

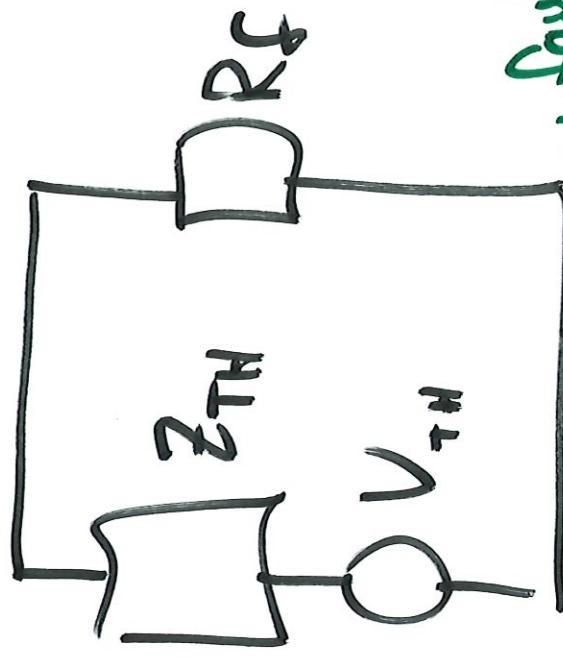
ECE 523
Symmetrical Components
Session 15

Three phase faults

\rightarrow pos sep

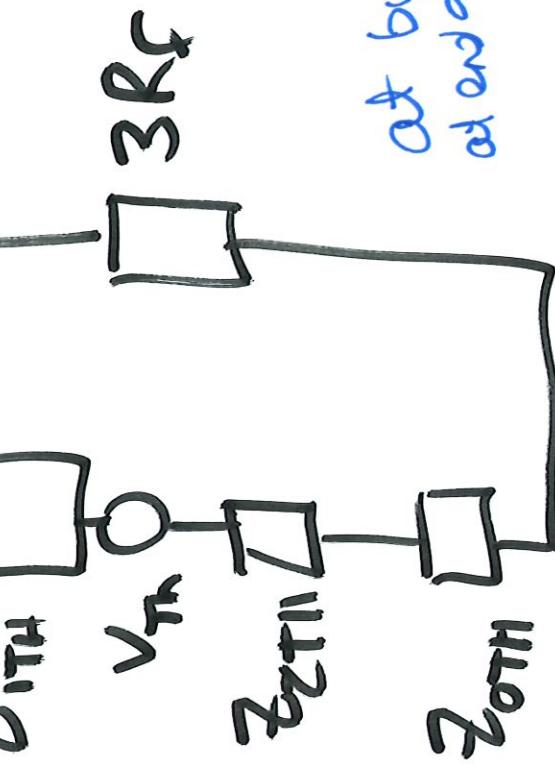
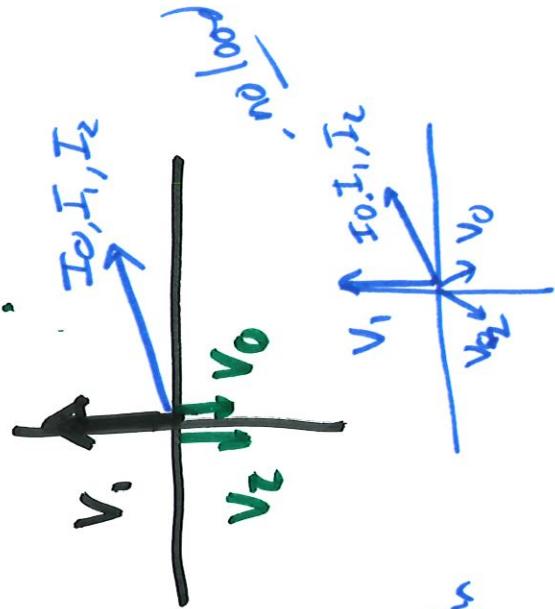
$\rightarrow Z_f$

$$I_0 = I_1 = I_2$$



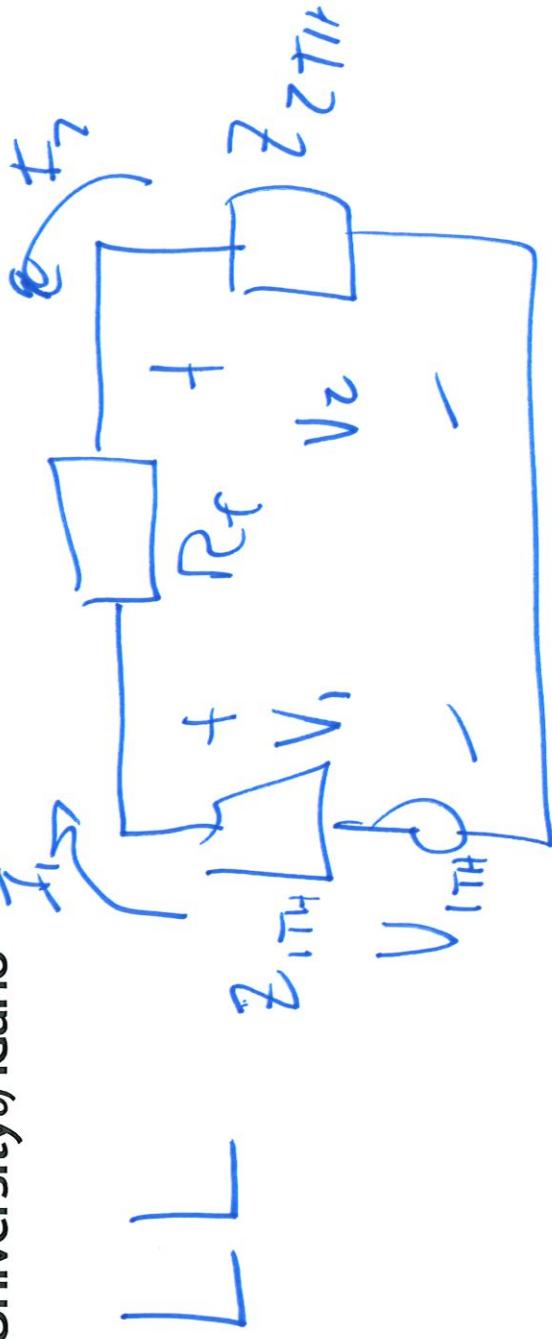
$$\text{At fault loc.}$$

$$V_1 + V_2 + V_0 = 3I_0 R_f$$



SLG

at bus
of end bus



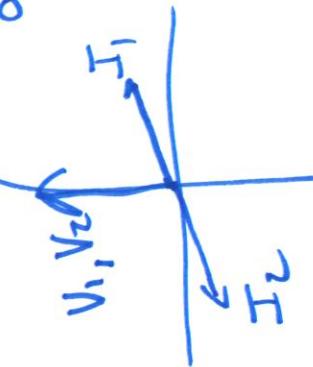
$$\text{If } R_f = 0 \quad V_1 = V_2 \text{ at fault point}$$

$$R_f \neq 0 \quad V_1 \neq V_2 = I_1 R_f$$

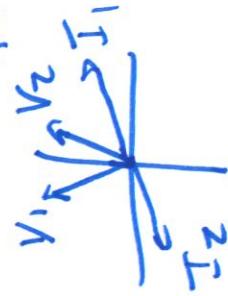
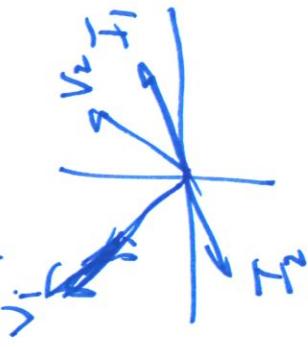
$R_f = 0$ at fault point

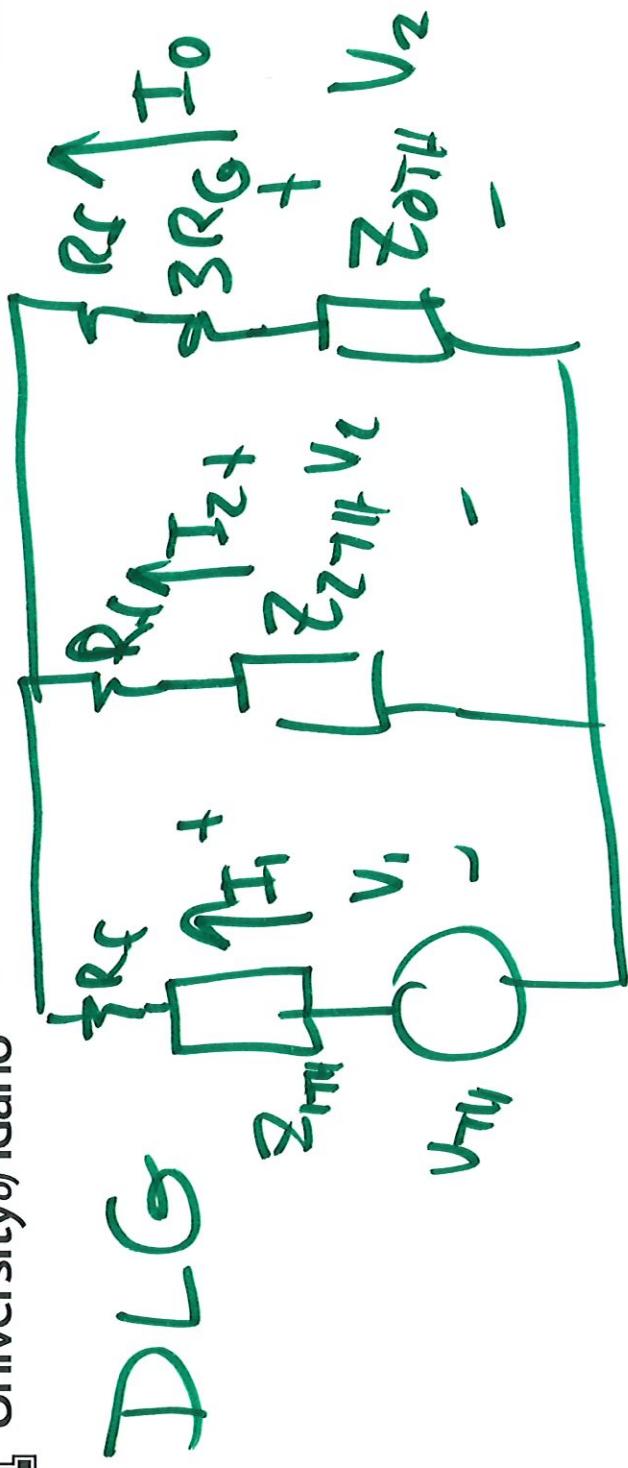
$$I_1 = -I_2$$

$$R_f \neq 0 \text{ at fault point}$$



All Gens





$$I_1 = I_2 + I_0 \quad \text{if } R_f = R_L = 0 \quad V_1 = V_2 = V_0$$

at fault point

Additive Power Flow in Fault Analysis

Important when:

1. Reproducing a field event

2. Studies with longer faults
- resistance \rightarrow or relay studies

3. for lines that have high load current

Introduces lower currents
load \downarrow

- especially weak system
 $(\frac{Z_s}{Z_{line}})$
high source impedance ratio

currents

Modelling load flow

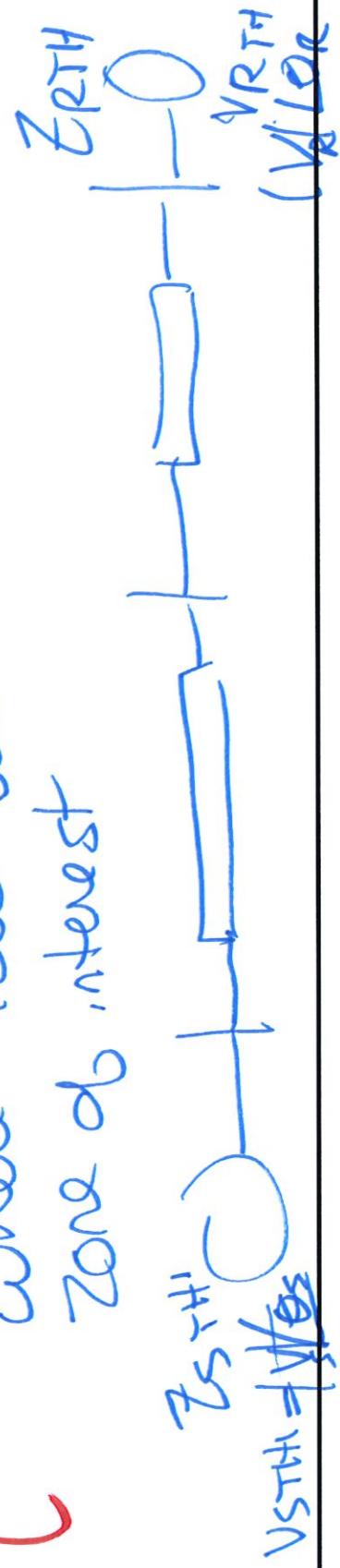
A. Put load impedances at specific buses

Positive feedbacks

→ solve Power flow
 $\Rightarrow |V|, \theta$ at each bus
 — including fault point

B. Use prefault power results where load buses outside of zone of interest

Positive feedbacks



Set line impedance parameters (set zero sequence line impedances to 3 times the positive sequence values):

$$Z_{L23} := j \cdot 0.05 \text{pu}$$

$$Z_{L25} := j \cdot 0.35 \text{pu}$$

$$Z_{L35} := j \cdot 0.35 \text{pu}$$

Transformer 1 Change of base (zero sequence impedances match positive sequence):

$$X_{t11} := 0.1 \left(\frac{S_B}{200 \text{MVA}} \right) \quad X_{t11} = 0.05 \text{pu} \quad X_{t10} := X_{t11}$$

Transformer 2 Change of base (zero sequence impedances match positive sequence):

$$X_{t21} := 0.1 \left(\frac{S_B}{200 \text{MVA}} \right) \quad X_{t21} = 0.05 \text{pu} \quad X_{t20} := X_{t11}$$

A. Assuming the load at Bus 5 is 100MW at 0.9 lagging power factor, perform a power flow solution. Use the voltage at Bus 3 as your angle reference

Options,

- (1) Solve the power flow equations for the entire system using Mathcad solve blocks
- (2) Use Powerworld or a similar load flow problem (at least to check results)

- In order to use V3 as the reference angle, use the angle for V3 from the power flow solution and shift the slack bus angle such that the new angle at Bus 3 is 0.

(1) Solve Full Powerflow Solution Using MathCAD Solve Block

- Positive sequence Y bus for power flow calculations (ignore phase shifts for the moment):

$$Y_{11} := \frac{1}{j \cdot X_{t11}} \quad Y_{12} := \frac{-1}{j \cdot X_{t11}}$$

• Symmetry assumed

power flow \rightarrow source impedance not included

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$$Y_{22} := \frac{1}{j \cdot X_{t11}} + \frac{1}{Z_{L23}} + \frac{1}{Z_{L25}} + \frac{j \cdot B_{c23}}{2} + \frac{j \cdot B_{c25}}{2} \quad Y_{23} := \frac{-1}{Z_{L23}} \quad Y_{25} := \frac{-1}{Z_{L25}}$$

$$Y_{33} := \frac{1}{j \cdot X_{t21}} + \frac{1}{Z_{L23}} + \frac{1}{Z_{L35}} + \frac{j \cdot B_{c23}}{2} + \frac{j \cdot B_{c35}}{2} \quad Y_{34} := \frac{-1}{j \cdot X_{t21}} \quad Y_{35} := \frac{-1}{Z_{L35}}$$

$$Y_{44} := \frac{1}{j \cdot X_{t11}} \quad Y_{55} := \frac{1}{Z_{L25}} + \frac{1}{Z_{L35}} + \left(\frac{j \cdot B_{c25}}{2} + \frac{j \cdot B_{c35}}{2} \right)$$

$$Y_{busPF} := \begin{pmatrix} Y_{11} & Y_{12} & 0 & 0 & 0 \\ Y_{12} & Y_{22} & Y_{23} & 0 & Y_{25} \\ 0 & Y_{23} & Y_{33} & Y_{34} & Y_{35} \\ 0 & 0 & Y_{34} & Y_{44} & 0 \\ 0 & Y_{25} & Y_{35} & 0 & Y_{55} \end{pmatrix} \quad Y_{busPF} = \begin{pmatrix} -20i & 20i & 0 & 0 & 0 \\ 20i & -42.8371i & 20i & 0 & 2.8571i \\ 0 & 20i & -42.8371i & 20i & 2.8571i \\ 0 & 0 & 20i & -20i & 0 \\ 0 & 2.8571i & 2.8571i & 0 & -5.6793i \end{pmatrix}$$

Initial values

$$P2 := 0 \quad P3 := 0 \quad P4 := 0.5pu \quad P5 := -1.0pu$$

$$Q2 := 0 \quad Q3 := 0 \quad Q5 := P5 \cdot \tan(\arccos(0.9)) \quad Q5 = -0.4843 \cdot pu$$

$$V1 := 1 \quad V4 := 1 \quad a1 := 0$$

- P5 and Q5 are negative injections since they represent a load

- Initial guesses

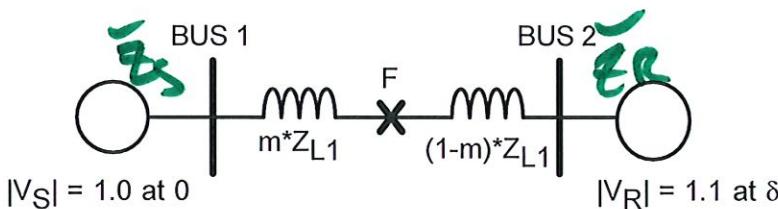
$$V2 := 1 \quad a2 := 0 \text{deg}$$

$$V3 := 1 \quad a3 := 0 \text{deg}$$

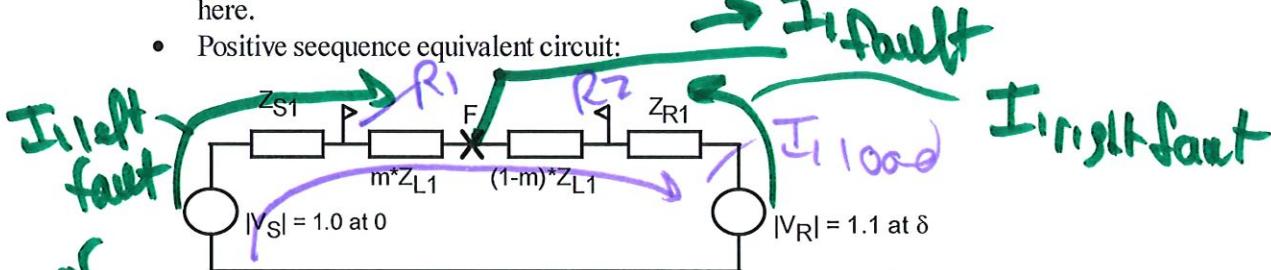
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Fault Calculations with Power Flow

- This is much easier to do with Zbus methods. This approach only works for simple systems
- The system below has a prefault power flow condition due to the angle and magnitude differences between the sources.
- The fault calculations need to change a little to ensure that the positive sequence current reflects this power flow in the case of a fault where power flow can continue to flow.
- Lets look at a SLG fault case.



- The negative and zero sequence circuits will be the same as one would in a case where the sources have equal angles and magnitudes, so they will not be described here.
- Positive sequence equivalent circuit:



- There are effectively two components to the current seen at each relay, and they can be determined using superposition.
 - The fault current that flows due to the fault and leave this network at point F and reenters from the neutral plane.
 - The current that flows between the two sources, the load current.

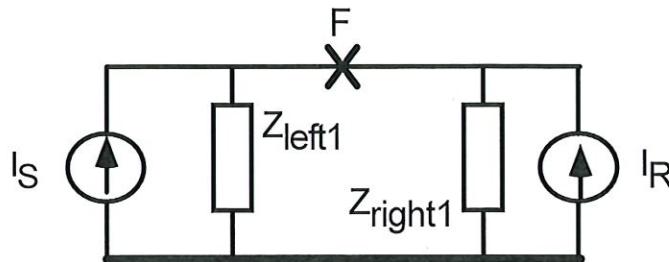
1. Determining fault current

- We need to find a Thevenin equivalent circuit.
- The process is actually a standard circuit analysis approach (Millman's Theorem), but is typically avoided if the voltage sources are all assumed to have the same magnitude and angle.
 - Convert the two sources to their Norton equivalents, using the impedance between the source and the fault point. Note that these are phasor calculations.



$$Z_{right1} = Z_{R1} + (1 - m) \cdot Z_{L1}$$

$$I_{Norton_right} = \frac{V_{S1}}{Z_{right1}}$$

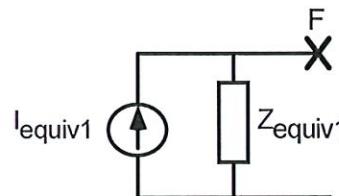


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2. Note that the impedances are in parallel and the current sources are effectively in parallel

- Combine the impedances in parallel
- Combine the two current sources (note that this is not limited to two sources)

$$Z_{\text{equiv1}} = \left(\frac{1}{Z_{\text{left1}}} + \frac{1}{Z_{\text{right1}}} \right)^{-1}$$

$$I_{\text{equiv1}} = I_{\text{Norton_left}} + I_{\text{Norton_right}}$$

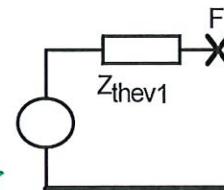


- Then convert back to a Thevenin equivalent

$$Z_{\text{thev1}} = Z_{\text{equiv1}}$$

$$V_{\text{thev1}} = I_{\text{equiv1}} \cdot Z_{\text{equiv1}}$$

voltage at fault point



- This Thevenin equivalent source is used for the fault calculations. **But not for the power flow calculation**
 - Note that the Thevenin impedance is the same as we always do.
 - Now the voltage source has a magnitude and angle that reflects the difference between the two sources.
 - If the sources both have the same magnitude and angle, the resulting Thevenin voltage source will match that.

2. Determining power flow current

- This is just like any other power flow calculation. In this case you can look between the two known source voltages and the **total impedance** between them. In other cases you might need to find V1 and V2 and just use the line impedance.

$$I_{12} = \frac{V_{S1} - V_{R1}}{Z_{S1} + Z_{L1} + Z_{R1}}$$

$$I_{21} = \frac{V_{R1} - V_{S1}}{(Z_{S1} + Z_{L1} + Z_{R1})}$$

- Notes:
 1. The fault location doesn't matter in this calculation
 2. The Thevenin equivalent source from above is not used
 3. I_{12} flows in the opposite direction I_{21}

3. Total sequence currents

- The positive sequence current for the relay at bus 1 (phasor sums):

$$I_{\text{Relay1}} = I_{f_relay1} + I_{12_relay1}$$

$$I_{\text{Relay2}} = I_{f_relay2} - I_{12_relay1}$$

- I_{f_relay1} and I_{f_relay2} come from current dividers as usual
- The negative and zero sequence currents do not include an load flow current and are simply from current dividers from the fault calculation.

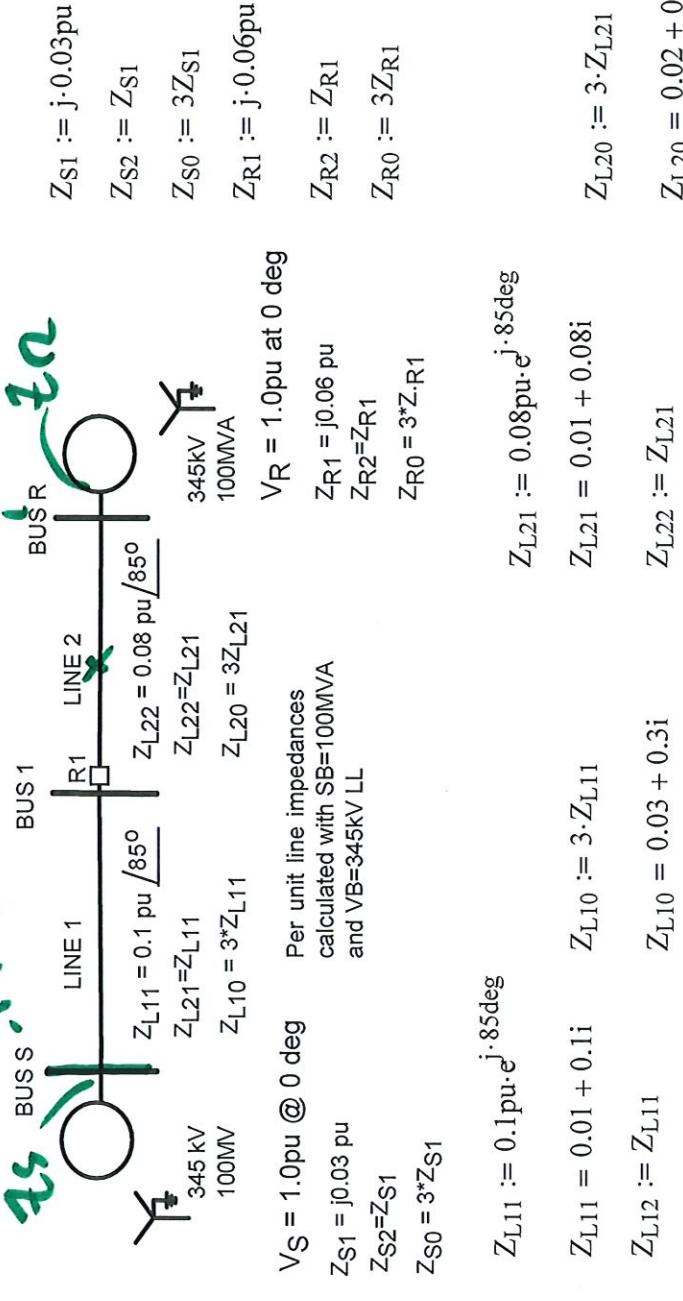
No load flow 0+Z networks

Fault Analysis with Power Flow on the System

$$\text{pu} := 1 \quad \text{MVA} := 1000 \text{kW}$$

$$a := 1e^{j \cdot 120\text{deg}} \quad A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

- Example with two sources:



- For faults on Line 2:

$$Z_{L2_1_thev}(n) := \left[\frac{1}{Z_{S1} + Z_{L11} + n \cdot Z_{L21}} + \frac{1}{(1-n) \cdot Z_{L21} + Z_{R1}} \right]^{-1}$$

$$Z_{L2_2_thev}(n) := \left[\frac{1}{Z_{S2} + Z_{L12} + n \cdot Z_{L22}} + \frac{1}{(1-n) \cdot Z_{L22} + Z_{R2}} \right]^{-1}$$

$$Z_{L2_0_thev}(n) := \left[\frac{1}{Z_{S0} + Z_{L10} + n \cdot Z_{L20}} + \frac{1}{(1-n) \cdot Z_{L20} + Z_{R0}} \right]^{-1}$$

Impedance Matrix Approach

- Need positive, negative and zero sequence matrices

$$Y_{bus1}(m) := \begin{bmatrix} \frac{1}{Z_{S1}} + \frac{1}{Z_{L11}} & -\frac{1}{Z_{L11}} & 0 \\ -\frac{1}{Z_{L11}} & \frac{1}{Z_{L11}} + \frac{1}{m \cdot Z_{L21}} & 0 \\ 0 & 0 & \frac{(1-m) \cdot Z_{L21}}{Z_{R1}} + \frac{1}{m \cdot Z_{L21}} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -\frac{1}{(1-m) \cdot Z_{L21}} \\ 0 & \frac{-1}{(1-m) \cdot Z_{L21}} & \frac{1}{m \cdot Z_{L21}} + \frac{1}{(1-m) \cdot Z_{L21}} \end{bmatrix}$$

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$$Z_{\text{bus}1}(m) := Y_{\text{bus}1}(m)^{-1}$$

$$Y_{\text{bus}2}(m) := \begin{bmatrix} \frac{1}{Z_{S2}} + \frac{1}{Z_{L12}} & -\frac{1}{Z_{L12}} & 0 & 0 \\ -\frac{1}{Z_{L12}} & \frac{1}{Z_{L12}} + \frac{1}{m \cdot Z_{L22}} & 0 & -\frac{1}{m \cdot Z_{L22}} \\ 0 & 0 & \frac{1}{(1-m) \cdot Z_{L22}} + \frac{1}{Z_{R2}} & -\frac{1}{(1-m) \cdot Z_{L22}} \\ 0 & -\frac{1}{m \cdot Z_{L22}} & \frac{-1}{(1-m) \cdot Z_{L22}} & \frac{1}{m \cdot Z_{L22}} + \frac{1}{(1-m) \cdot Z_{L22}} \end{bmatrix}$$

ordinate
bus 0, 1, 2

$$Z_{\text{bus}2}(m) := Y_{\text{bus}2}(m)^{-1}$$

$$Y_{\text{bus}0}(m) := \begin{bmatrix} \frac{1}{Z_{S0}} + \frac{1}{Z_{L10}} & -\frac{1}{Z_{L10}} & 0 & 0 \\ -\frac{1}{Z_{L10}} & \frac{1}{Z_{L10}} + \frac{1}{m \cdot Z_{L20}} & 0 & -\frac{1}{m \cdot Z_{L20}} \\ 0 & 0 & \frac{1}{(1-m) \cdot Z_{L20}} + \frac{1}{Z_{R0}} & -\frac{1}{(1-m) \cdot Z_{L20}} \\ 0 & -\frac{1}{m \cdot Z_{L20}} & \frac{-1}{(1-m) \cdot Z_{L20}} & \frac{1}{m \cdot Z_{L20}} + \frac{1}{(1-m) \cdot Z_{L20}} \end{bmatrix}$$

$$Z_{\text{bus}0}(m) := Y_{\text{bus}0}(m)^{-1}$$

- SLG Fault

$$I_{f0}(m) := \frac{V_1_Thev(m)}{Z_{L2_1_thev}(m) + Z_{L2_2_thev}(m) + Z_{L2_0_thev}(m)}$$

$$\begin{aligned} I_{f1}(m) &:= I_{f0}(m) \\ I_{f2}(m) &:= I_{f0}(m) \end{aligned}$$

$$\arg(I_{f0}(0.5)) = -95.09\text{-deg}$$

- Fault Currents at Relay 1:

$$I_{fA1}(m) := I_{f1}(m) \cdot \left[\frac{Z_{R1} + (1-m) \cdot Z_{L21}}{(Z_{S1} + Z_{L11} + m \cdot Z_{L21}) + [Z_{R1} + (1-m) \cdot Z_{L21}]} \right]$$

$$|I_{fA1}(0.5)| = 1.19 \quad \arg(I_{fA1}(0.5)) = -93.76\text{-deg}$$

flow

However, the positive sequence current seen by the relay will include the load current

$$I_{relayA1}(m) := I_{fA1}(m) + ISR1$$

$$I_{relayA1}(0.5) = 0.91 - 1.08i$$

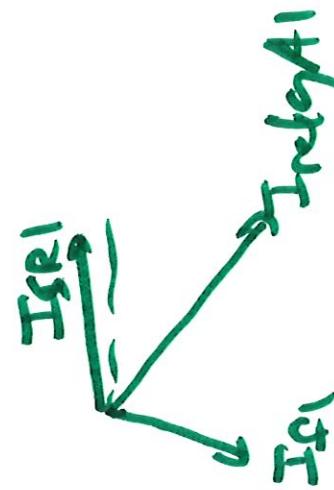
$$|I_{relayA1}(0.5)| = 1.41$$

$$\arg(I_{relayA1}(0.5)) = -49.96\text{-deg}$$

Negative sequence and zero sequence currents don't see load current..

$$I_{relayA2}(m) := I_{f2}(m) \cdot \left[\frac{(1-m) \cdot Z_{L22} + Z_{R2}}{(Z_{S2} + Z_{L12} + m \cdot Z_{L22}) + [Z_{R2} + (1-m) \cdot Z_{L22}]} \right]$$

$$|I_{relayA2}(0.5)| = 1.19 \quad \arg(I_{relayA2}(0.5)) = -93.76\text{-deg}$$



Now add power flow based on phase angle differences

- Make Bus S the slack bus at 1.0pu
- Set Bus R magnitude and angle

$$V_{S1} := 1.0 \text{pu} e^{j \cdot 0 \text{deg}}$$

- This case is simple enough that we don't need to do a normal power flow solution.

$$I_{SR1} := \frac{V_{S1} - V_{R1}}{Z_{L11} + Z_{L21}} \quad |I_{SR1}| = 0.99 \cdot \text{pu} \quad \arg(I_{SR1}) = 6.37 \cdot \text{deg}$$

$$V_{B1} := V_{S1} - I_{SR1} \cdot Z_{L11} \quad |V_{B1}| = 1.01 \cdot \text{pu} \quad \arg(V_{B1}) = -5.65 \cdot \text{deg}$$

$\overline{V_{S1}}$
 $\overline{V_{R1}}$
 $\overline{V_{B1}}$

Over V_{src_S}

- But for fault analysis we need the voltages behind the source impedances

$$\begin{aligned} V_{src_S} &:= V_{S1} + I_{SR1} \cdot Z_{S1} & |V_{src_S}| &= 0.997 & \arg(V_{src_S}) &= 1.7 \cdot \text{deg} \\ V_{src_R} &:= V_{R1} - I_{SR1} \cdot Z_{R1} & |V_{src_R}| &= 1.038 & \arg(V_{src_R}) &= -13.22 \cdot \text{deg} \end{aligned}$$

$$I_{s1Nor}(m) := \frac{V_{src_S}}{Z_{S1} + Z_{L11} + m \cdot Z_{L21}} \quad I_{r1Nor}(m) := \frac{V_{src_R}}{Z_{R1} + (1 - m) \cdot Z_{L21}}$$

$$I_1 \text{Nor}(m) := I_{s1Nor}(m) + I_{r1Nor}(m)$$

$$V_1 \text{Thev}(m) := I_1 \text{Nor}(m) \cdot Z_{L2_1 \text{thev}}(m) \quad |V_1 \text{Thev}(0.5)| = 1.01 \cdot \text{pu} \quad \arg(V_1 \text{Thev}(0.5)) = -7.87 \cdot \text{deg}$$

at fault point
 $m = 0.5$

$$I_{\text{relayA}0}(m) := I_{f0}(m) \cdot \left[\frac{(1-m) \cdot Z_{L20} + Z_{R0}}{Z_{S0} + Z_{L10} + m \cdot Z_{L20} + [(1-m) \cdot Z_{L20} + Z_{R0}]} \right]$$

$$|I_{\text{relayA}0}(0.5)| = 1.19 \quad \arg(I_{\text{relayA}0}(0.5)) = -93.76 \cdot \text{deg}$$

$$V_{\text{relayA}1}(m) := V_{\text{src_S}} - I_{\text{relayA}1}(m) \cdot (Z_{S1} + Z_{L11})$$

$$\arg(V_{\text{relayA}1}(0.5)) = 0.85 \cdot \text{pu}$$

$$V_{\text{relayA}2}(m) := -I_{\text{relayA}2}(m) \cdot (Z_{S2} + Z_{L12})$$

$$|V_{\text{relayA}2}(0.5)| = 0.15 \cdot \text{pu}$$

$$V_{\text{relayA}0}(m) := -I_{\text{relayA}0}(m) \cdot (Z_{S0} + Z_{L10})$$

$$|V_{\text{relayA}0}(0.5)| = 0.46 \cdot \text{pu}$$

$$\arg(V_{\text{relayA}0}(0.5)) = 172.4 \cdot \text{deg}$$

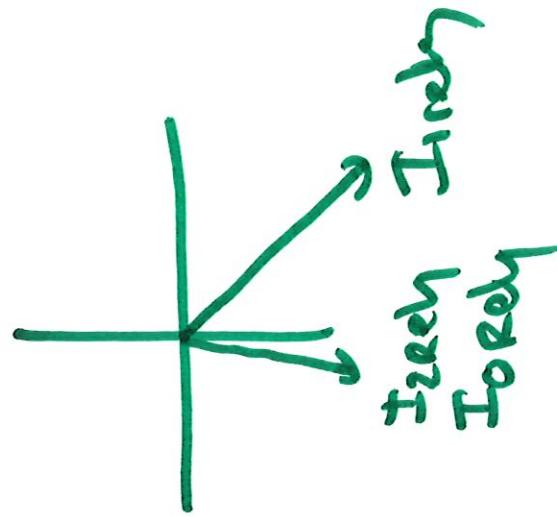
$$I_{ABC_RA}(m) := A_{012} \cdot \begin{pmatrix} I_{\text{relayA}0}(m) \\ I_{\text{relayA}1}(m) \\ I_{\text{relayA}2}(m) \end{pmatrix}$$

$$V_{ABC_RA}(m) := A_{012} \cdot \begin{pmatrix} V_{\text{relayA}0}(m) \\ V_{\text{relayA}1}(m) \\ V_{\text{relayA}2}(m) \end{pmatrix}$$

$$|I_{ABC_RA}(0.5)| = \begin{pmatrix} 3.54 \\ 0.99 \\ 0.99 \end{pmatrix} \cdot \text{pu}$$

$$\arg(I_{ABC_RA}(0.5)) = \begin{pmatrix} -77.75 \\ -113.63 \\ 126.37 \end{pmatrix} \cdot \text{deg}$$

labeled current of source



$$|V_{ABC_RA}(0.5)| = \begin{pmatrix} 0.23 \\ 1.18 \\ 1.2 \end{pmatrix} \cdot pu$$

Without load flow:

$$\begin{aligned} I_A &:= 3.83pu \cdot e^{-j \cdot 85.88\deg} & V_A &:= 0.24pu \cdot e^{j \cdot 0.88\deg} \\ I_B &:= 0 & V_B &:= 1.18pu \cdot e^{-j \cdot 132.89\deg} \\ I_C &:= 0 & V_C &:= 1.18pu \cdot e^{j \cdot 132.99\deg} \end{aligned}$$

Now repeat using Z_{bus} :

$$I_0_SLG(m) := \frac{V_{1_Thev}(m)}{Z_{bus1}(m)_{3,3} + Z_{bus2}(m)_{3,3} + Z_{bus0}(m)_{3,3}}$$

$$I_1_SLG(m) := I_0_SLG(m) \quad |I_0_SLG(0.5)| = 3.22 \quad \arg(I_0_SLG(0.5)) = -95.09\deg$$

$$I_{ABC_SLG}(m) := A_{012} \cdot \begin{pmatrix} I_0_SLG(m) \\ I_1_SLG(m) \\ I_2_SLG(m) \end{pmatrix} = \begin{pmatrix} 9.66 \\ 0 \\ 0 \end{pmatrix} \cdot pu \quad \xrightarrow{\arg(I_{ABC_SLG}(0.5)) = \begin{pmatrix} -95.09 \\ 19.98 \\ 19.98 \end{pmatrix}} \begin{pmatrix} 19.98 \\ 19.98 \\ 19.98 \end{pmatrix} \cdot deg$$

- Now find the voltages:

$$\Delta V1(m) := Z_{bus1}(m) \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ -I_{1_SLG}(m) \end{pmatrix}$$

$$V1(m) := \begin{pmatrix} V_{S1} \\ V_{B1} \\ V_{R1} \\ V_{1_Thev}(m) \end{pmatrix} + \Delta V1(m)$$

$$\Delta V2(m) := Z_{bus2}(m) \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ -I_{2_SLG}(m) \end{pmatrix}$$

$$V2(m) := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \Delta V2(m)$$

$$\Delta V0(m) := Z_{bus0}(m) \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ -I_{0_SLG}(m) \end{pmatrix}$$

$$V0(m) := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \Delta V0(m)$$

- Relay 1 voltage:

$$V_{ABC_B1}(m) := A_{012} \cdot \begin{pmatrix} V0(m)_1 \\ V1(m)_1 \\ V2(m)_1 \end{pmatrix}$$

$$\overrightarrow{|V_{ABC_B1}(0.5)|} = \begin{pmatrix} 0.23 \\ 1.18 \\ 1.2 \end{pmatrix} \text{pu}$$

$$\overrightarrow{\arg(V_{ABC_B1}(0.5))} = \begin{pmatrix} 0.82 \\ -138.99 \\ 127 \end{pmatrix} \text{deg}$$

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- Relay 1 current:

$$I_{B1_F_1}(m) := \frac{V1(m)_1 - V1(m)_3}{m \cdot Z_{L21}}$$

$$\left| I_{B1_F_1}(0.5) \right| = 1.41 \quad \arg(I_{B1_F_1}(0.5)) = -49.96 \cdot \text{deg}$$

$$I_{B1_F_2}(m) := \frac{V2(m)_1 - V2(m)_3}{m \cdot Z_{L22}}$$

$$\left| I_{B1_F_2}(0.5) \right| = 1.19 \quad \arg(I_{B1_F_2}(0.5)) = -93.76 \cdot \text{deg}$$

$$I_{B1_F_0}(m) := \frac{V0(m)_1 - V0(m)_3}{m \cdot Z_{L20}}$$

$$\left| I_{B1_F_0}(0.5) \right| = 1.19 \quad \arg(I_{B1_F_0}(0.5)) = -93.76 \cdot \text{deg}$$

Same as above....

don't need to
do $I_{1F} + I_{1PF}$

$$I_{ABC_R1}(m) := A_{012} \cdot \begin{pmatrix} I_{B1_F_0}(m) \\ I_{B1_F_1}(m) \\ I_{B1_F_2}(m) \end{pmatrix} \xrightarrow{\left| I_{ABC_R1}(0.5) \right| = \begin{pmatrix} 3.54 \\ 0.99 \\ 0.99 \end{pmatrix} \cdot \text{pu}} \xrightarrow{\arg(I_{ABC_R1}(0.5)) = \begin{pmatrix} -77.75 \\ -113.63 \\ 126.37 \end{pmatrix} \cdot \text{deg}}$$

Same as above....

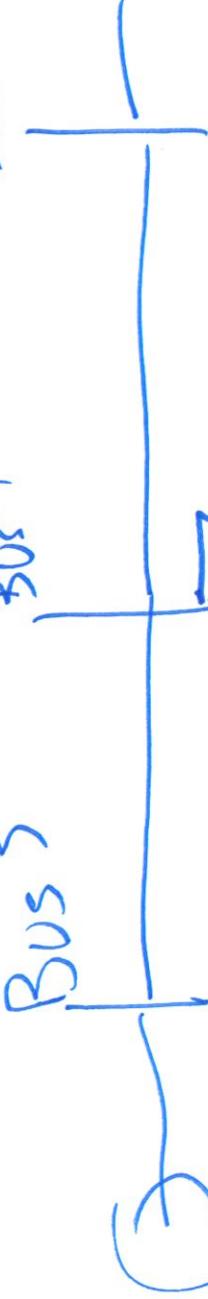
Next Time

We will add 1000 at a

specific bus

Bus 1

Bus n



$E'' = V_{BUS} -$  In Power Flow Solution -
 $P, Q (P_f, P_f)$

\rightarrow constant P
 \rightarrow constant I
 $=$ constant E
 $-$ motor

$$\begin{aligned} S &= V \cdot I^* \\ P_d &= V \cdot \frac{V^*}{Z^*} = \frac{V^2}{Z^*} \end{aligned}$$

How does
 P, Q vary
 with V ?
 at bus