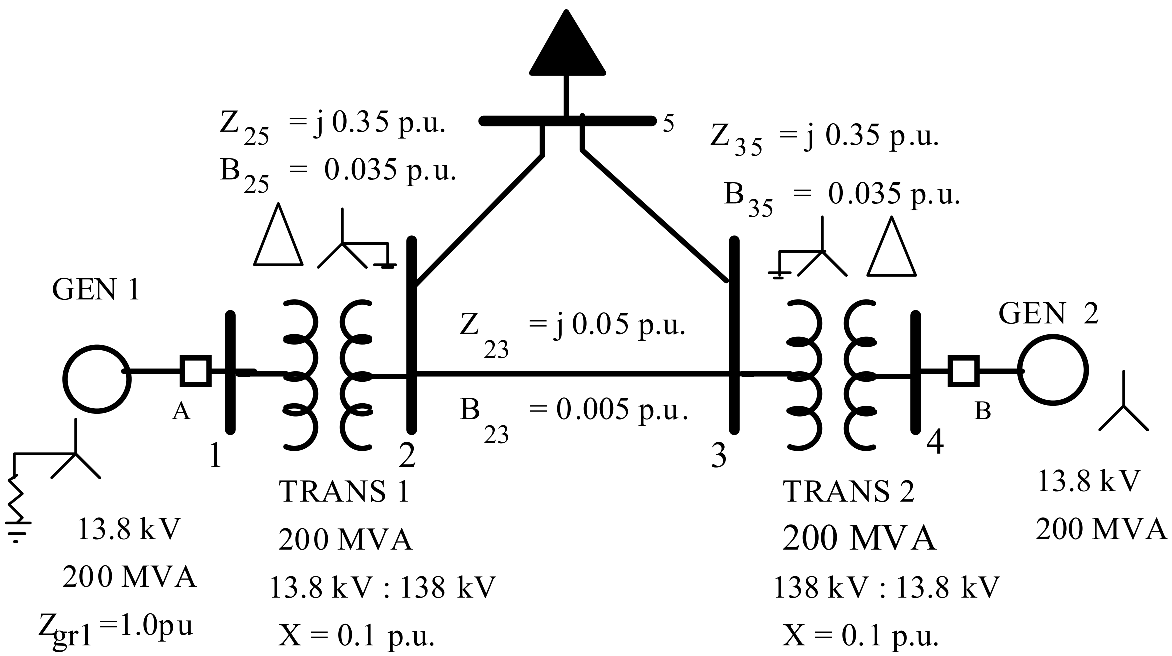
Powerflow Example

$a := 1 \cdot e^{j \cdot 120deg}$ $\quad A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$ $\quad pu := 1 \quad MVA := 1000kW$

$\angle(mag, ang) := mag \cdot \cos(ang \cdot deg) + j \cdot mag \cdot \sin(ang \cdot deg)$

The system shown below has two synchronous generators.

- Line impedances are on a 138kV, 100MVA base



$S_B := 100MVA$

- Bus 1 is slack bus
- Set |V1| = 1.0pu

- Set PG2 to 50MW
- Set |V4| to 1.0pu (generator 2 terminal voltage)

Set line impedance parameters (set zero sequence line impedances to 3 times the positive sequence values):

$Z_{L23} := j \cdot 0.05pu$ $\qquad B_{c23} := 0.005 \cdot pu$

$Z_{L25} := j \cdot 0.35pu$ $\qquad B_{c25} := 0.035 \cdot pu$

$Z_{L35} := j \cdot 0.35pu$ $\qquad B_{c35} := 0.035 \cdot pu$

Transformer 1 Change of base (zero sequence impedances match positive sequence):

$$X_{t11} := 0.1 \cdot \left(\frac{S_B}{200MVA} \right) \qquad X_{t11} = 0.05 \cdot pu \qquad X_{t10} := X_{t11}$$

Transformer 2 Change of base (zero sequence impedances match positive sequence):

$$X_{t21} := 0.1 \cdot \left(\frac{S_B}{200MVA} \right) \qquad X_{t21} = 0.05 \cdot pu \qquad X_{t20} := X_{t11}$$

A. Assuming, the load at Bus 5 is 100MW at 0.9 lagging power factor, perform a power flow solution. Use the voltage at Bus 3 as your angle reference

Options,

(1) Solve the power flow equations for the entire system using Mathcad solve blocks
(2) Use Powerworld or a similar load flow problem (at least to check results)

- In order to use V3 as the reference angle, use the angle for V3 from the power flow solution and shift the slack bus angle such that the new angle at Bus 3 is 0.

(1) Solve Full Powerflow Solution Using MathCAD Solve Block

- Positive sequence Y bus for power flow calculations (ignore phase shifts for the moment):

$$Y_{11} := \frac{1}{j \cdot X_{t11}} \qquad Y_{12} := \frac{-1}{j \cdot X_{t11}}$$

- Symmetry assumed

$$Y_{22} := \frac{1}{j \cdot X_{t11}} + \frac{1}{Z_{L23}} + \frac{1}{Z_{L25}} + \frac{j \cdot B_{c23}}{2} + \frac{j \cdot B_{c25}}{2} \qquad Y_{23} := \frac{-1}{Z_{L23}} \qquad Y_{25} := \frac{-1}{Z_{L25}}$$

$$Y_{33} := \frac{1}{j \cdot X_{t21}} + \frac{1}{Z_{L23}} + \frac{1}{Z_{L35}} + \frac{j \cdot B_{c23}}{2} + \frac{j \cdot B_{c35}}{2} \qquad Y_{34} := \frac{-1}{j \cdot X_{t21}} \qquad Y_{35} := \frac{-1}{Z_{L35}}$$

$$Y_{44} := \frac{1}{j \cdot X_{t11}} \qquad Y_{55} := \frac{1}{Z_{L25}} + \frac{1}{Z_{L35}} + \left(\frac{j \cdot B_{c25}}{2} + \frac{j \cdot B_{c35}}{2} \right)$$

$$Y_{busPF} := \begin{pmatrix} Y_{11} & Y_{12} & 0 & 0 & 0 \\ Y_{12} & Y_{22} & Y_{23} & 0 & Y_{25} \\ 0 & Y_{23} & Y_{33} & Y_{34} & Y_{35} \\ 0 & 0 & Y_{34} & Y_{44} & 0 \\ 0 & Y_{25} & Y_{35} & 0 & Y_{55} \end{pmatrix} \qquad Y_{busPF} = \begin{pmatrix} -20i & 20i & 0 & 0 & 0 \\ 20i & -42.8371i & 20i & 0 & 2.8571i \\ 0 & 20i & -42.8371i & 20i & 2.8571i \\ 0 & 0 & 20i & -20i & 0 \\ 0 & 2.8571i & 2.8571i & 0 & -5.6793i \end{pmatrix}$$

$P2 := 0 \qquad P3 := 0 \qquad P4 := 0.5pu \qquad P5 := -1.0pu$

$Q2 := 0 \qquad Q3 := 0 \qquad Q5 := P5 \cdot \tan(\operatorname{acos}(0.9)) \qquad Q5 = -0.4843 \cdot pu$

$V1 := 1 \qquad V4 := 1 \qquad a1 := 0$

- P5 and Q5 are negative injections since they represent a load

- Initial guesses

  $V2 := 1 \qquad a2 := 0deg$

  $V3 := 1 \qquad a3 := 0deg$

$a4 := 0$

$V5 := 1 \qquad a5 := 0\text{deg}$

Given

$$P2 = V2 \cdot V1 \cdot \text{Im}(Y_{12}) \cdot \sin(a2 - a1) + V2 \cdot V3 \cdot \text{Im}(Y_{23}) \cdot \sin(a2 - a3) + V2 \cdot V5 \cdot \text{Im}(Y_{25}) \cdot \sin(a2 - a5)$$

$$P3 = V3 \cdot V4 \cdot \text{Im}(Y_{34}) \cdot \sin(a3 - a4) + V3 \cdot V2 \cdot \text{Im}(Y_{23}) \cdot \sin(a3 - a2) + V3 \cdot V5 \cdot \text{Im}(Y_{35}) \cdot \sin(a3 - a5)$$

$$P4 = V4 \cdot V3 \cdot \text{Im}(Y_{34}) \cdot \sin(a4 - a3)$$

- Need to add additional terms to include transformer phase shift

$$P5 = V5 \cdot V2 \cdot \text{Im}(Y_{25}) \cdot \sin(a5 - a2) + V5 \cdot V3 \cdot \text{Im}(Y_{35}) \cdot \sin(a5 - a2)$$

$$Q2 = -V2^2 \text{Im}(Y_{22}) - V2 \cdot V1 \cdot \text{Im}(Y_{12}) \cdot \cos(a2 - a1) - V2 \cdot V3 \cdot \text{Im}(Y_{23}) \cdot \cos(a2 - a3) - V2 \cdot V5 \cdot \text{Im}(Y_{25}) \cdot \cos(a2 - a5)$$

$$Q3 = -V3^2 \text{Im}(Y_{33}) - V3 \cdot V4 \cdot \text{Im}(Y_{34}) \cdot \cos(a3 - a4) - V3 \cdot V2 \cdot \text{Im}(Y_{23}) \cdot \cos(a3 - a2) - V3 \cdot V5 \cdot \text{Im}(Y_{35}) \cdot \cos(a3 - a5)$$

$$Q5 = -V5^2 \text{Im}(Y_{55}) - V5 \cdot V2 \cdot \text{Im}(Y_{25}) \cdot \cos(a5 - a2) - V5 \cdot V3 \cdot \text{Im}(Y_{35}) \cdot \cos(a5 - a2)$$

$X := \text{Find}(V2, a2, V3, a3, a4, V5, a5)$

$mV2 := X_0 \qquad mV2 = 0.9819 \cdot pu \qquad \theta_2 := X_1 \qquad \theta_2 = -1.459 \cdot deg$

$mV3 := X_2 \qquad mV3 = 0.9819 \cdot pu \qquad \theta_3 := X_3 \qquad \theta_3 = -1.459 \cdot deg$

$mV5 := X_5 \qquad mV5 = 0.8687 \cdot pu \qquad \theta_4 := X_4 \qquad \theta_4 = 0 \cdot deg$

$\theta_5 := X_6 \qquad \theta_5 = -13.2983 \cdot deg$

- Note, that if we include the transformer phase shift, we need to subtract 30 degrees from the voltages at Bus 1 and Bus 4:

$$\theta_{1\_LV} := a1 - 30deg \qquad \theta_{1\_LV} = -30 \cdot deg$$

$$\theta_{4\_LV} := \theta_4 - 30deg \qquad \theta_{4\_LV} = -30 \cdot deg$$

- Finally, we need to shift the angle, by subtracting the angle of θ_3 to each of the bus angles such that angle at Bus 3 = 0.

$$\theta_{1\_p} := \theta_{1\_LV} - \theta_3 \qquad \theta_{1\_p} = -28.541 \cdot deg$$

$$\theta_{2\_p} := \theta_2 - \theta_3 \qquad \theta_{2\_p} = 0 \cdot deg$$

$$\theta_{3\_p} := \theta_3 - \theta_3 \qquad \theta_{3\_p} = 0 \cdot deg$$

$$\theta_{4\_p} := \theta_{4\_LV} - \theta_3 \qquad \theta_{4\_p} = -28.541 \cdot deg$$

$$\theta_{5\_p} := \theta_5 - \theta_3 \qquad \theta_{5\_p} = -11.8393 \cdot deg$$

- Calculate generator currents (note that you need to account for transformer phase shift)

$$I_{gen1} := \frac{\left[V1 \cdot e^{j \cdot \theta_{1\_p}} - mV2 \cdot e^{j \cdot \theta_{2\_p}} \cdot \left(e^{-j \cdot 30deg}\right)\right]}{\left(j \cdot X_{t11}\right)} \qquad \left|I_{gen1}\right| = 0.6216 \cdot pu \qquad \arg\left(I_{gen1}\right) = -64.9864 \cdot deg$$

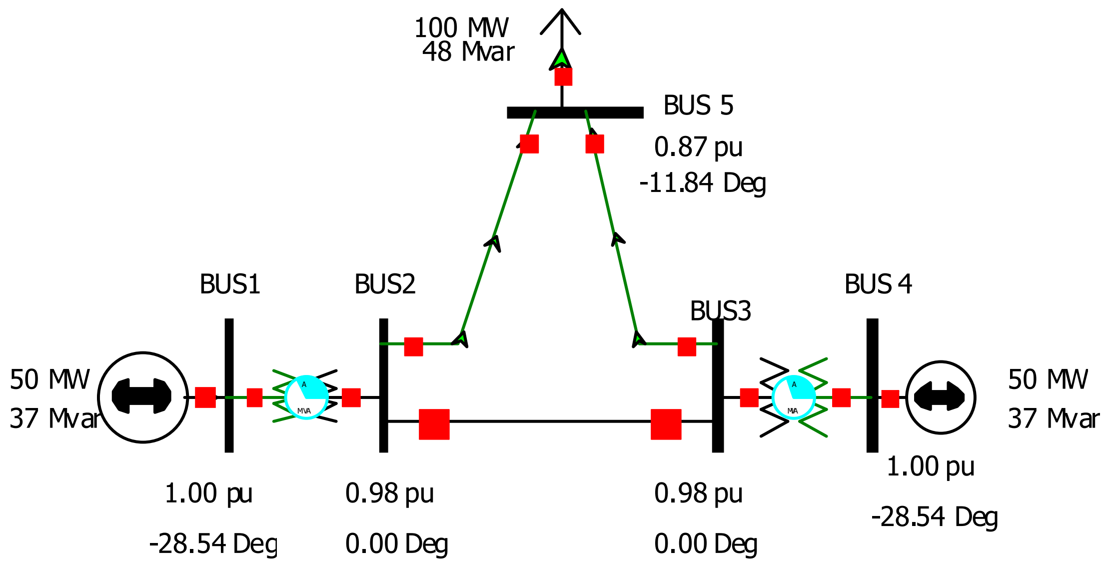
$$S_{gen1} := V1 \cdot e^{j \cdot \theta_{1\_p}} \cdot \overline{I_{gen1}} \qquad S_{gen1} = (0.5 + 0.3692i) \cdot pu$$

$$I_{gen2} := \frac{\left[V4 \cdot e^{j \cdot \theta_{4\_p}} - mV3 \cdot e^{j \cdot \theta_{3\_p}} \cdot \left(e^{-j \cdot 30deg}\right)\right]}{\left(j \cdot X_{t21}\right)} \qquad \left|I_{gen2}\right| = 0.6216 \cdot pu \qquad \arg\left(I_{gen2}\right) = -64.9864 \cdot deg$$

$$S_{gen2} := V4 \cdot e^{j \cdot \theta_{4\_p}} \cdot \overline{I_{gen2}} \qquad S_{gen2} = (0.5 + 0.3692i) \cdot pu$$

Powerworld Implementation and Results

- The system was entered into Powerworld, with the system MVA base set to 200MVA
- Slack bus angle (angle of V1) was changed to - 28.538 degrees so Bus 3 would be at 0 degrees after first solution.



- Power flow results:

Earlier we had

$V_{Bus1} := 1.0 \angle -28.54$	$V1 = 1$	$\theta_{1\_p} = -28.541 \cdot deg$
$V_{Bus2} := 0.9819 \angle 0.00$	$mV2 = 0.9819$	$\theta_{2\_p} = 0 \cdot deg$
$V_{Bus3} := 0.9819 \angle 0.00$	$mV3 = 0.9819$	$\theta_{3\_p} = 0 \cdot deg$
$V_{Bus4} := 1.0 \angle -28.54$	$V4 = 1$	$\theta_{4\_p} = -28.541 \cdot deg$
$V_{Bus5} := 0.8688 \angle -11.84$	$mV5 = 0.8687$	$\theta_{5\_p} = -11.8393 \cdot deg$

- The results match, so the currents will match

Y_{bus} with phase shifts included:

$$Y_{1Bus} := \begin{pmatrix} Y_{11} & \frac{-1\cdot e^{-j\cdot 30deg}}{j\cdot X_{t11}} & 0 & 0 & 0 \\ \frac{-1\cdot e^{j\cdot 30deg}}{j\cdot X_{t11}} & Y_{22} & \frac{-1}{Z_{L23}} & 0 & \frac{-1}{Z_{L25}} \\ 0 & \frac{-1}{Z_{L23}} & Y_{33} & \frac{-1\cdot e^{j\cdot 30deg}}{jX_{t21}} & \frac{-1}{Z_{L35}} \\ 0 & 0 & \frac{-1\cdot e^{-j\cdot 30deg}}{jX_{t21}} & Y_{44} & 0 \\ 0 & \frac{-1}{Z_{L25}} & \frac{-1}{Z_{L35}} & 0 & Y_{55} \end{pmatrix}$$

$$Y_{1Bus} = \begin{pmatrix} -20i & 10+17.3205i & 0 & 0 & 0 \\ -10+17.3205i & -42.8371i & 20i & 0 & 2.8571i \\ 0 & 20i & -42.8371i & -10+17.3205i & 2.8571i \\ 0 & 0 & 10+17.3205i & -20i & 0 \\ 0 & 2.8571i & 2.8571i & 0 & -5.6793i \end{pmatrix}$$

	Number	Name	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5
1	1	BUS1	0.00 - j20.00	10.00 + j17.32			
2	2	BUS2	-10.00 + j17.32	0.00 - j42.84	-0.00 + j20.00		-0.00 + j2.86
3	3	BUS3		-0.00 + j20.00	0.00 - j42.84	-10.00 + j17.32	-0.00 + j2.86
4	4	BUS 4			10.00 + j17.32	0.00 - j20.00	
5	5	BUS 5		-0.00 + j2.86	-0.00 + j2.86		0.00 - j5.68