ECE 523
Symmetrical Components

Session 14

1/20

L14

Derivation of the DLG Fault Sequence Connections

A. ABC Domain Boundary Conditions

$I_A = 0$ $I_B = I_{Bf}$ $I_C = I_{Cf}$

$V_{BG} = I_B \cdot (R_f) + (I_B + I_C) \cdot R_g$

$= I_B \cdot (R_f + R_g) + I_C \cdot R_g$ (1)

$V_{CG} = (I_B + I_C) \cdot R_g + I_C \cdot (R_f)$

$= I_B \cdot R_g + I_C \cdot (R_f + R_g)$ (2)

B. Transform Boundary Conditions to Sequence Domain (phase A components)

$I_A = 0 = I_0 + I_1 + I_2$

$I_B = I_0 + a^2 \cdot I_1 + a \cdot I_2$

$I_C = I_0 + a \cdot I_1 + a^2 \cdot I_2$

$V_{BG} = V_0 + a^2 \cdot V_1 + a \cdot V_2$

$V_{CG} = V_0 + a \cdot V_1 + a^2 \cdot V_2$

C. Now rewrite equations (1) and (2) in the sequence domain

$V_{BG} = V_0 + a^2 \cdot V_1 + a \cdot V_2 = I_B \cdot (R_f + R_g) + I_C \cdot R_g$

$= (I_0 + a^2 \cdot I_1 + a \cdot I_2) \cdot (R_f + R_g) + (I_0 + a \cdot I_1 + a^2 \cdot I_2) \cdot R_g$ (3)

$V_{CG} = V_0 + a \cdot V_1 + a^2 \cdot V_2 = I_B \cdot R_g + I_C \cdot (R_f + R_g)$

$= (I_0 + a^2 \cdot I_1 + a \cdot I_2) \cdot R_g + (I_0 + a \cdot I_1 + a^2 \cdot I_2) \cdot (R_f + R_g)$ (4)

D. Now subtract equation (4) from equation (3):

$V_{BG} - V_{CG} = (V_0 - V_0) + (a^2 - a) \cdot V_1 + (a - a^2) \cdot V_2$

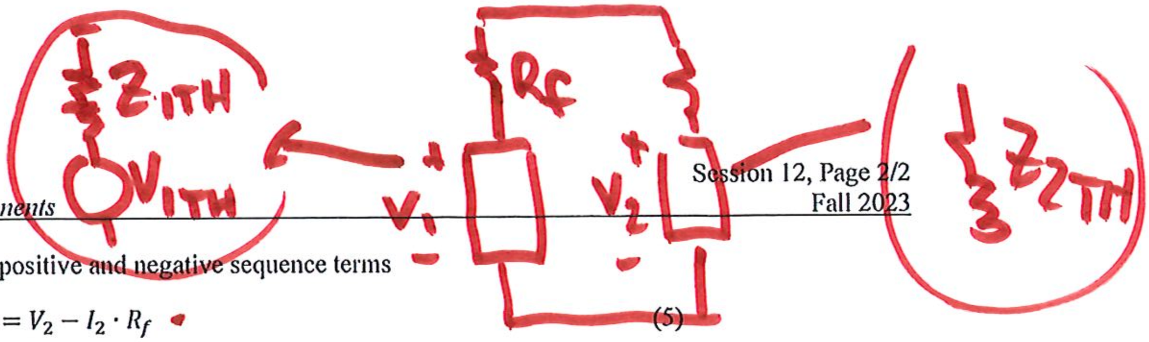
$= (I_0 - I_0) \cdot (R_f + 2 \cdot R_g) + (a^2 - a) \cdot I_1 \cdot R_f + (a - a^2) \cdot I_2 \cdot R_f + (a^2 + a - (a^2 + a)) \cdot (I_1 + I_2) \cdot R_g$ L13

- Simplifies to:

$(a^2 - a) \cdot V_1 - (a^2 - a) \cdot V_2 = (a^2 - a) \cdot I_1 \cdot R_f - (a^2 - a) \cdot I_2 \cdot R_f$ 14/16

- Divide by $(a^2 - a)$:

$V_1 - V_2 = (I_1 - I_2) \cdot R_f$ (5)



- Collect positive and negative sequence terms

$$V_1 - I_1 \cdot R_f = V_2 - I_2 \cdot R_f \qquad (5)$$

E. Now add equation (4) to equation (3):

$$V_{BG} + V_{CG} = 2 \cdot V_0 + (a^2 + a) \cdot V_1 + (a + a^2) \cdot V_2$$

$$= (2 \cdot I_0) \cdot (R_f + 2 \cdot R_g) + (a^2 + a) \cdot I_1 \cdot R_f + (a + a^2) \cdot I_2 \cdot R_f + (2 \cdot (a^2 + a)) \cdot (I_1 + I_2) \cdot R_g$$

- Collect terms:

$$2 \cdot V_0 + (a^2 + a) \cdot (V_1 + V_2) = (R_f + 2 \cdot R_g) \cdot (2 \cdot I_0 + (a^2 + a) \cdot (I_1 + I_2))$$

- Substitute in the following relationship:

$$(a^2 + a) = -1 \text{ which comes from } (1 + a^2 + a) = 0$$

- Resulting equation

$$2 \cdot V_0 - (V_1 + V_2) = (R_f + 2 \cdot R_g) \cdot (2 \cdot I_0 - (I_1 + I_2))$$

- Collect all zero sequence terms on the left-hand side, and positive and negative on right:

$$2 \cdot V_0 - 2 \cdot I_0 \cdot R_f - 4 \cdot I_0 \cdot R_g = (V_1 + V_2) - (I_1 + I_2) \cdot (R_f) - (I_1 + I_2) \cdot (2 \cdot R_g)$$

- Use the boundary condition for the currents:

$I_A =$ $I_1 + I_2 + I_0 = 0$ which implies: $I_1 + I_2 = -I_0$

- Substitute this only for the R_g term

$$2 \cdot V_0 - 2 \cdot I_0 \cdot R_f - 4 \cdot I_0 \cdot R_g = (V_1 + V_2) - (I_1 + I_2) \cdot (R_f) + (I_0) \cdot (2 \cdot R_g)$$

- Again, collect all zero sequence terms on the left-hand side:

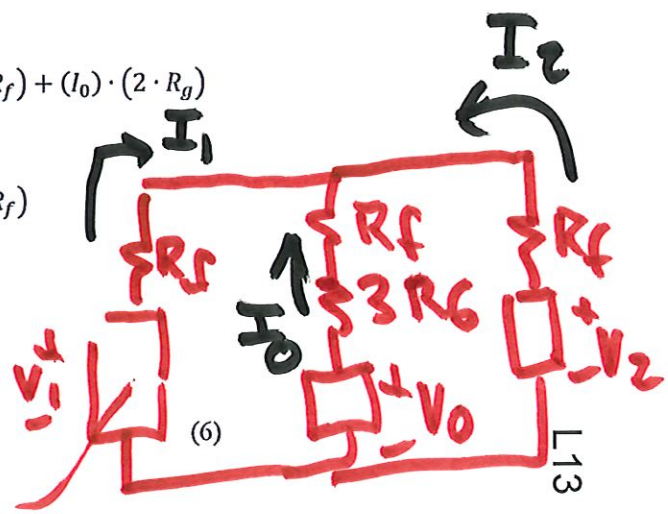
$$2 \cdot V_0 - 2 \cdot I_0 \cdot R_f - 6 \cdot I_0 \cdot R_g = (V_1 + V_2) - (I_1 + I_2) \cdot (R_f)$$

- Substitute in equation (5) on the right hand, resulting in:

$$2 \cdot V_0 - 2 \cdot I_0 \cdot R_f - 6 \cdot I_0 \cdot R_g = 2 \cdot (V_1 - I_1 \cdot R_f)$$

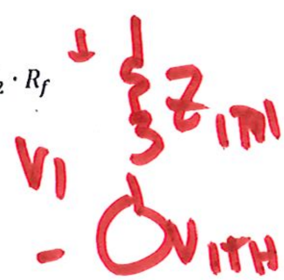
- Divide by 2, leaving:

$$V_0 - I_0 \cdot R_f - 3 \cdot I_0 \cdot R_g = V_1 - I_1 \cdot R_f \qquad (6)$$



F. Final result:

$$V_0 - I_0 \cdot R_f - 3 \cdot I_0 \cdot R_g = V_1 - I_1 \cdot R_f = V_2 - I_2 \cdot R_f$$



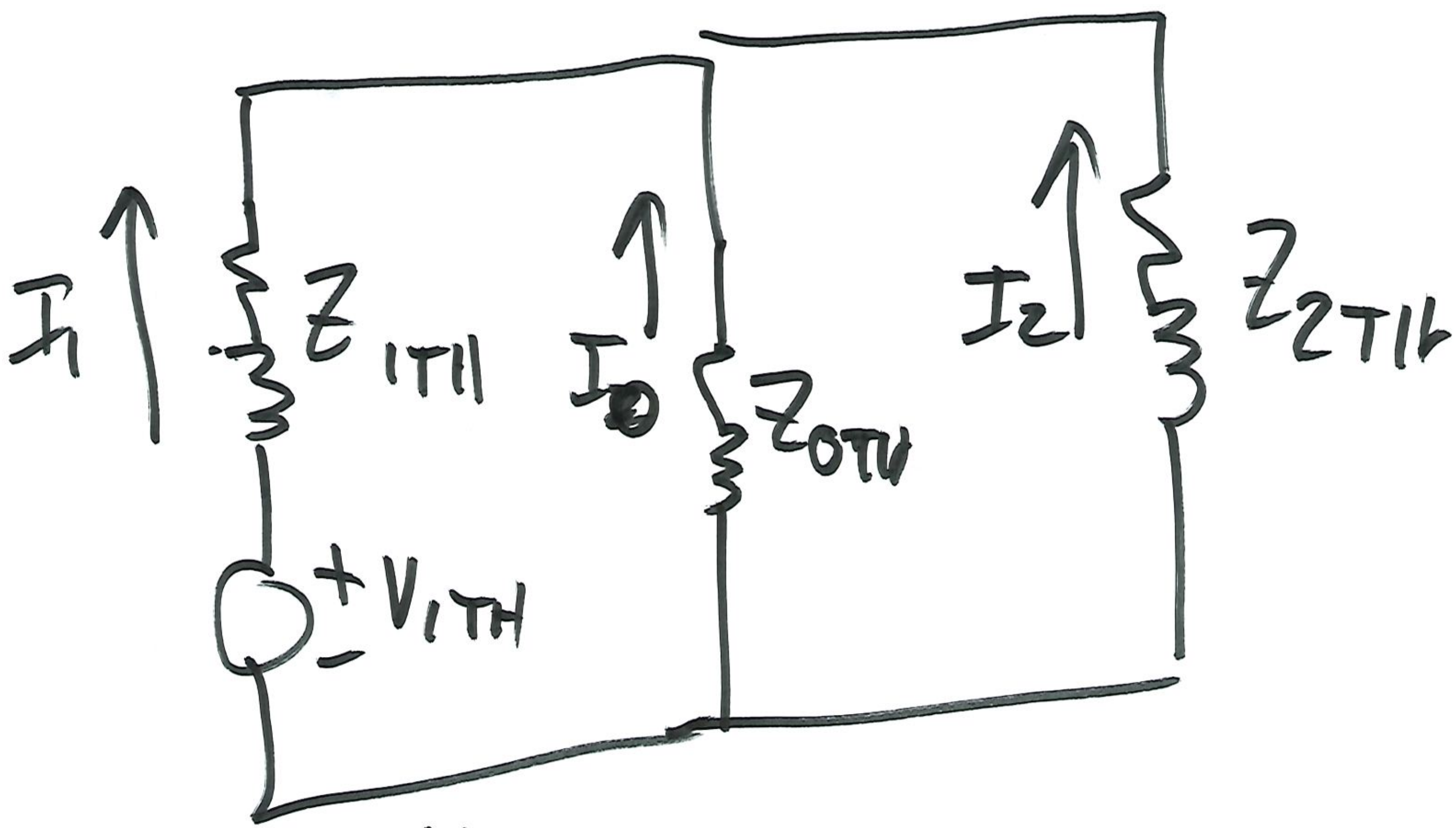
voltages

$I_A = I_0 + I_1 + I_2 = 0$

L13 15/16

L14 2/20

If R_f & $R_G = 0$



$$I_1 = \frac{V_{1TH}}{Z_{1TH} + (Z_{0TH} \| Z_{2TH})}$$

Use current dividers for I_2 + I_0

$$\bar{I}_2 = -\bar{I}_1 \left(\frac{\bar{Z}_{0TH}}{\bar{Z}_{2TH} + \bar{Z}_{0TH}} \right)$$

$$\bar{I}_0 = -I_1 \left(\frac{Z_{2TH}}{Z_{2TH} + Z_{0TH}} \right)$$

With fault resistances included

$$\bar{I}_1 = \frac{V_{1TH}}{(Z_{1TH} + R_f) + (Z_{2TH} + R_f) + (Z_{0TH} + R_f + 3R_G)}$$

$$\bar{I}_2 = -\bar{I}_1 \left(\frac{Z_{0TH} + R_f + 3R_f}{Z_{2TH} + R_f + Z_{0TH} + R_f + 3R_G} \right)$$

$$\bar{I}_0 = -\bar{I}_1 \left(\frac{(Z_{2TH} + R_f)}{(Z_{2TH} + R_f + Z_{0TH} + R_f + 3R_G)} \right)$$

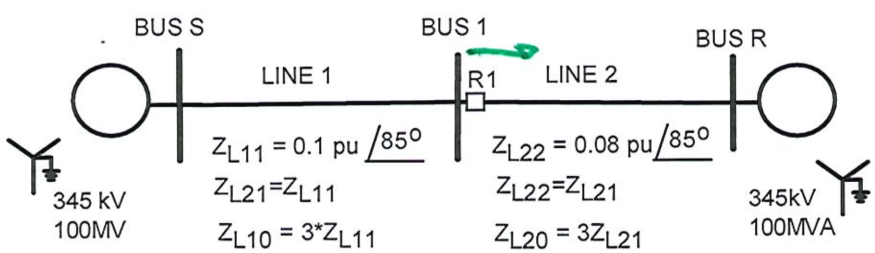
Liu 6/20

$$V_{abc\_SLGHV} := A_{012} \cdot \begin{pmatrix} V_{0\_SLGHV} \\ V_{1\_SLGHV} \\ V_{2\_SLGHV} \end{pmatrix}$$

$$\overrightarrow{\left|V_{abc\_SLGHV}\right|} = \begin{pmatrix} 0.62 \\ 1 \\ 0.62 \end{pmatrix} \cdot pu$$

$$\overrightarrow{\arg\left(V_{abc\_SLGHV}\right)} = \begin{pmatrix} 54.06 \\ -90 \\ 125.94 \end{pmatrix} \cdot deg$$

- ***Example with two sources:***



V_S = 1.0pu @ 0 deg

Z_{S1} = j0.03 pu

Z_{S2}=Z_{S1}

Z_{S0} = 3*Z_{S1}

Per unit line impedances calculated with SB=100MVA and VB=345kV LL

V_R = 1.0pu at 0 deg

Z_{R1} = j0.06 pu

Z_{R2}=Z_{R1}

Z_{R0} = 3*Z.$_{R1}$

$Z_{S1} := j \cdot 0.03pu$

$Z_{S2} := Z_{S1}$

$Z_{L11} := 0.1pu \cdot e^{j \cdot 85deg}$

$Z_{L11} = 0.01 + 0.1i$

$Z_{L21} := 0.08pu \cdot e^{j \cdot 85deg}$

$Z_{L21} = 0.01 + 0.08i$

Liu 7/20

$$V_{ABC\_R1}(m) := A_{012}\cdot\begin{pmatrix} V_{R1\_0}(m) \\ V_{R1\_1}(m) \\ V_{R1\_2}(m) \end{pmatrix} \qquad \overrightarrow{\left|V_{ABC\_R1}(0.5)\right|} = \begin{pmatrix} 0.24 \\ 1.18 \\ 1.18 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(V_{ABC\_R1}(0.5)\right)} = \begin{pmatrix} -0.88 \\ -132.89 \\ 132.99 \end{pmatrix}\cdot deg$$

$$I_{ABC\_R1}(m) := A_{012}\cdot\begin{pmatrix} I_{left\_0}(m) \\ I_{left\_1}(m) \\ I_{left\_2}(m) \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_R1}(0.5)\right|} = \begin{pmatrix} 3.53 \\ 0 \\ 0 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_R1}(0.5)\right)} = \begin{pmatrix} -85.88 \\ 49.05 \\ 49.05 \end{pmatrix}\cdot deg$$

- ***DLG faults:***

→ Prefault volt at fault location

Total sequence currents at fault location

$$I_{f1}(m) := \frac{V_f}{Z_{L2\_1\_thev}(m) + \left(\frac{1}{Z_{L2\_2\_thev}(m)} + \frac{1}{Z_{L2\_0\_thev}(m)}\right)^{-1}}$$

($Z_2 \| Z_0$)

$$I_{f2}(m) := -I_{f1}(m)\cdot\left(\frac{Z_{L2\_0\_thev}(m)}{Z_{L2\_2\_thev}(m) + Z_{L2\_0\_thev}(m)}\right)$$

$$I_{f0}(m) := -I_{f1}(m)\left(\frac{Z_{L2\_2\_thev}(m)}{Z_{L2\_2\_thev}(m) + Z_{L2\_0\_thev}(m)}\right)$$

POS

$$I_{left\_1}(m) := \frac{I_{f1}(m)\cdot\left[(1-m)\cdot Z_{L21} + Z_{R1}\right]}{Z_{S1} + Z_{L11} + m\cdot Z_{L21} + \left[(1-m)\cdot Z_{L21} + Z_{R1}\right]}$$

(at meter location

Liu 8/20

neg

$$I_{left\_2}(m) := \frac{I_{f2}(m)\cdot\left[(1-m)\cdot Z_{L22} + Z_{R2}\right]}{Z_{S2} + Z_{L12} + m\cdot Z_{L22} + \left[(1-m)\cdot Z_{L22} + Z_{R2}\right]}$$

zero

$$I_{left\_0}(m) := \frac{I_{f0}(m)\cdot\left[(1-m)\cdot Z_{L20} + Z_{R0}\right]}{Z_{S0} + Z_{L10} + m\cdot Z_{L20} + \left[(1-m)\cdot Z_{L20} + Z_{R0}\right]}$$

find seq voltc at R1

$$V_{R1\_1}(m) := V_f - I_{left\_1}(m)\cdot\left(Z_{S1} + Z_{L11}\right)$$

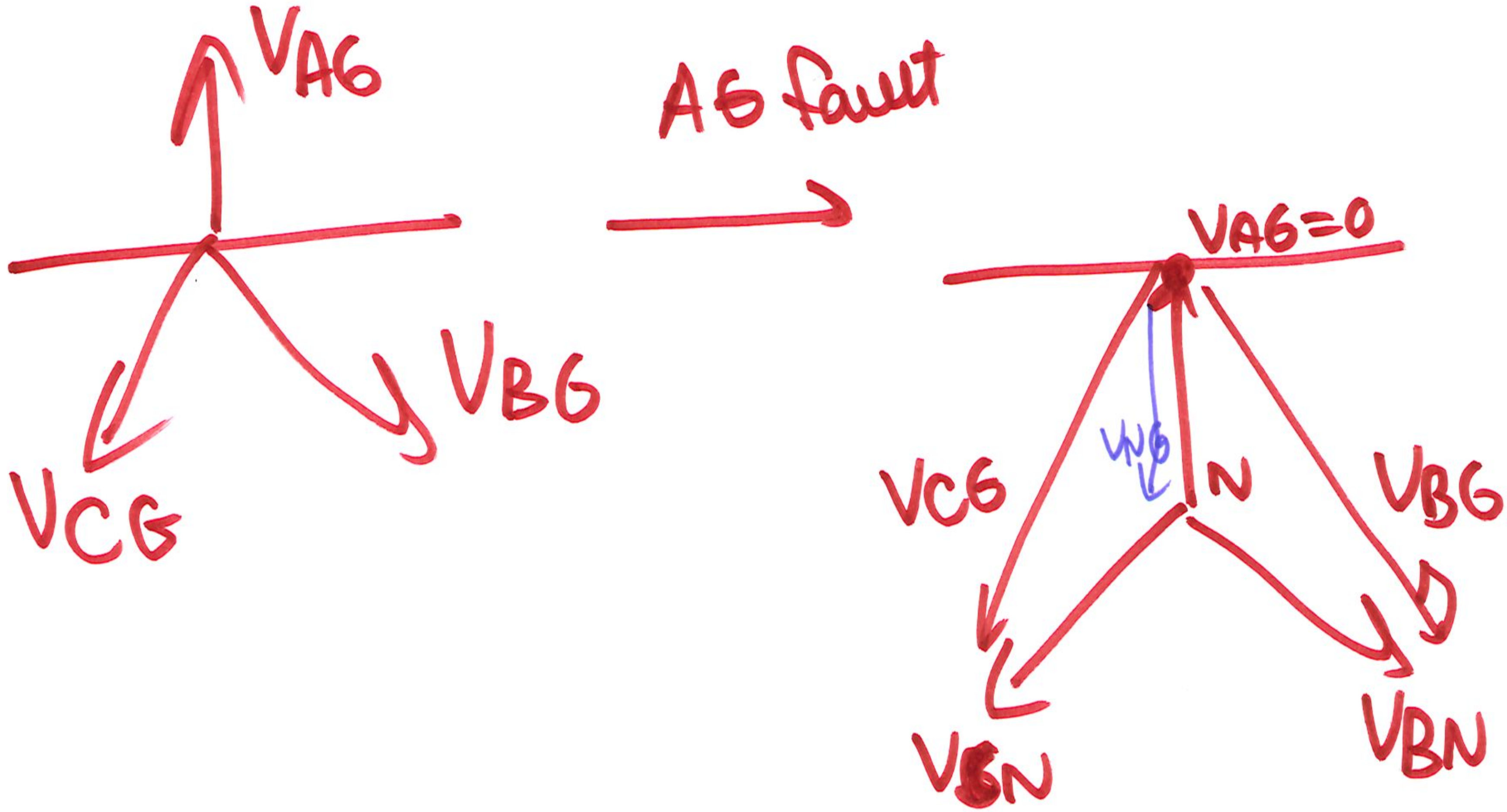
$$V_{R1\_2}(m) := 0 - I_{left\_2}(m)\cdot\left(Z_{S2} + Z_{L12}\right)$$

$$V_{R1\_0}(m) := 0 - I_{left\_0}(m)\cdot\left(Z_{S0} + Z_{L10}\right)$$

$$V_{ABC\_R1}(m) := A_{012}\cdot\begin{pmatrix} V_{R1\_0}(m) \\ V_{R1\_1}(m) \\ V_{R1\_2}(m) \end{pmatrix} \qquad \overrightarrow{\left|V_{ABC\_R1}(0.5)\right|} = \begin{pmatrix} 1.22 \\ 0.24 \\ 0.24 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(V_{ABC\_R1}(0.5)\right)} = \begin{pmatrix} 0.05 \\ -120.88 \\ 119.12 \end{pmatrix}\cdot deg$$

$$I_{ABC\_R1}(m) := A_{012}\cdot\begin{pmatrix} I_{left\_0}(m) \\ I_{left\_1}(m) \\ I_{left\_2}(m) \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_R1}(0.5)\right|} = \begin{pmatrix} 0 \\ 5.25 \\ 5.25 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_R1}\left(0.5 + 10^{-15}\right)\right)} = \begin{pmatrix} 93.58 \\ 170.22 \\ 18.02 \end{pmatrix}\cdot deg$$

- Adding the 10^{-15} avoided a divide by 0 problem in angle calculation


VAG
VBG
VCG
AG fault
VAG=0
N
VNG
VCG
VBG
VCN
VBN

DLG Fault:

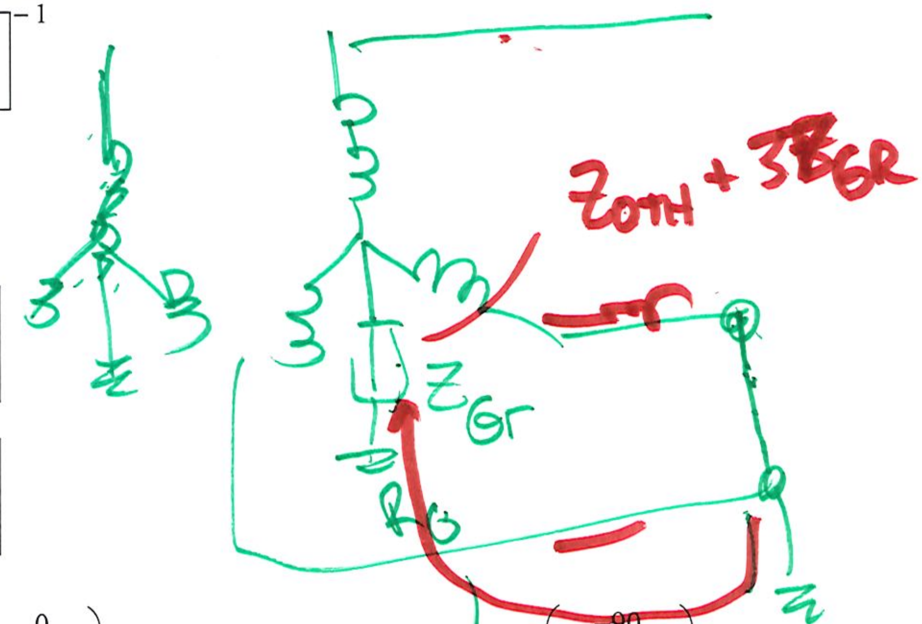
$$I_{1\_DLG}(M,R_f) := \frac{V_f}{\left(Z_1(M)_{4,4}\right) + \left[\frac{1}{Z_2(M)_{4,4}} + \frac{1}{\left(Z_0(M)_{4,4} + 3\cdot R_f\right)}\right]^{-1}}$$

$$\left|I_{1\_DLG}(0.5,0)\right| = 2.7269 \qquad \arg\left(I_{1\_DLG}(0.5,0)\right) = -90\cdot deg$$

Current dividers for I_0 and I_2

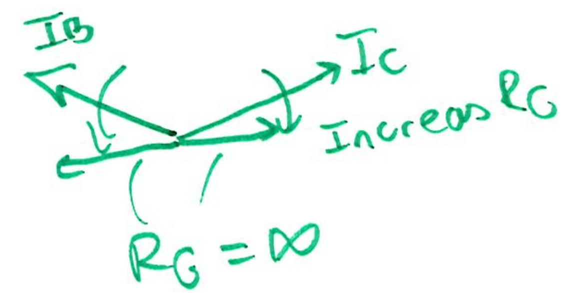
$$I_{0\_DLG}(M,R_f) := -I_{1\_DLG}(M,R_f)\cdot\left[\frac{Z_2(M)_{4,4}}{Z_2(M)_{4,4} + \left(Z_0(M)_{4,4} + 3\cdot R_f\right)}\right]$$

$$I_{2\_DLG}(M,R_f) := -I_{1\_DLG}(M,R_f)\cdot\left[\frac{Z_0(M)_{4,4} + 3\cdot R_f}{Z_2(M)_{4,4} + \left(Z_0(M)_{4,4} + 3\cdot R_f\right)}\right]$$



$$I_{ABC\_DLG}(M,R_f) := A_{012}\cdot\begin{pmatrix} I_{0\_DLG}(M,R_f) \\ I_{1\_DLG}(M,R_f) \\ I_{2\_DLG}(M,R_f) \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_DLG}(0.5,0)\right|} = \begin{pmatrix} 0 \\ 4.1902 \\ 4.1902 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_DLG}(0.5,0)\right)} = \begin{pmatrix} -90 \\ 162.5359 \\ 17.4641 \end{pmatrix}\cdot deg$$

- Angles meaningless when magnitude is 0



- Now find voltages in each sequence component)

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Unbalanced Fault Analysis Examples

Define units: $MVA := 1000kW$ $MVAR := MVA$ $a := 1 \cdot e^{j \cdot 120deg}$ $A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$

$MW := MVA$ $pu := 1$

$\angle(mag, ang) := mag \cdot \cos(ang \cdot deg) + j \cdot mag \cdot \sin(ang \cdot deg)$

System Description:

13.8kV
200MVA

Z_{S1} = j0.1 pu
Z_{S2}=Z_{S1}
Z_{S0} = 5.0 + j0.05pu

13.8kV: 115kV
200 MVA
X = 5%
X/R = 25
Δ-Yg

Z_{L1} = j0.5 pu
Z_{L2}=Z_{L1}
Z_{L0} = j1.8 pu
B_{c1}= 0.02pu
B_{c2} = B_{c1}
B_{c0} = 0.0133 pu
on 100MVA and 115 kV

115:24kV
75MVA
X = 10%
X/R = 25
Yg-Δ

24kV
75MVA

Z_{R1} = j0.2 pu
Z_{R2}=Z_{R1}
Z_{R0} = 10 + j0.05pu

Per Unit Change of Base Calculations (hidden, to view double click on the arrow)

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I_{2} I_{0} I_{1}

$$I_{0\_B1.DLG}(M,R_f) := \frac{V0_{DLG}(M,R_f)_1 - V0_{DLG}(M,R_f)_4}{M \cdot Z_{L10}} \qquad \left|I_{0\_B1.DLG}(0.5,0)\right| = 0.4424 \qquad \arg\left(I_{0\_B1.DLG}(0.5,0)\right) = 90 \cdot deg$$

I_0

$$I_{ABC\_B1.DLG}(M,R_f) := A_{012} \cdot \begin{pmatrix} I_{0\_B1.DLG}(M,R_f) \\ I_{1\_B1.DLG}(M,R_f) \\ I_{2\_B1.DLG}(M,R_f) \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_B1.DLG}(0.5,0)\right|} = \begin{pmatrix} 0.1165 \\ 2.7607 \\ 2.7607 \end{pmatrix} \cdot pu \qquad \arg\left(I_{ABC\_B1.DLG}(0.5,0)\right) = \begin{pmatrix} -90 \\ 164.8436 \\ 15.1564 \end{pmatrix} \cdot deg$$

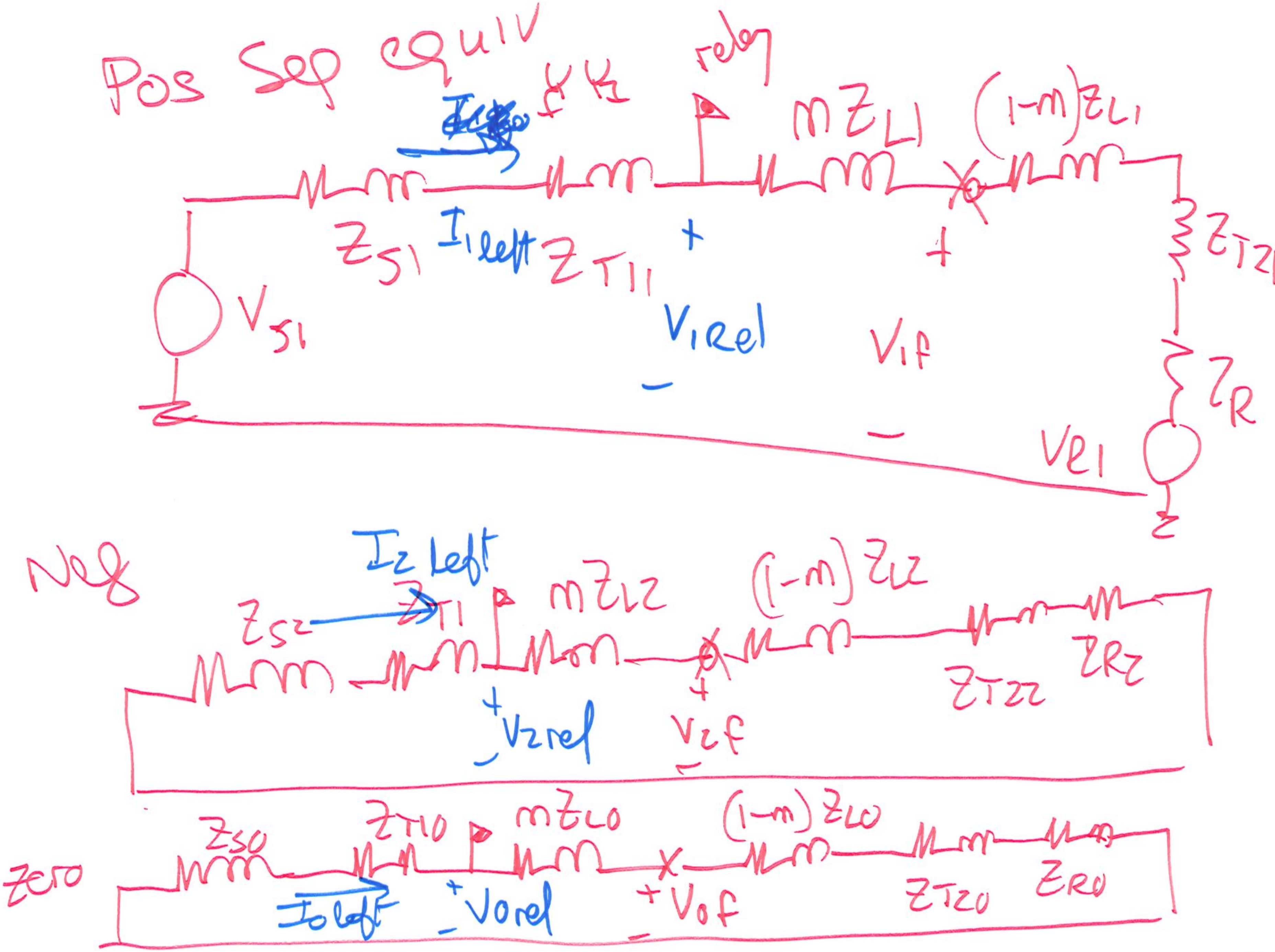
- LVside of transformer (Bus S)

$$I_{1\_BS.DLG}(M,R_f) := \frac{V_S \cdot e^{-j \cdot 30deg} - V1_{DLG}(M,R_f)_0}{jX_{G11}} \qquad \left|I_{1\_BS.DLG}(0.5,0)\right| = 1.8179 \cdot pu \qquad \arg\left(I_{1\_BS.DLG}(0.5,0)\right) = -120 \cdot deg$$

$$I_{2\_BS.DLG}(M,R_f) := \frac{0 - V2_{DLG}(M,R_f)_0}{jX_{G12}} \qquad \left|I_{2\_BS.DLG}(0.5,0)\right| = 1.259 \cdot pu \qquad \arg\left(I_{2\_BS.DLG}(0.5,0)\right) = 120 \cdot deg$$

$$I_{0\_BS.DLG} := 0$$

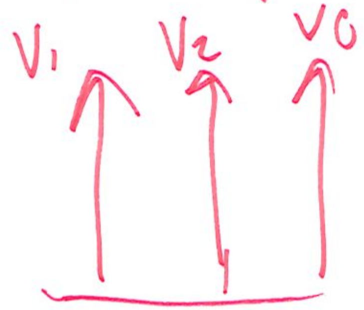
$$I_{ABC\_BS.DLG}(M,R_f) := A_{012} \cdot \begin{pmatrix} I_{0\_BS.DLG} \\ I_{1\_BS.DLG}(M,R_f) \\ I_{2\_BS.DLG}(M,R_f) \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_BS.DLG}(0.5,0)\right|} = \begin{pmatrix} 1.6128 \\ 1.6128 \\ 3.0769 \end{pmatrix} \cdot pu \qquad \arg\left(I_{ABC\_BS.DLG}(0.5,0)\right) = \begin{pmatrix} -162.5359 \\ 162.5359 \\ 0 \end{pmatrix} \cdot deg$$


Pos Seq equiv
relay
mZ_{L1}
$(1-m)Z_{L1}$
Z_{S1}
I_{1left}
Z_{T11}
Z_{T21}
V_{S1}
V_{1Rel}
V_{1f}
Z_R
V_{R1}
Neg
I_2 Left
Z_{S2}
Z_{T1}
mZ_{L2}
$(1-m)Z_{L2}$
Z_{T22}
Z_{R2}
V_{2rel}
V_{2f}
Zero
Z_{S0}
Z_{T10}
mZ_{L0}
$(1-m)Z_{L0}$
Z_{T20}
Z_{R0}
I_0 left
V_{0rel}
V_{0f}

Symmetrical components

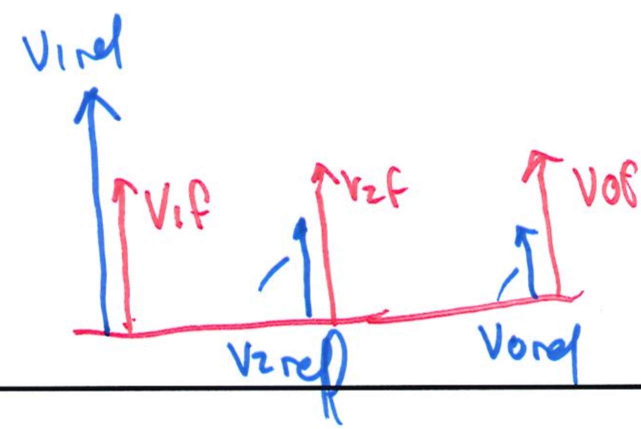
DLG

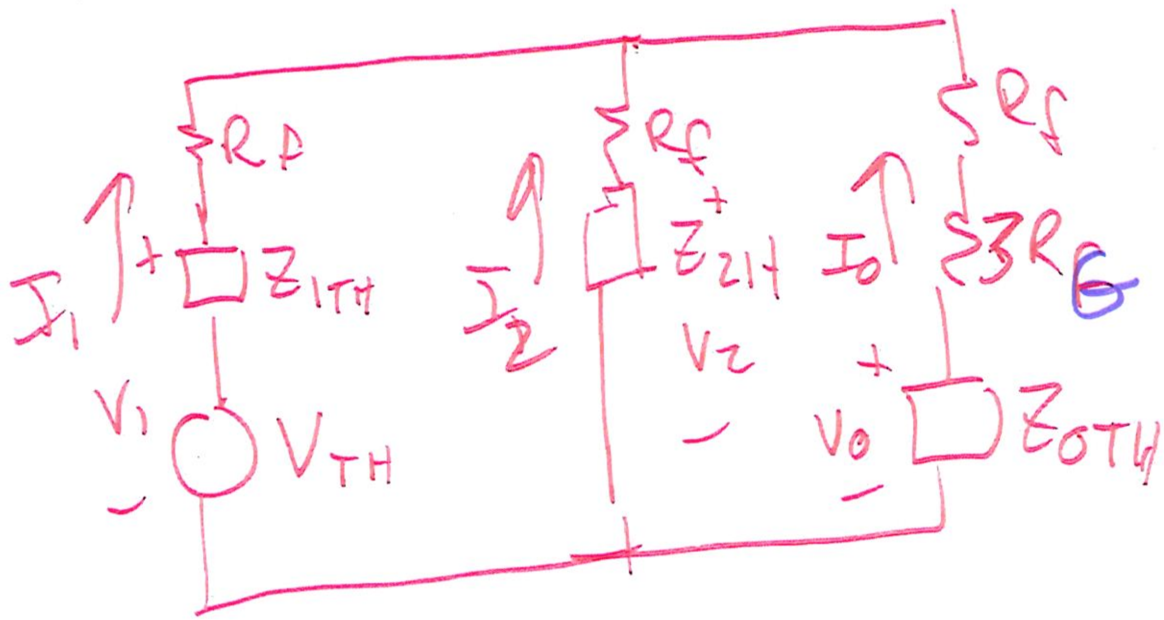
If $R_f = R_G = 0$



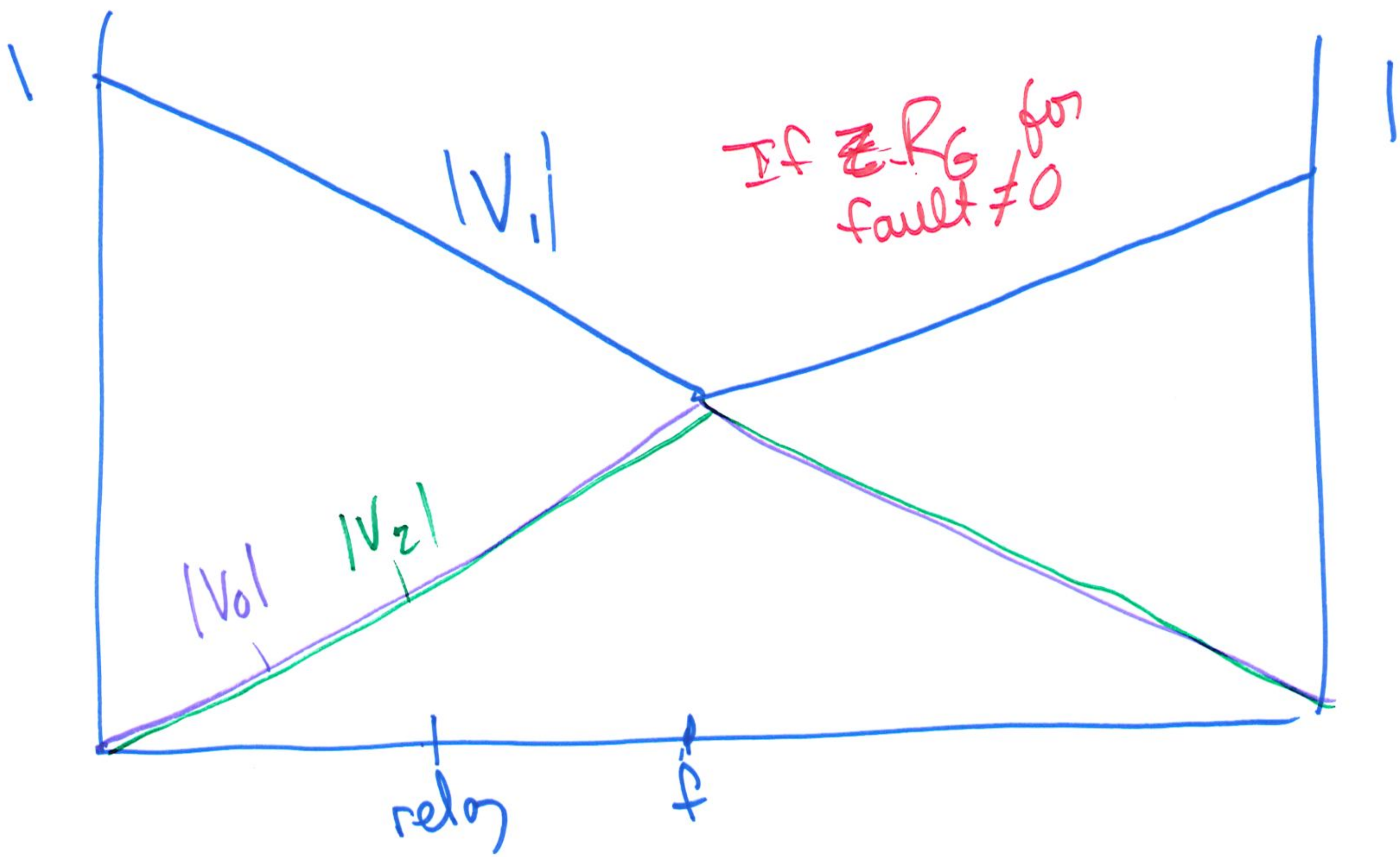
at fault location

At a relay at end of a line

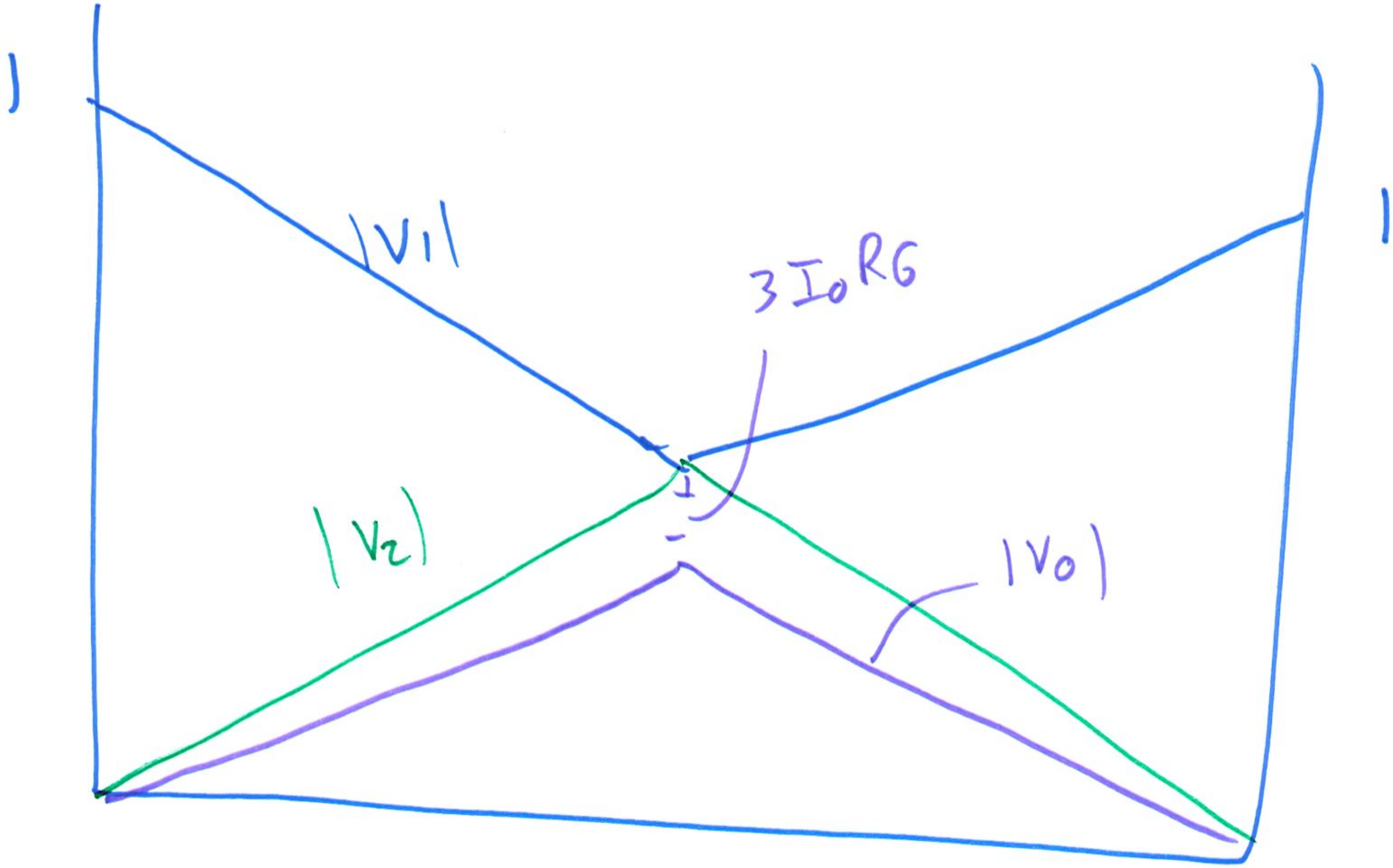



Rf
Rf
Rf
3Rf
I_1
I_2
I_0
Z_{1TH}
Z_{2TH}
Z_{OTH}
V_1
V_2
V_0
V_{TH}

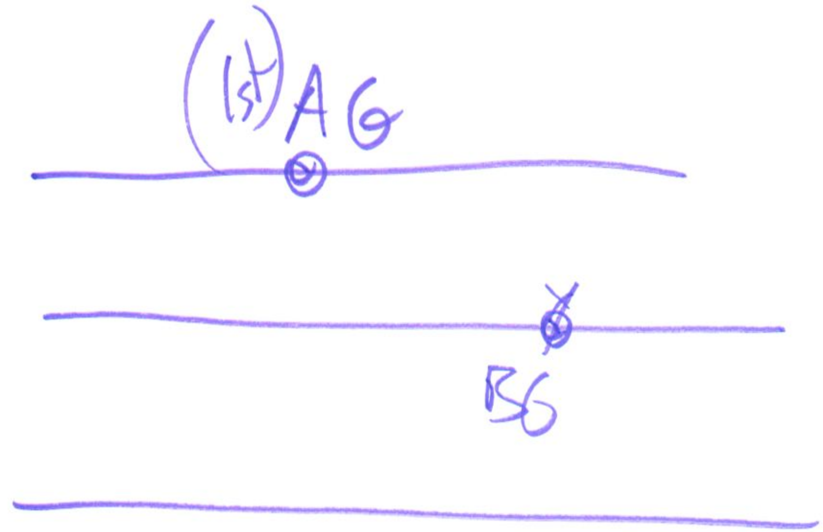
Voltage Profile



$R_G \neq 0$




Zf


(1st) AG
BG

University of Idaho

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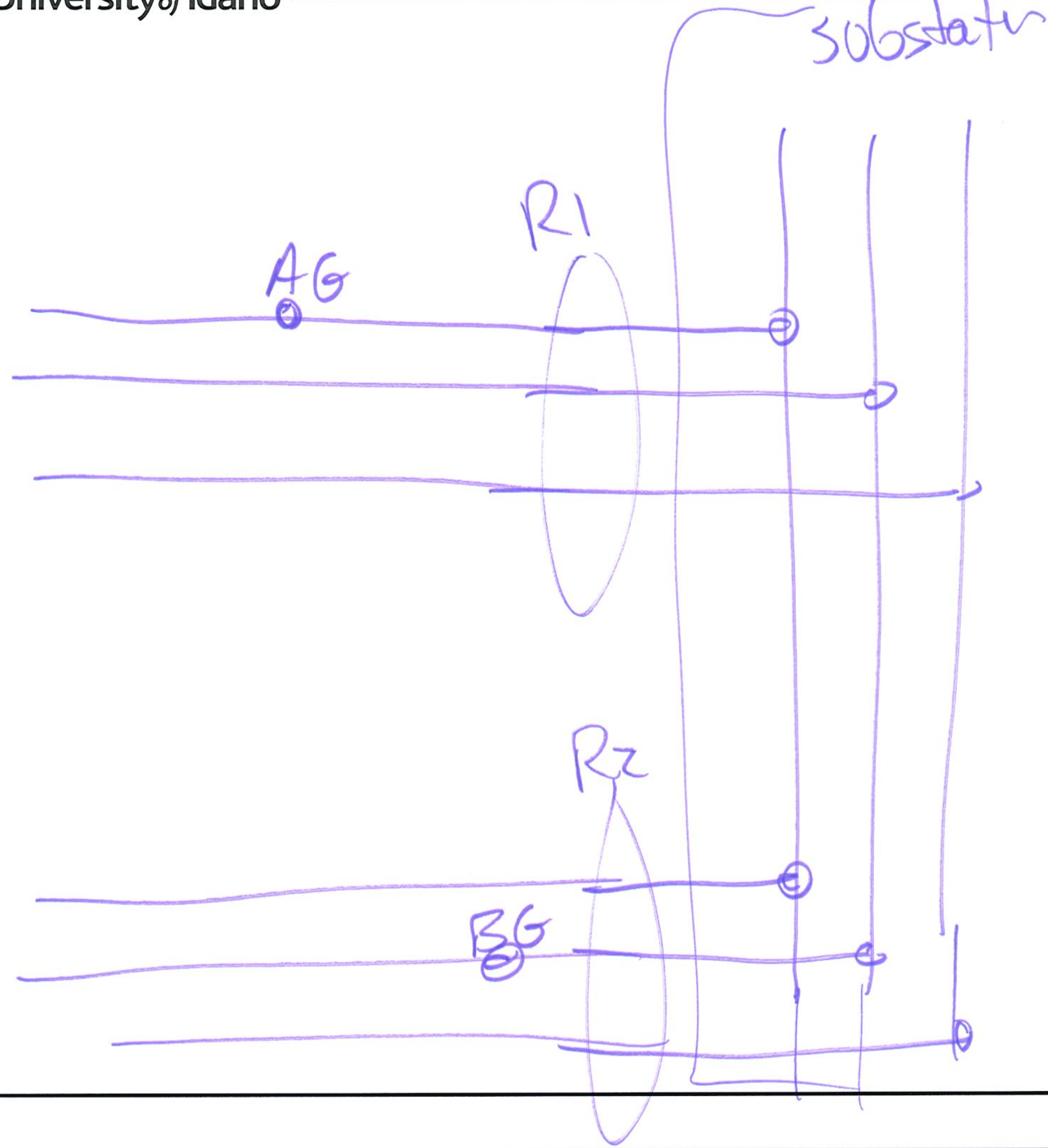

$R_f \neq 0$

1.