0 & 0 & 0 & \frac{1}{Z_{G20}} + \frac{1}{(1-M) \cdot Z_{L10}} \end{bmatrix}$$

$Z_0(M) := Y_0(M)^{-1}$

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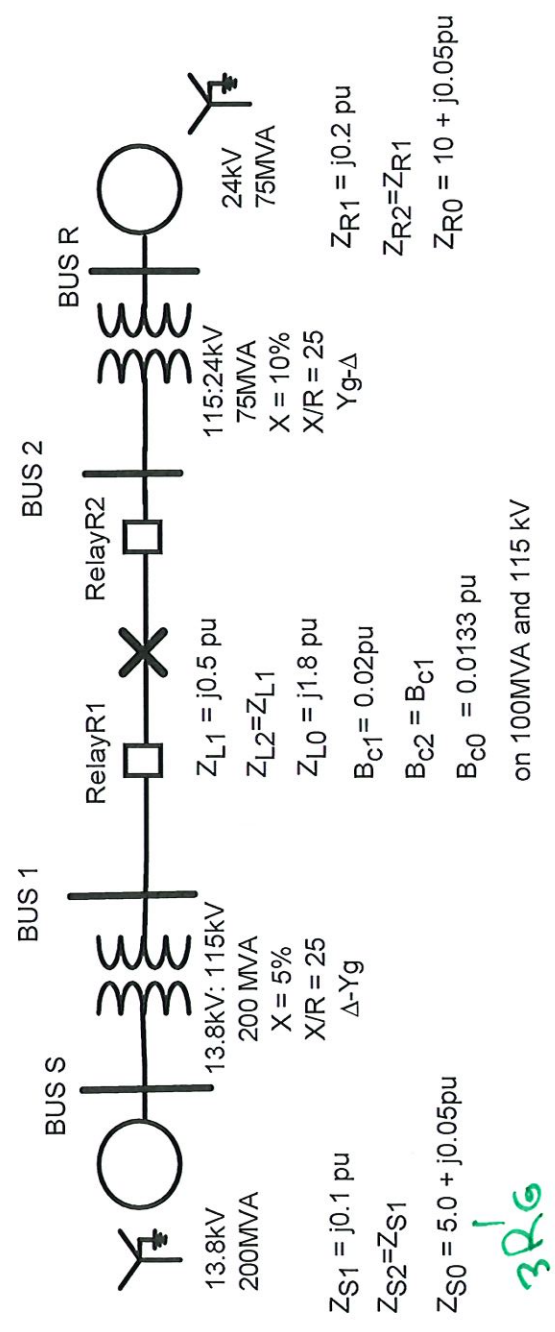
Unbalanced Fault Analysis Examples

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}$$

Define units: MVA := 1000kW MVAR := MVA
 MW := MVA pu := 1
 $a := 1 \cdot e^{j \cdot 120 \text{deg}}$

$$\angle(\text{mag, ang}) := \text{mag} \cdot \cos(\text{ang} \cdot \text{deg}) + j \cdot \text{mag} \cdot \sin(\text{ang} \cdot \text{deg})$$

System Description:



Per Unit Change of Base Calculations (hidden, to view double click on the arrow)

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Single phase

SLG Fault:

$$I_{0_SLG}(M, R_f) := \frac{V_f}{Z_1(M)_{4,4} + Z_2(M)_{4,4} + Z_0(M)_{4,4} + 3 \cdot R_f}$$

$$|I_{0_SLG}(0.5, 0)| = 1.0853$$

$$\arg(I_{0_SLG}(0.5, 0)) = -90 \cdot \text{deg}$$

$$I_{1_SLG}(M, R_f) := I_{0_SLG}(M, R_f)$$

$$I_{2_SLG}(M, R_f) := I_{0_SLG}(M, R_f)$$

$$I_{ABC_SLG}(M, R_f) := A_{012} \cdot \begin{pmatrix} I_{0_SLG}(M, R_f) \\ I_{1_SLG}(M, R_f) \\ I_{2_SLG}(M, R_f) \end{pmatrix}$$

$$|I_{ABC_SLG}(0.5, 0)| = \begin{pmatrix} 3.2558 \\ 0 \\ 0 \end{pmatrix} \cdot \text{pu}$$

$$\arg(I_{ABC_SLG}(0.5, 0)) = \begin{pmatrix} -90 \\ 18.4349 \\ 18.4349 \end{pmatrix} \cdot \text{deg}$$

- Angles meaningless when magnitude is 0

- Now find voltages in each sequence component)

$$\Delta V_{1_SLG}(M, R_f) := Z_1(M) \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_{1_SLG}(M, R_f) \end{pmatrix}$$

$$|\Delta V_{1_SLG}(0.5, 0)| = \begin{pmatrix} 0.0362 \\ 0.0543 \\ 0.1447 \\ 0.0965 \\ 0.2351 \end{pmatrix}$$

$$\arg(\Delta V_{1_SLG}(0.5, 0)) = \begin{pmatrix} 150 \\ 180 \\ -180 \\ 150 \\ 180 \end{pmatrix} \cdot \text{deg}$$

I1 fault

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- Now when we find V1 we need to include the transformer phase shift in the prefault voltages

$$V1_{SLG}(M, R_f) := \begin{pmatrix} 1.0e^{-j \cdot 30deg} \\ 1.0 \\ 1.0 \\ 1.0 \cdot e^{-j \cdot 30deg} \\ 1.0 \end{pmatrix} + \Delta V1_{SLG}(M, R_f) \xrightarrow{\arg(V1_{SLG}(0.5, 0))} = \begin{pmatrix} 0.9638 \\ 0.9457 \\ 0.8553 \\ 0.9035 \\ 0.7649 \end{pmatrix} \cdot deg$$

V1 prefault

$$\Delta V2_{SLG}(M, R_f) := Z2(M) \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I2_{SLG}(M, R_f) \end{pmatrix} \xrightarrow{\arg(\Delta V2_{SLG}(0.5, 0))} = \begin{pmatrix} 0.0362 \\ 0.0543 \\ 0.1447 \\ 0.0965 \\ 0.2351 \end{pmatrix} \cdot deg$$

I2 fault

Note the -30 degree shift...

$$V2_{SLG}(M, R_f) := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \Delta V2_{SLG}(M, R_f) \cdot \begin{matrix} \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \end{matrix} \begin{matrix} \text{No prefault voltage, so enter 0.} \\ \text{No prefault voltage, so enter 0.} \\ \text{No prefault voltage, so enter 0.} \\ \text{No prefault voltage, so enter 0.} \\ \text{No prefault voltage, so enter 0.} \end{matrix}$$

V2 prefault = 0

$$V1_f + V2_f + V0_f = 0.7649 - 0.2351 - 0.5297 = 0 = IAF \cdot PF$$

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$$\Delta V_{0\text{SLG}}(M, R_f) := Z_0(M) \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_{0\text{SLG}}(M, R_f) \end{pmatrix} + \Delta V_{0\text{SLG}}(M, R_f)$$

BUS 5
BUS 1
BUS 2
BUS 0
Z BUS 5

$$= \begin{pmatrix} 0 \\ -0.0143 \\ -0.0683 \\ 0 \\ -0.5297 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Again, no pre-fault voltage

- ABC Voltages at Bus 1

$$V_{\text{ABCB1.SLG}}(M, R_f) := A_{012} \cdot \begin{pmatrix} V_{0\text{SLG}}(M, R_f)_1 \\ V_{1\text{SLG}}(M, R_f)_1 \\ V_{2\text{SLG}}(M, R_f)_1 \end{pmatrix} \xrightarrow{|V_{\text{ABCB1.SLG}}(0.5, 0)|} = \begin{pmatrix} 0.8772 \\ 0.9806 \\ 0.9806 \end{pmatrix} \cdot \text{pu} \quad \arg(V_{\text{ABCB1.SLG}}(0.5, 0)) = \begin{pmatrix} 0 \\ -117.9783 \\ 117.9783 \end{pmatrix} \cdot \text{deg}$$

conversion of bus 1

- ABC Voltages at Bus S

$$V_{\text{ABCBS.SLG}}(M, R_f) := A_{012} \cdot \begin{pmatrix} V_{0\text{SLG}}(M, R_f)_0 \\ V_{1\text{SLG}}(M, R_f)_0 \\ V_{2\text{SLG}}(M, R_f)_0 \end{pmatrix} \xrightarrow{|V_{\text{ABCBS.SLG}}(0.5, 0)|} = \begin{pmatrix} 0.9463 \\ 0.9463 \\ 1 \end{pmatrix} \cdot \text{pu} \quad \arg(V_{\text{ABCBS.SLG}}(0.5, 0)) = \begin{pmatrix} -31.8973 \\ -148.1027 \\ 90 \end{pmatrix} \cdot \text{deg}$$

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Bus 1 Bus 2

- Branch currents
- Relay I currents

$$I_{1_B1.SLG}(M, R_f) := \frac{V1_{SLG}(M, R_f)_1 - V1_{SLG}(M, R_f)_4}{M \cdot Z_{L11}}$$

$$I_{2_B1.SLG}(M, R_f) := \frac{V2_{SLG}(M, R_f)_1 - V2_{SLG}(M, R_f)_4}{M \cdot Z_{L12}}$$

$$I_{0_B1.SLG}(M, R_f) := \frac{V0_{SLG}(M, R_f)_1 - V0_{SLG}(M, R_f)_4}{M \cdot Z_{L10}}$$

$$|I_{1_B1.SLG}(0.5, 0)| = 0.7235 \quad \arg(I_{1_B1.SLG}(0.5, 0)) = -90 \cdot \text{deg}$$

$$|I_{2_B1.SLG}(0.5, 0)| = 0.7235 \quad \arg(I_{2_B1.SLG}(0.5, 0)) = -90 \cdot \text{deg}$$

$$|I_{0_B1.SLG}(0.5, 0)| = 0.5727 \quad \arg(I_{0_B1.SLG}(0.5, 0)) = -90 \cdot \text{deg}$$

$$I_{ABC_B1.SLG}(M, R_f) := A_{012} \cdot \begin{pmatrix} I_{0_B1.SLG}(M, R_f) \\ I_{1_B1.SLG}(M, R_f) \\ I_{2_B1.SLG}(M, R_f) \end{pmatrix}$$

$$|I_{ABC_B1.SLG}(0.5, 0)| = \begin{pmatrix} 2.0197 \\ 0.1509 \\ 0.1509 \end{pmatrix} \cdot \text{pu} \quad \arg(I_{ABC_B1.SLG}(0.5, 0)) = \begin{pmatrix} -90 \\ 90 \\ 90 \end{pmatrix} \cdot \text{deg}$$

I0 - circulating

- LV side of transformer (Bus S)

$$I_{1_BS.SLG}(M, R_f) := \frac{V_S \cdot e^{-j \cdot 30 \text{deg}} - V1_{SLG}(M, R_f)_0}{jX_{G11}}$$

$$|I_{1_BS.SLG}(0.5, 0)| = 0.7235 \cdot \text{pu} \quad \arg(I_{1_BS.SLG}(0.5, 0)) = -120 \cdot \text{deg}$$

$$I_{2_BS.SLG}(M, R_f) := \frac{0 - V2_{SLG}(M, R_f)_0}{jX_{G12}}$$

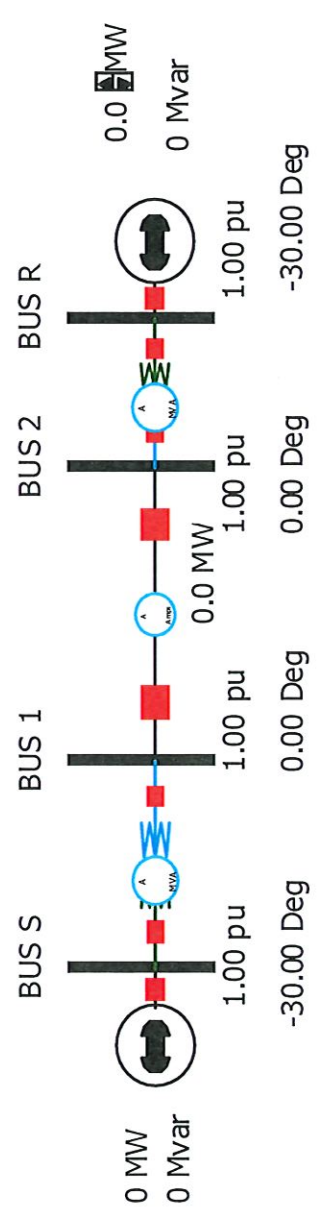
$$|I_{2_BS.SLG}(0.5, 0)| = 0.7235 \cdot \text{pu} \quad \arg(I_{2_BS.SLG}(0.5, 0)) = -60 \cdot \text{deg}$$

$$I_{0_BS.SLG} := 0$$

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$$I_{ABC_BS.SLG}(M, R_f) := A_{012} \cdot \begin{pmatrix} I_{0_BS.SLG} \\ I_{1_BS.SLG}(M, R_f) \\ I_{2_BS.SLG}(M, R_f) \end{pmatrix} \rightarrow \begin{pmatrix} 1.2532 \\ 1.2532 \cdot pu \\ 0 \end{pmatrix} = \begin{pmatrix} -90 \\ 90 \\ 38.6598 \end{pmatrix} \cdot deg$$

• Circuit implementation in Powerworld



• Powerworld Results

$I_{AFSLG} := 3.256 \angle (-90deg)$

consistency with above

Fault Data - Buses						
Name	Phase Volt A	Phase Ang A	Phase Volt B	Phase Ang B	Phase Volt C	Phase Ang C
BUS S	0.94625	-31.9	0.94625	-148.1	1	90
BUS 1	0.87716	0	0.98064	-117.98	0.98064	117.98
BUS 2	0.64224	0	0.96409	-116.07	0.96409	116.07
BUS R	0.85937	-35.58	0.85937	-144.42	1	90
FaultPt	0	0	1.1753	-132.54	1.1753	132.54

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From Name	To Name	Phase Cur A From	Phase Ang A From	Phase Cur B From	Phase Ang B From	Phase Cur C From	Phase Ang C From
BUS S	BUS 1	1.25318	-90	1.25318	90	0	0
BUS 1	BUS 2	0	0	0	0	0	0
BUS 1	FaultPt	2.0197	-90	0.15086	90	0.15086	90
BUS R	BUS 2	0.62659	-90	0.62659	90	0	0
FaultPt	BUS 2	1.23615	90	0.15086	90	0.15086	90

Good match with calculations above.

LL Fault:

$$I_{1_LL}(M, R_f) := \frac{V_f}{Z_1(M)_{4,4} + Z_2(M)_{4,4} + R_f}$$

$$|I_{1_LL}(0.5, 0)| = 2.3077$$

$$I_{2_LL}(M, R_f) := -I_{1_LL}(M, R_f)$$

$$\arg(I_{1_LL}(0.5, 0)) = -90 \cdot \text{deg}$$

$$I_{0_LL} := 10^{-15}$$

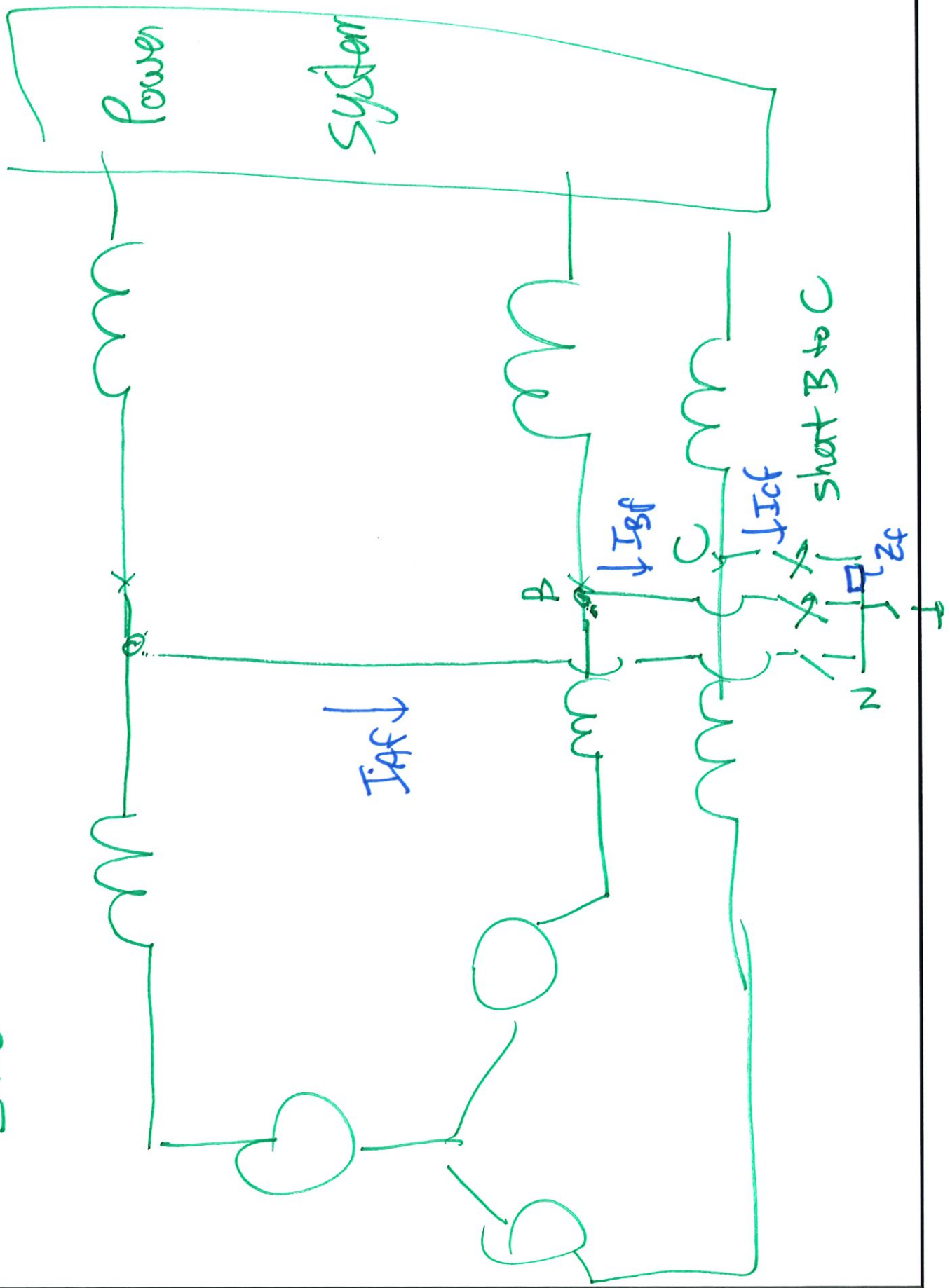
Approximately zero, works better with MathCAD.

$$I_{ABC_LL}(M, R_f) := A_{012} \cdot \begin{pmatrix} I_{0_LL} \\ I_{1_LL}(M, R_f) \\ I_{2_LL}(M, R_f) \end{pmatrix}$$

$$|I_{ABC_LL}(0.5, 0)| = \begin{pmatrix} 0 \\ 3.997 \\ 3.997 \end{pmatrix} \cdot \text{pu}$$

$$\arg(I_{ABC_LL}(0.5, 0)) = \begin{pmatrix} 0 \\ 180 \\ 0 \end{pmatrix} \cdot \text{deg}$$

Line to Line Faults



Constraints at fault point

$$I_{Af} = 0$$

$$I_{Bf} = -I_{Cf}$$

$$\text{at fault location } V_{Bf} - V_{Gf} = V_{Cf} = I_{Bf} \cdot Z_f$$

$$I_{f} Z_f = 0 \quad V_{Bf} = V_{Gf} \Rightarrow V_{Cf} = 0$$

In Sequence Domain

$$\begin{bmatrix} I_{AO} \\ I_{AI} \\ I_{AZ} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ I_{BF} \\ -I_{BF} \end{bmatrix} \leftarrow I_{CF}$$

↙

If $I_{SO} = 0$

$I_0 \Rightarrow I_{AO}$

$I_1 \Rightarrow I_{AI}$

$I_2 \Rightarrow I_{AZ}$

$$I_0 = \frac{1}{3} (0 + I_{BF} - I_{CF}) = 0$$

$$I_1 = \frac{1}{3} (0 + a I_{BF} - a^2 I_{CF}) = \left(\frac{a - a^2}{3} \right) I_{BF}$$

$$I_2 = \frac{1}{3} (0 + a^2 I_{BF} - a I_{CF}) = \left(\frac{a^2 - a}{3} \right) I_{BF}$$

$$I_2 = -I_1$$

voltages

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ a & a^2 & a \\ a^2 & a & a \end{bmatrix} \begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{BG} - I_{BF} Z_f \end{bmatrix}$$

$$\frac{I_0 = 0 \Rightarrow \begin{cases} I_0 \\ I_0 \end{cases} \uparrow \begin{cases} Z_{TH0} \\ V_0 \end{cases}}$$

$$V_{B0} - V_{CG} = I_{BF} Z_f$$

$$V_{CG} = V_{BG} - I_{BF} Z_f$$

$$V_1 = \frac{1}{3} [V_{AG} + a V_{BG} + a^2 (V_{BG} - I_{BF} Z_f)]$$

$$V_2 = \frac{1}{3} [V_{AG} + a^2 V_{BG} + a (V_{BG} - I_{BF} Z_f)]$$

Simplify $\rightarrow Z_f = 0$

$$V_1 = \frac{1}{3} [V_{A6} + \alpha V_{B6} + \alpha^2 V_{B6}]$$

$$V_2 = \frac{1}{3} [V_{A6} + \alpha^2 V_{B6} + \alpha V_{B6}] = V_1$$

$$V_1 = V_2$$

$$I_1 = -I_2$$

