Derivation of the DLG Fault Sequence Connections

A. ABC Domain Boundary Conditions

$$I_A = 0$$

$$V_{BG} = I_B \cdot (R_f) + (I_B + I_C) \cdot R_g$$

$$= I_B \cdot (R_f + R_g) + I_C \cdot R_g \qquad (1)$$

$$V_{CG} = (I_B + I_C) \cdot R_g + I_C \cdot (R_f)$$

$$= I_B \cdot R_g + I_C \cdot (R_f + R_g) \qquad (2)$$

B. Transform Boundary Conditions to Sequence Domain (phase A components)

$$I_A = 0 = I_0 + I_1 + I_2$$

$$I_B = I_0 + a^2 \cdot I_1 + a \cdot I_2$$

$$I_C = I_0 + a \cdot I_1 + a^2 \cdot I_2$$

$$V_{BG} = V_0 + a^2 \cdot V_1 + a \cdot V_2$$

$$V_{CG} = V_0 + a \cdot V_1 + a^2 \cdot V_2$$

C. Now rewrite equations (1) and (2) in the sequence domain

$$V_{BG} = V_0 + a^2 \cdot V_1 + a \cdot V_2 = I_B \cdot (R_f + R_g) + I_C \cdot R_g$$

$$= (I_0 + a^2 \cdot I_1 + a \cdot I_2) \cdot (R_f + R_g) + (I_0 + a \cdot I_1 + a^2 \cdot I_2) \cdot R_g \qquad (3)$$

$$V_{CG} = V_0 + a \cdot V_1 + a^2 \cdot V_2 = I_B \cdot R_g + I_C \cdot (R_f + R_g)$$

$$= (I_0 + a^2 \cdot I_1 + a \cdot I_2) \cdot R_g + (I_0 + a \cdot I_1 + a^2 \cdot I_2) \cdot (R_f + R_g) \qquad (4)$$

D. Now subtract equation (4) from equation (3):

$$V_{BG} - V_{CG} = (V_0 - V_0) + (a^2 - a) \cdot V_1 + (a - a^2) \cdot V_2$$

$$= (I_0 - I_0) \cdot (R_f + 2 \cdot R_g) + (a^2 - a) \cdot I_1 \cdot R_f + (a - a^2) \cdot I_2 \cdot R_f + (a^2 + a - (a^2 + a)) \cdot (I_1 + I_2) \cdot R_g$$

- Simplifies to:

$$(a^2 - a) \cdot V_1 - (a^2 - a) \cdot V_2 = (a^2 - a) \cdot I_1 \cdot R_f - (a^2 - a) \cdot I_2 \cdot R_f$$

- Divide by $(a^2 - a)$:

$$V_1 - V_2 = (I_1 - I_2) \cdot R_f$$

- Collect positive and negative sequence terms

$$V_1 - I_1 \cdot R_f = V_2 - I_2 \cdot R_f \quad (5)$$

E. Now add equation (4) to equation (3):

$$V_{BG} + V_{CG} = 2 \cdot V_0 + (a^2 + a) \cdot V_1 + (a + a^2) \cdot V_2$$

$$= (2 \cdot I_0) \cdot (R_f + 2 \cdot R_g) + (a^2 + a) \cdot I_1 \cdot R_f + (a + a^2) \cdot I_2 \cdot R_f + (2 \cdot (a^2 + a)) \cdot (I_1 + I_2) \cdot R_g$$

- Collect terms:

$$2 \cdot V_0 + (a^2 + a) \cdot (V_1 + V_2) = (R_f + 2 \cdot R_g) \cdot (2 \cdot I_0 + (a^2 + a) \cdot (I_1 + I_2))$$

- Substitute in the following relationship:

$(a^2 + a) = -1$ which comes from $(1 + a^2 + a) = 0$

- Resulting equation

$$2 \cdot V_0 - (V_1 + V_2) = (R_f + 2 \cdot R_g) \cdot (2 \cdot I_0 - (I_1 + I_2))$$

- Collect all zero sequence terms on the left-hand side, and positive and negative on right:

$$2 \cdot V_0 - 2 \cdot I_0 \cdot R_f - 4 \cdot I_0 \cdot R_g = (V_1 + V_2) - (I_1 + I_2) \cdot (R_f) - (I_1 + I_2) \cdot (2 \cdot R_g)$$

- Use the boundary condition for the currents:

$I_1 + I_2 + I_0 = 0$ which implies: $I_1 + I_2 = -I_0$

- Substitute this only for the R_g term

$$2 \cdot V_0 - 2 \cdot I_0 \cdot R_f - 4 \cdot I_0 \cdot R_g = (V_1 + V_2) - (I_1 + I_2) \cdot (R_f) + (I_0) \cdot (2 \cdot R_g)$$

- Again, collect all zero sequence terms on the left-hand side:

$$2 \cdot V_0 - 2 \cdot I_0 \cdot R_f - 6 \cdot I_0 \cdot R_g = (V_1 + V_2) - (I_1 + I_2) \cdot (R_f)$$

- Substitute in equation (5) on the right hand, resulting in:

$$2 \cdot V_0 - 2 \cdot I_0 \cdot R_f - 6 \cdot I_0 \cdot R_g = 2 \cdot (V_1 - I_1 \cdot R_f)$$

- Divide by 2, leaving:

$$V_0 - I_0 \cdot R_f - 3 \cdot I_0 \cdot R_g = V_1 - I_1 \cdot R_f \quad (6)$$

F. Final result:

$$V_0 - I_0 \cdot R_f - 3 \cdot I_0 \cdot R_g = V_1 - I_1 \cdot R_f = V_2 - I_2 \cdot R_f$$