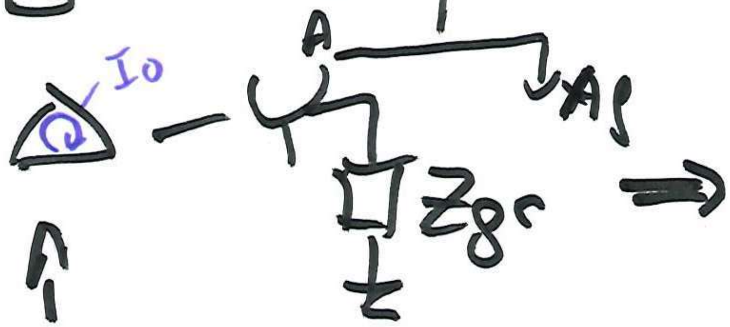
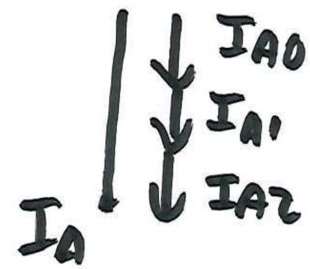
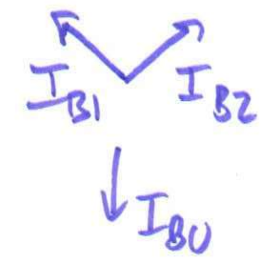
ECE 523
Symmetrical Components

Session 13

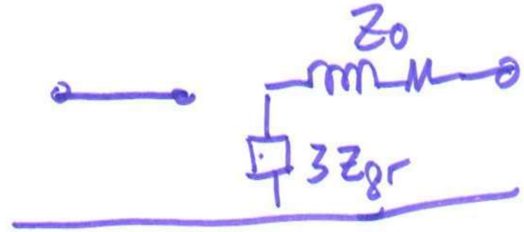
Zero sequence model







- no connection to ground
- I_0 circulated in Δ if flows on Y side



SLG Fault:

fault bus

- prefault volt at fault point

$$I_{0\_SLG}(M,R_f) := \frac{V_f}{Z_1(M)_{4,4} + Z_2(M)_{4,4} + Z_0(M)_{4,4} + 3\cdot R_f}$$

Then equiv at fault point

$$\left|I_{0\_SLG}(0.5,0)\right| = 1.0853$$

$$\arg\left(I_{0\_SLG}(0.5,0)\right) = -90\cdot deg$$

$$I_{1\_SLG}(M,R_f) := I_{0\_SLG}(M,R_f)$$

$$I_{2\_SLG}(M,R_f) := I_{0\_SLG}(M,R_f)$$

$$I_{ABC\_SLG}(M,R_f) := A_{012}\cdot\begin{pmatrix} I_{0\_SLG}(M,R_f) \\ I_{1\_SLG}(M,R_f) \\ I_{2\_SLG}(M,R_f) \end{pmatrix}$$

$$\overrightarrow{\left|I_{ABC\_SLG}(0.5,0)\right|} = \begin{pmatrix} 3.2558 \\ 0 \\ 0 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_SLG}(0.5,0)\right)} = \begin{pmatrix} -90 \\ 18.4349 \\ 18.4349 \end{pmatrix}\cdot deg$$

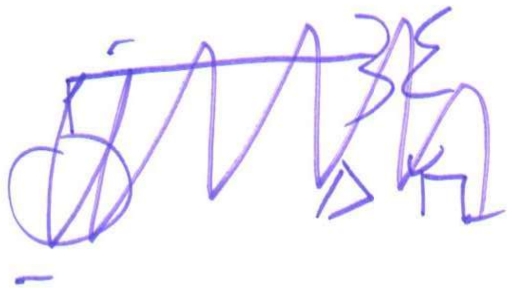
- Angles meaningless when magnitude is 0

- Now find voltages in each sequence component)

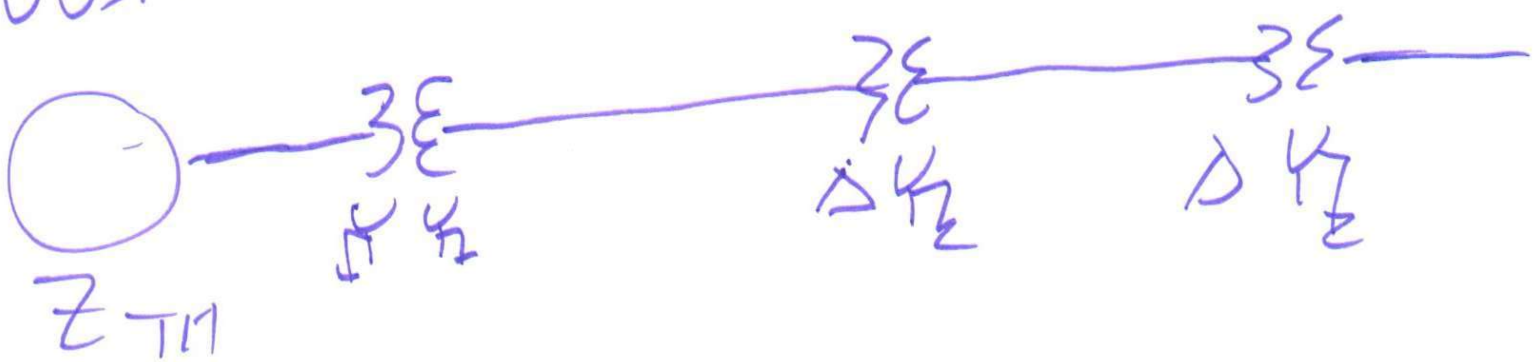
$$\Delta V1_{SLG}(M,R_f) := Z_1(M)\cdot\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_{1\_SLG}(M,R_f) \end{pmatrix}$$

$$\overrightarrow{\left|\Delta V1_{SLG}(0.5,0)\right|} = \begin{pmatrix} 0.0362 \\ 0.0543 \\ 0.1447 \\ 0.0965 \\ 0.2351 \end{pmatrix} \qquad \overrightarrow{\arg\left(\Delta V1_{SLG}(0.5,0)\right)} = \begin{pmatrix} 150 \\ 180 \\ -180 \\ 150 \\ 180 \end{pmatrix}\cdot deg$$

$-I_{fault}$



substation



Simplify $\rightarrow Z_f = 0$

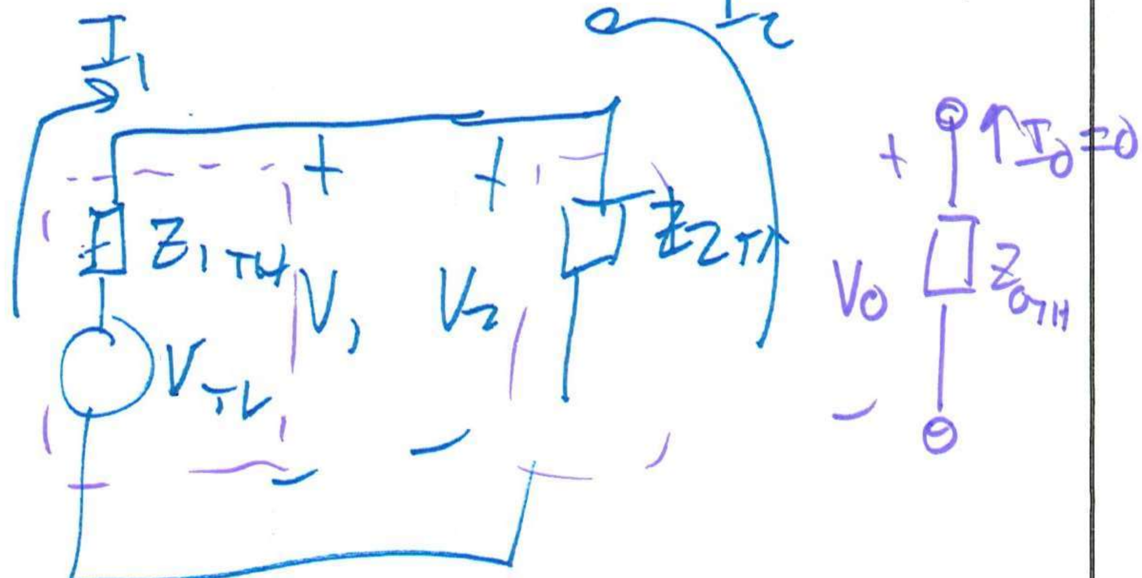
$(I_{Bf} \cdot Z_f = 0)$

$V_{BG} = V_{CG}$

$$V_1 = \frac{1}{3}\left[V_{AG} + a V_{BG} + a^2 V_{BG}\right)$$

$$V_2 = \frac{1}{3}\left[V_{AG} + a^2 V_{BG} + a V_{BG}\right) = V_1$$

$$\boxed{\begin{aligned} V_1 &= V_2 \\ I_1 &= -I_2 \end{aligned}}$$



$$\bar{I}_1 = \frac{V_{TH}}{Z_{1TH} + Z_{2TH}}$$

$$\bar{I}_2 = -\bar{I}_1$$

$$I_0 = 0 \qquad V_0 = 0$$

$Z_f \neq 0$

$V_{B1} = a^2 V_{A1}$

$V_{C1} = a V_{A1}$

$V_{B2} = a V_{A2}$

$V_{BG} - V_{CG} = Z_F \cdot I_{Bf}$

In terms of A reference symmetrical components

$$(V_{BG} - V_{CG}) = (V_{A0} - V_{A0}) + V_{A1}(a^2 - a) + V_{A2}(a - a^2)$$

$$= Z_f\Big(\underbrace{\cancel{I_{A0}}}_{0} + \underbrace{a^2 I_{A1}}_{B1} + \underbrace{a I_{A2}}_{I_{B2}}\Big) \quad -aI_{A1}$$

$$= Z_f\left(I_{A1} \cdot (a^2 - a)\right)$$

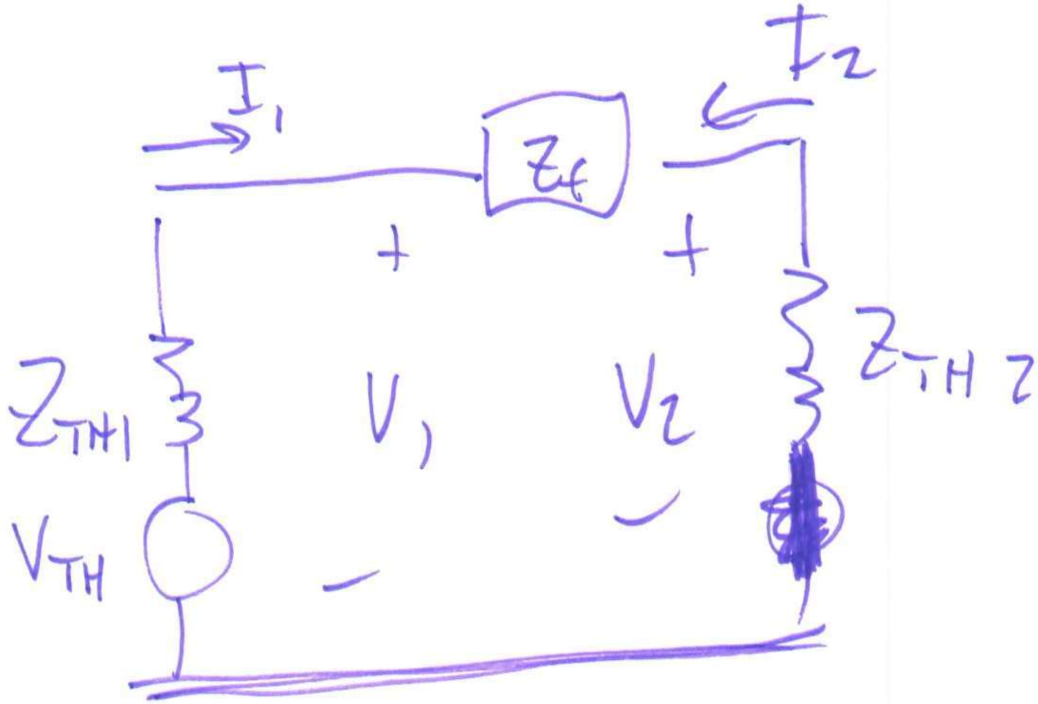
$$V_{BG} - V_{CG} = Z_f I_{Bf}$$

$$\Downarrow$$

$$\cancel{(a^2 - a)}(V_{A1} - V_{A2}) = \cancel{(a^2 - a)} I_{A1} \cdot Z_f$$

$$(V_{A1} - V_{A2}) = I_{A1} Z_f$$

$$I_{A1} = -I_{A2}$$



$$I_1 = \frac{V_{TH}}{Z_{1TH} + Z_{2TH} + Z_f}$$

faulted branch - which is replaced

Fault Data - Lines							
From Name	To Name	Phase Cur A From	Phase Ang A From	Phase Cur B From	Phase Ang B From	Phase Cur C From	Phase Ang C From
BUS S	BUS 1	1.25318	-90	1.25318	90	0	0
~~BUS 1~~	~~BUS 2~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~
BUS 1	FaultPt	2.0197	-90	0.15086	90	0.15086	90
BUS R	BUS 2	0.62659	-90	0.62659	90	0	0
FaultPt	BUS 2	1.23615	90	0.15086	90	0.15086	90

Good match with calculations above.

LL Fault:

$$I_{1\_LL}(M,R_f) := \frac{V_f}{Z_1(M)_{4,4} + Z_2(M)_{4,4} + R_f}$$

M, Rf

$$|I_{1\_LL}(0.5,0)| = 2.3077$$

$$\arg(I_{1\_LL}(0.5,0)) = -90\cdot deg$$

$$I_{2\_LL}(M,R_f) := -I_{1\_LL}(M,R_f)$$

$I_{0\_LL} := 10^{-15}$ Approximately zero, works better with MathCAD.

$$I_{ABC\_LL}(M,R_f) := A_{012}\cdot\begin{pmatrix} I_{0\_LL} \\ I_{1\_LL}(M,R_f) \\ I_{2\_LL}(M,R_f) \end{pmatrix}$$

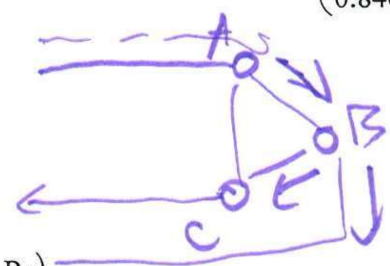
$$\overrightarrow{|I_{ABC\_LL}(0.5,0)|} = \begin{pmatrix} 0 \\ 3.997 \\ 3.997 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg(I_{ABC\_LL}(0.5,0))} = \begin{pmatrix} 0 \\ 180 \\ 0 \end{pmatrix}\cdot deg$$

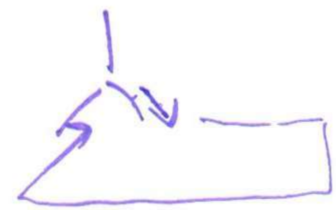
$$\frac{I_B}{I_1} = -j\sqrt{3}$$

$$V_{ABCB1.LL}(M,R_f) := A_{012}\cdot\begin{pmatrix} V0_{LL}(M,R_f)_1 \\ V1_{LL}(M,R_f)_1 \\ V2_{LL}(M,R_f)_1 \end{pmatrix} \quad \overrightarrow{\left|V_{ABCB1.LL}(0.5,0)\right|} = \begin{pmatrix} 1 \\ 0.8329 \\ 0.8329 \end{pmatrix}\cdot pu \quad \arg\left(V_{ABCB1.LL}(0.5,0)\right) = \begin{pmatrix} 0 \\ -126.8903 \\ 126.8903 \end{pmatrix}\cdot deg$$

- ABC Voltages at Bus S

$$V_{ABCBS.LL}(M,R_f) := A_{012}\cdot\begin{pmatrix} V0_{LL}(M,R_f)_0 \\ V1_{LL}(M,R_f)_0 \\ V2_{LL}(M,R_f)_0 \end{pmatrix} \quad \overrightarrow{\left|V_{ABCBS.LL}(0.5,0)\right|} = \begin{pmatrix} 0.9638 \\ 0.9638 \\ 0.8462 \end{pmatrix}\cdot pu \quad \arg\left(V_{ABCBS.LL}(0.5,0)\right) = \begin{pmatrix} -26.0368 \\ -153.9632 \\ 90 \end{pmatrix}\cdot deg$$





- Branch currents
  - Relay 1 currents

$$I_{1\_B1.LL}(M,R_f) := \frac{V1_{LL}(M,R_f)_1 - V1_{LL}(M,R_f)_4}{M\cdot Z_{L11}} \qquad \left|I_{1\_B1.LL}(0.5,0)\right| = 1.5385 \qquad \arg\left(I_{1\_B1.LL}(0.5,0)\right) = -90\cdot deg$$

$$I_{2\_B1.LL}(M,R_f) := \frac{V2_{LL}(M,R_f)_1 - V2_{LL}(M,R_f)_4}{M\cdot Z_{L12}} \qquad \left|I_{2\_B1.LL}(0.5,0)\right| = 1.5385 \qquad \arg\left(I_{2\_B1.LL}(0.5,0)\right) = 90\cdot deg$$

$$I_{0\_B1.LL}(M,R_f) := \frac{V0_{LL}(M,R_f)_1 - V0_{LL}(M,R_f)_4}{M \cdot Z_{L10}}$$ $\left|I_{0\_B1.LL}(0.5,0)\right| = 0$ $\arg\left(I_{0\_B1.LL}(0.5,0)\right) = 0 \cdot deg$

$$I_{ABC\_B1.LL}(M,R_f) := A_{012} \cdot \begin{pmatrix} I_{0\_B1.LL}(M,R_f) \\ I_{1\_B1.LL}(M,R_f) \\ I_{2\_B1.LL}(M,R_f) \end{pmatrix}$$ $\overrightarrow{\left|I_{ABC\_B1.LL}(0.5,0)\right|} = \begin{pmatrix} 0 \\ 2.6647 \\ 2.6647 \end{pmatrix} \cdot pu$ $\arg\left(I_{ABC\_B1.LL}(0.5,0)\right) = \begin{pmatrix} 21.2839 \\ 180 \\ 0 \end{pmatrix} \cdot deg$

rounding error

- LVside of transformer (Bus S)

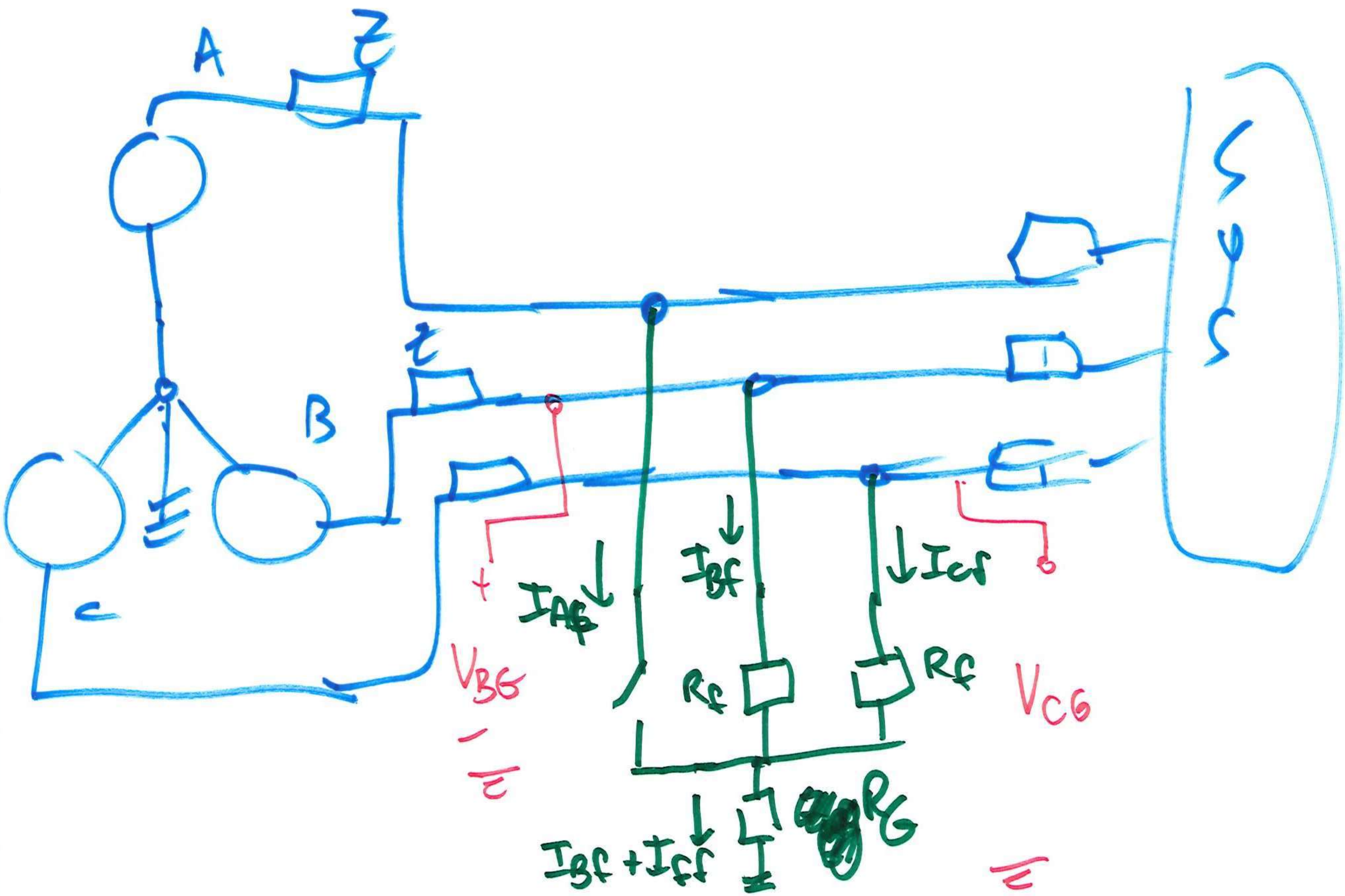
$$I_{1\_BS.LL}(M,R_f) := \frac{V_S \cdot e^{-j \cdot 30deg} - V1_{LL}(M,R_f)_0}{jX_{G11}}$$ $\left|I_{1\_BS.LL}(0.5,0)\right| = 1.5385 \cdot pu$ $\arg\left(I_{1\_BS.LL}(0.5,0)\right) = -120 \cdot deg$

$$I_{2\_BS.LL}(M,R_f) := \frac{0 - V2_{LL}(M,R_f)_0}{jX_{G12}}$$ $\left|I_{2\_BS.LL}(0.5,0)\right| = 1.5385 \cdot pu$ $\arg\left(I_{2\_BS.LL}(0.5,0)\right) = 120 \cdot deg$

$I_{0\_BS.LL} := 0$

$$I_{ABC\_BS.LL}(M,R_f) := A_{012} \cdot \begin{pmatrix} I_{0\_BS.LL} \\ I_{1\_BS.LL}(M,R_f) \\ I_{2\_BS.LL}(M,R_f) \end{pmatrix}$$ $\overrightarrow{\left|I_{ABC\_BS.LL}(0.5,0)\right|} = \begin{pmatrix} 1.5385 \\ 1.5385 \\ 3.0769 \end{pmatrix} \cdot pu$ $\arg\left(I_{ABC\_BS.LL}(0.5,0)\right) = \begin{pmatrix} 180 \\ 180 \\ 0 \end{pmatrix} \cdot deg$

Δ side the largest phase current is larger than any of Y side for LL fault


A
Z
Z
B
C
I_{Af}
I_{Bf}
I_{Cf}
R_f
R_f
R_G
$I_{Bf} + I_{Cf}$
V_{BG}
V_{CG}
+
−

constraints

$$I_{Af} = 0$$

we don't know I_B relative to I_C

$$V_{BG} = I_{Bf} \cdot R_f + (I_{Bf} + I_{Cf}) R_G$$

$$V_{CG} = I_{Cf} \cdot R_f + (I_{Bf} + I_{Cf}) R_G$$

Derivation of the DLG Fault Sequence Connections

A. ABC Domain Boundary Conditions

$I_A = 0$ $I_B = I_{Bf}$ $I_C = I_{Cf}$

$$V_{BG} = I_B \cdot (R_f) + (I_B + I_C) \cdot R_g$$

$$= I_B \cdot (R_f + R_g) + I_C \cdot R_g \quad (1)$$

$$V_{CG} = (I_B + I_C) \cdot R_g + I_C \cdot (R_f)$$

$$= I_B \cdot R_g + I_C \cdot (R_f + R_g) \quad (2)$$

B. Transform Boundary Conditions to Sequence Domain (phase A components)

$$I_A = 0 = I_0 + I_1 + I_2$$

$$I_B = I_0 + a^2 \cdot I_1 + a \cdot I_2$$

$$I_C = I_0 + a \cdot I_1 + a^2 \cdot I_2$$

$$V_{BG} = V_0 + a^2 \cdot V_1 + a \cdot V_2$$

$$V_{CG} = V_0 + a \cdot V_1 + a^2 \cdot V_2$$

C. Now rewrite equations (1) and (2) in the sequence domain

$$V_{BG} = V_0 + a^2 \cdot V_1 + a \cdot V_2 = I_B \cdot (R_f + R_g) + I_C \cdot R_g$$

$$= (I_0 + a^2 \cdot I_1 + a \cdot I_2) \cdot (R_f + R_g) + (I_0 + a \cdot I_1 + a^2 \cdot I_2) \cdot R_g \quad (3)$$

$$V_{CG} = V_0 + a \cdot V_1 + a^2 \cdot V_2 = I_B \cdot R_g + I_C \cdot (R_f + R_g)$$

$$= (I_0 + a^2 \cdot I_1 + a \cdot I_2) \cdot R_g + (I_0 + a \cdot I_1 + a^2 \cdot I_2) \cdot (R_f + R_g) \quad (4)$$

D. Now subtract equation (4) from equation (3):

$$V_{BG} - V_{CG} = (V_0 - V_0) + (a^2 - a) \cdot V_1 + (a - a^2) \cdot V_2$$

$$= (I_0 - I_0) \cdot (R_f + 2 \cdot R_g) + (a^2 - a) \cdot I_1 \cdot R_f + (a - a^2) \cdot I_2 \cdot R_f + (a^2 + a - (a^2 + a)) \cdot (I_1 + I_2) \cdot R_g$$

- Simplifies to:

$$(a^2 - a) \cdot V_1 - (a^2 - a) \cdot V_2 = (a^2 - a) \cdot I_1 \cdot R_f - (a^2 - a) \cdot I_2 \cdot R_f$$

- Divide by $(a^2 - a)$:

$$V_1 - V_2 = (I_1 - I_2) \cdot R_f$$

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- Collect positive and negative sequence terms

$$V_1 - I_1 \cdot R_f = V_2 - I_2 \cdot R_f \qquad (5)$$

E. Now add equation (4) to equation (3):

$$V_{BG} + V_{CG} = 2 \cdot V_0 + (a^2 + a) \cdot V_1 + (a + a^2) \cdot V_2$$

$$= (2 \cdot I_0) \cdot (R_f + 2 \cdot R_g) + (a^2 + a) \cdot I_1 \cdot R_f + (a + a^2) \cdot I_2 \cdot R_f + (2 \cdot (a^2 + a)) \cdot (I_1 + I_2) \cdot R_g$$

- Collect terms:

$$2 \cdot V_0 + (a^2 + a) \cdot (V_1 + V_2) = (R_f + 2 \cdot R_g) \cdot (2 \cdot I_0 + (a^2 + a) \cdot (I_1 + I_2))$$

- Substitute in the following relationship:

$(a^2 + a) = -1$ which comes from $(1 + a^2 + a) = 0$

- Resulting equation

$$2 \cdot V_0 - (V_1 + V_2) = (R_f + 2 \cdot R_g) \cdot (2 \cdot I_0 - (I_1 + I_2))$$

- Collect all zero sequence terms on the left-hand side, and positive and negative on right:

$$2 \cdot V_0 - 2 \cdot I_0 \cdot R_f - 4 \cdot I_0 \cdot R_g = (V_1 + V_2) - (I_1 + I_2) \cdot (R_f) - (I_1 + I_2) \cdot (2 \cdot R_g)$$

- Use the boundary condition for the currents:

$I_1 + I_2 + I_0 = 0$ which implies: $I_1 + I_2 = -I_0$

- Substitute this only for the R_g term

$$2 \cdot V_0 - 2 \cdot I_0 \cdot R_f - 4 \cdot I_0 \cdot R_g = (V_1 + V_2) - (I_1 + I_2) \cdot (R_f) + (I_0) \cdot (2 \cdot R_g)$$

- Again, collect all zero sequence terms on the left-hand side:

$$2 \cdot V_0 - 2 \cdot I_0 \cdot R_f - 6 \cdot I_0 \cdot R_g = (V_1 + V_2) - (I_1 + I_2) \cdot (R_f)$$

- Substitute in equation (5) on the right hand, resulting in:

$$2 \cdot V_0 - 2 \cdot I_0 \cdot R_f - 6 \cdot I_0 \cdot R_g = 2 \cdot (V_1 - I_1 \cdot R_f)$$

- Divide by 2, leaving:

$$V_0 - I_0 \cdot R_f - 3 \cdot I_0 \cdot R_g = V_1 - I_1 \cdot R_f \qquad (6)$$

F. Final result:

$$V_0 - I_0 \cdot R_f - 3 \cdot I_0 \cdot R_g = V_1 - I_1 \cdot R_f = V_2 - I_2 \cdot R_f$$

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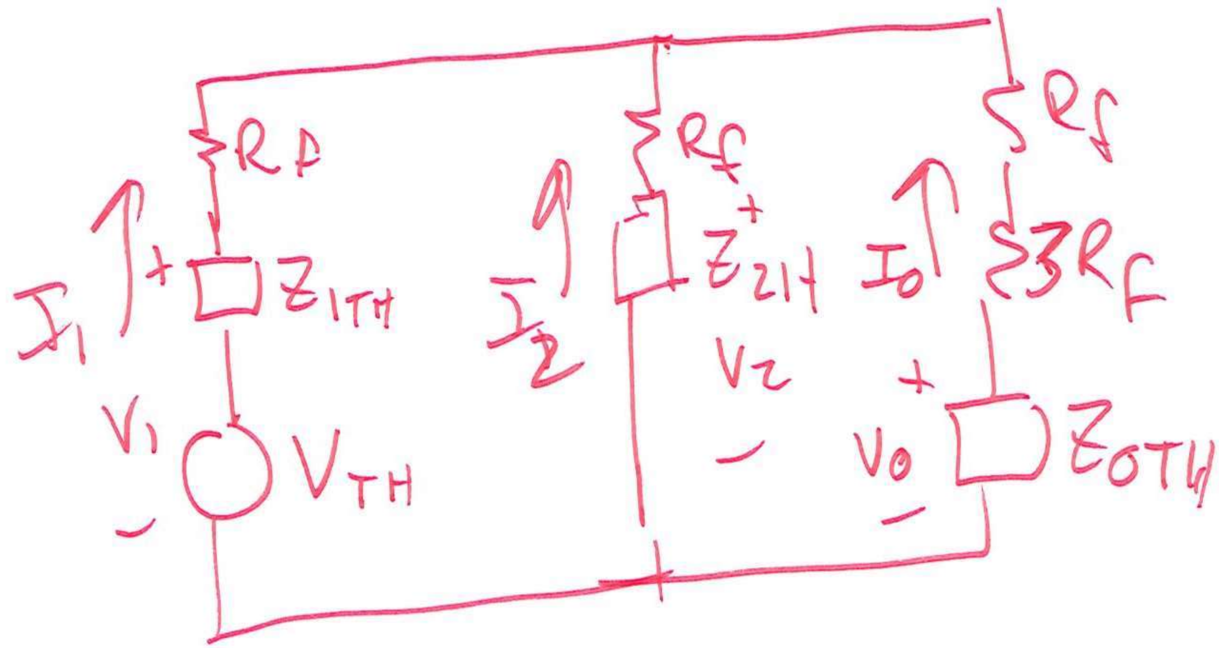
Rf
Rf
Rf
3Rf
I1
I2
Io
+
+
+
Z1TH
Z21H
ZOTH
V1
V2
Vo
VTH
-
-
-