Series Unbalance Derivation

First step:

$$V_{aa'0} - V_{aa'1} = \frac{1}{3}\cdot\left[I_0\cdot\left[\left(Z_A + 2\cdot Z_B\right) - \left(Z_A - Z_B\right)\right] + I_1\cdot\left[\left(Z_A - Z_B\right) - \left(Z_A + 2\cdot Z_B\right)\right] + I_2\cdot\left[\left(Z_A - Z_B\right) - \left(Z_A - Z_B\right)\right]\right]$$

- Simplifying

$$V_{aa'0} - V_{aa'1} = \frac{1}{3}\cdot\left[Z_B\cdot\left(3\cdot I_0 - 3\cdot I_1\right)\right] = Z_B\cdot\left(I_0 - I_1\right)$$

- Rearrange terms so all zero sequence terms on one side and positive sequence on the other:

$$V_{aa'0} - I_0\cdot Z_B = V_{aa'1} - I_1\cdot Z_B$$

- Similarly, using V0 - V2 we get:

$$V_{aa'0} - I_0\cdot Z_B = V_{aa'2} - I_2\cdot Z_B = V_{aa'1} - I_1\cdot Z_B$$

Second Step:

$$V_{aa'0} + V_{aa'1} = \frac{1}{3}\cdot\left[I_0\cdot\left[\left(Z_A + 2\cdot Z_B\right) + \left(Z_A - Z_B\right)\right] + I_1\cdot\left[\left(Z_A - Z_B\right) + \left(Z_A + 2\cdot Z_B\right)\right] + I_2\cdot\left[\left(Z_A - Z_B\right) + \left(Z_A - Z_B\right)\right]\right]$$

- Simplifying

$$V_{aa'0} + V_{aa'1} = \frac{1}{3}\cdot\left[\left(I_0 + I_1\right)\cdot\left(2\cdot Z_A + Z_B\right) + 2\cdot I_2\cdot\left(Z_A - Z_B\right)\right]$$

- Now substitute for $V_{aa'0}$ with

$$V_{aa'0} - I_0 \cdot Z_B = V_{aa'1} - I_1 \cdot Z_B$$

becomes: $$V_{aa'0} = V_{aa'1} - I_1 \cdot Z_B + I_0 \cdot Z_B$$

- Resulting in:

$$V_{aa'1} - I_1 \cdot Z_B + I_0 \cdot Z_B + V_{aa'1} = \frac{1}{3} \cdot \left[\left(I_0 + I_1 \right) \cdot \left(2 \cdot Z_A + Z_B \right) + 2 \cdot I_2 \cdot \left(Z_A - Z_B \right) \right]$$

- Rearrange terms over several steps::

$$2 \cdot V_{aa'1} - I_1 \cdot Z_B = \frac{1}{3} \cdot \left[\left(I_0 + I_1 \right) \cdot \left(2 \cdot Z_A + Z_B \right) + 2 \cdot I_2 \cdot \left(Z_A - Z_B \right) - 3 \cdot I_0 \cdot Z_B \right]$$

$$2 \cdot V_{aa'1} - I_1 \cdot Z_B = \frac{1}{3} \cdot \left[I_0 \cdot \left(2 \cdot Z_A + Z_B - 3 \cdot Z_B \right) + I_1 \cdot \left(2 \cdot Z_A + Z_B \right) + 2 \cdot I_2 \cdot \left(Z_A - Z_B \right) \right]$$

$$2 \cdot V_{aa'1} - I_1 \cdot Z_B = \frac{1}{3} \cdot \left[2 I_0 \cdot \left(Z_A - Z_B \right) + I_1 \cdot \left(2 \cdot Z_A + Z_B \right) + 2 \cdot I_2 \cdot \left(Z_A - Z_B \right) \right]$$

$$2 \cdot V_{aa'1} - 2 I_1 \cdot Z_B = \frac{1}{3} \cdot \left[2 I_0 \cdot \left(Z_A - Z_B \right) + I_1 \cdot \left(2 \cdot Z_A + Z_B \right) + 2 \cdot I_2 \cdot \left(Z_A - Z_B \right) \right] - I_1 \cdot Z_B$$

$$2 \cdot V_{aa'1} - 2 I_1 \cdot Z_B = \frac{1}{3} \cdot \left[2 I_0 \cdot \left(Z_A - Z_B \right) + I_1 \cdot \left(2 \cdot Z_A + Z_B - 3 Z_B \right) + 2 \cdot I_2 \cdot \left(Z_A - Z_B \right) \right]$$

$$2 \cdot V_{aa'1} - 2I_1 \cdot Z_B = \frac{1}{3} \cdot \left[2I_0 \cdot (Z_A - Z_B) + 2I_1 \cdot (Z_A - Z_B) + 2 \cdot I_2 \cdot (Z_A - Z_B) \right]$$

- Divide everything by 2:

$$V_{aa'1} - I_1 \cdot Z_B = \frac{1}{3} \cdot \left[I_0 \cdot (Z_A - Z_B) + I_1 \cdot (Z_A - Z_B) + I_2 \cdot (Z_A - Z_B) \right]$$

Finally we get:

$$\left(V_{aa'1} - I_1 \cdot Z_B\right) = \frac{1}{3} \cdot \left[(Z_A - Z_B) \cdot (I_0 + I_1 + I_2) \right]$$