University of Idaho

ECE 523
Symmetrical Components

Session 17

Exam 1

- Take home exam
- 72 hours
- Available Oct 27 ~ morning
  - complete by Nov 6 night
- Topics - up through series faults

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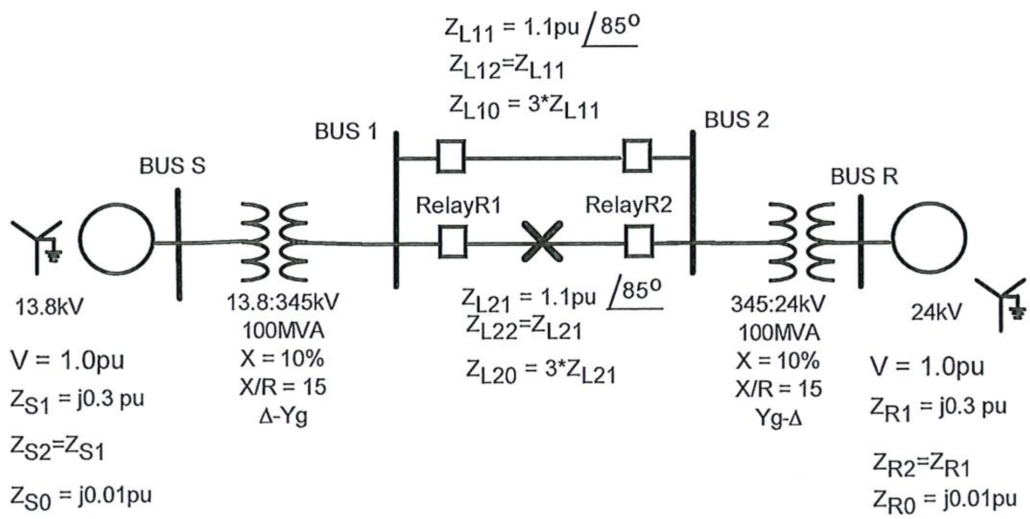
ECE 523: Homework #4

Due ~~Session 19 (October 31)~~ Session 20, Oct 26

1. Do the following for the circuit below using Z_{bus} matrix methods assuming faults 33% of the way down line 2 (the lower of the two lines). No change of base calculations needed.
   a. Set the voltage source at Bus S is 1.0 at -30 degrees (this is to account for the transformer phase shift), and the voltage at Bus R to be 1.0 is -50 degrees. Calculate the prefault voltage magnitude and angle at each bus, including the fault point based on the prefault power flow. Check your results with a Powerworld or a similar program.
   b. Calculate the voltages and currents in the sequence domain and in the abc domain at RelayR1 and RelayR2, for 3 phase, SLG, LL, and DLG faults with Rf = 0.3 pu (for the DLG put the resistance in the ground path). Again, check your results with Powerworld or a similar program.



2 series fault case

Series faults (abnormal conditions)

- in a line, transformer, etc
- large, temporary condition

Common cases

1. ~~ope~~ one phase abnormal
   other two normal

   ⇒ Single pole open (single phase open)

   A. Intentional – trip faulted phase for SLG at both ends

   B. Unintentional – Breaker failure – one end

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2. phased open → breaker failure
   - one pole fails to open

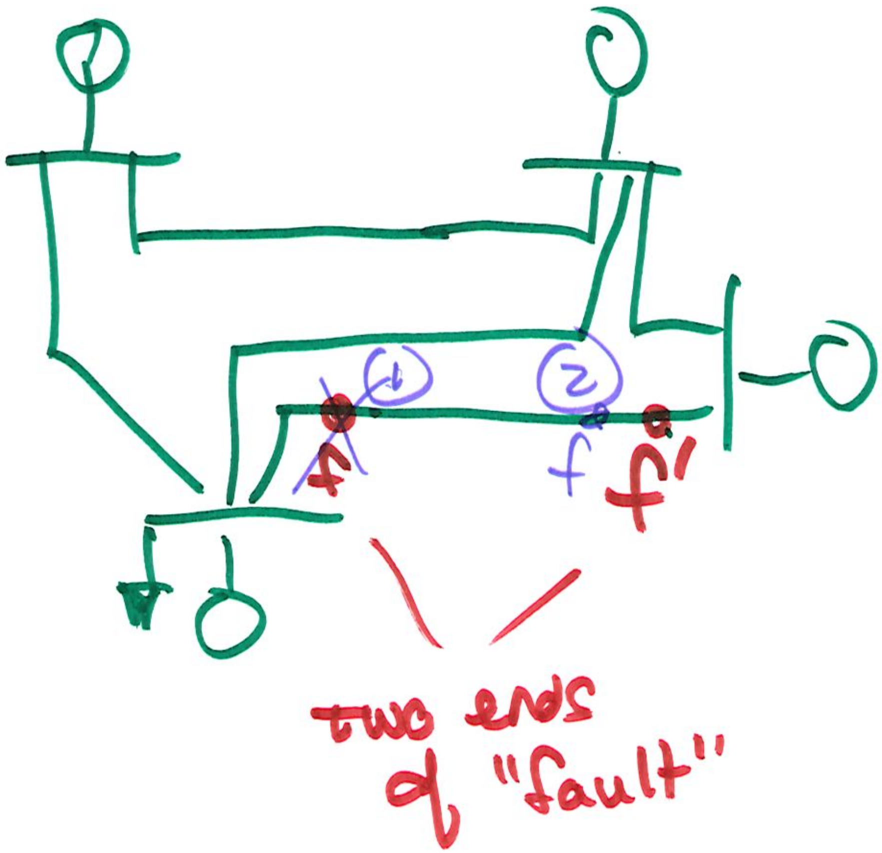
3. Unbalanced impedance
   A. → series capacitor
      → MOV

B. mismatched transformers

→ large power transformers

→ single phase

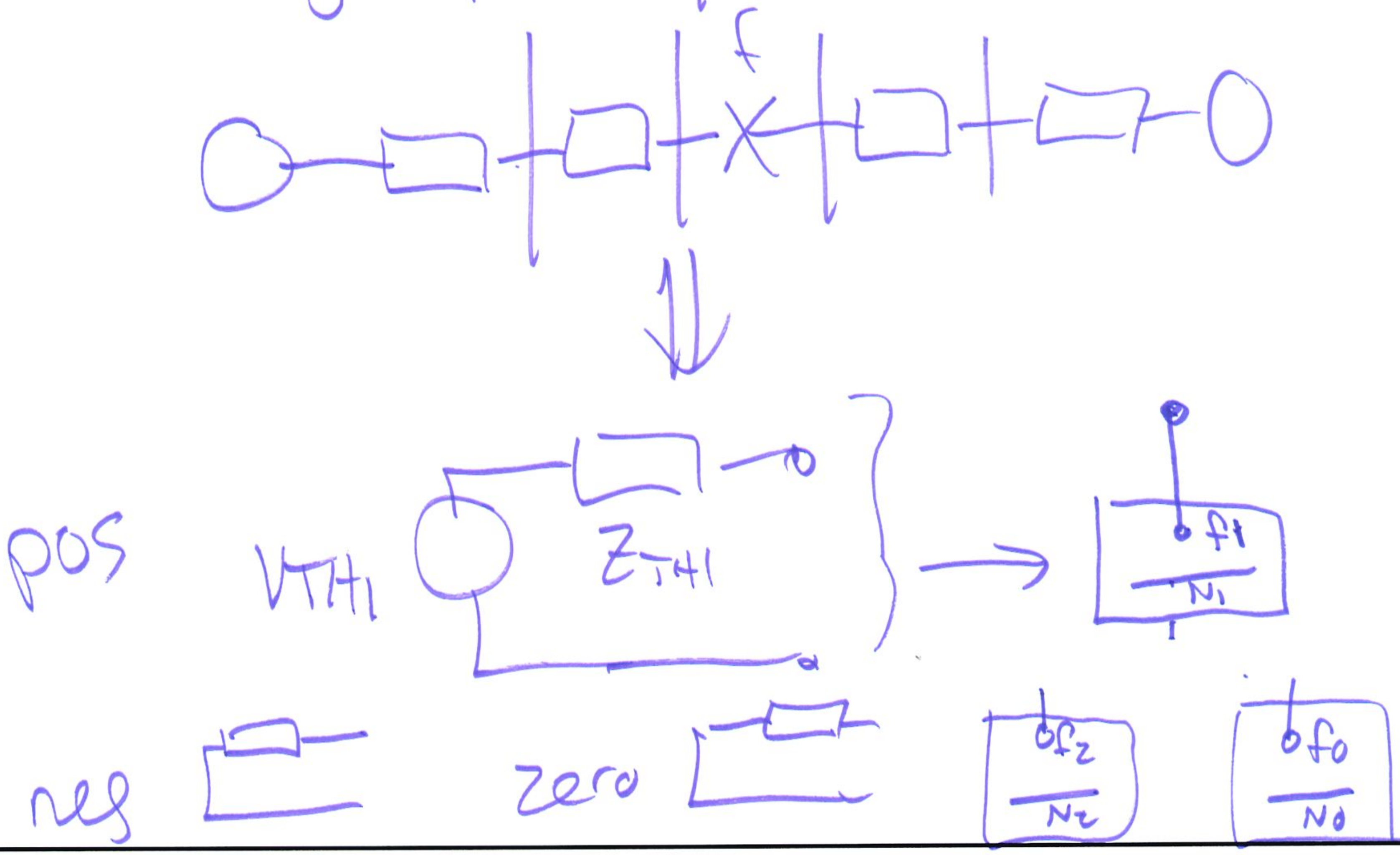
Analysis of series faults



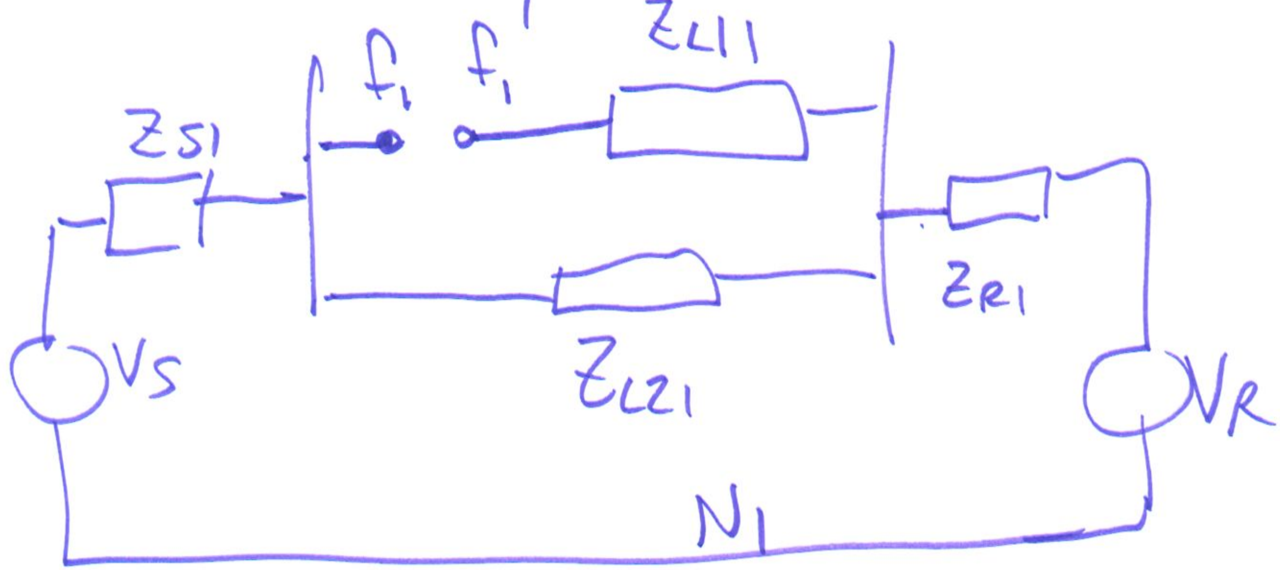
1. phasors
   - symmetrical components
   - Two port Thevenin equivalent from f-f'
2. Electromagnetic transients simulation

Thevenin Equivalent

1. Single port equivalent circuits



2. Two port Thevenin Equivalent



$\bar{V}_S \neq \bar{V}_R$

Need to have load flow on system

← more or less difference between V_s & V_R (w/o parallel line it is the diff)

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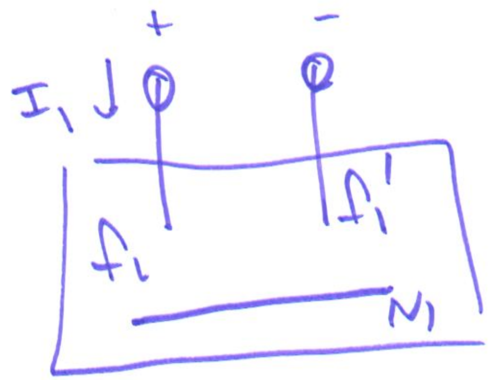
Symbol



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Series Fault Examples

$pu := 1 \qquad MVA := 1000kW$

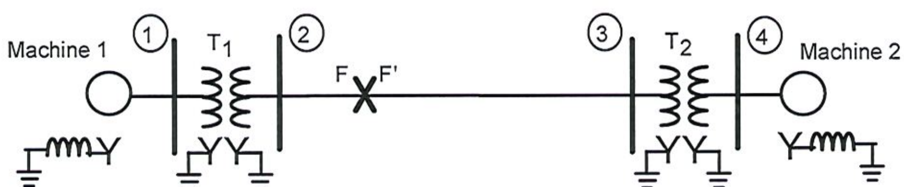
$a := 1e^{j \cdot 120deg}$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

Single Phase Open Examples

Example 1:

- System one-line diagram:



Machines 1 and 2: $S_{Mach} := 100MVA$ $\qquad V_{machine} := 20kV$

$X_{dMach''} := 20\%$ $\qquad X_{1Mach} := X_{dMach''}$ $\qquad X_{2Mach} := X_{1Mach}$

$X_{0Mach} := 4\%$ $\qquad X_{nMach} := 5\%$

Transformers T1 and T2: $S_{Tran} := 100\cancel{0}MVA$ $\qquad V_{HV} := 345kV$ $\qquad V_{LV} := 20kV$ $\qquad X_T := 8\%$ $= X_1 = X_2 = X_0$

$R = 0$

Transmission Line $\qquad X_{L1} := 15\%$ $\qquad X_{L2} := X_{L1}$ $\qquad X_{L0} := 50\%$

$S_{Base} := 100MVA$

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$V_{BLine} := 345kV \qquad V_{B\_mach} := V_{BLine} \cdot \left(\frac{V_{LV}}{V_{HV}}\right) \qquad V_{B\_mach} = 20 \cdot kV$

No change of base calculations are needed for this system.

Determine internal source voltages: voltage behind reactance for the sources

$magS_{pre} := 80MVA \qquad pf_{pre} := 0.85$ lagging $\qquad \theta_{pre} := acos(pf_{pre}) \qquad \theta_{pre} = 31.79 \cdot deg$

at Bus 3

$S_{pre} := \frac{magS_{pre}}{S_{Base}} \cdot e^{j \cdot \theta_{pre}} \qquad S_{pre} = (0.68 + 0.42i) \cdot pu \qquad |S_{pre}| = 0.8 \cdot pu$

Assume bus 3 voltage is 1.0 pu at and angle of 0 degrees.

$V_3 := 1.0$

$I_{load} := \overline{\left(\frac{S_{pre}}{V_3}\right)} \qquad I_{load} = 0.68 - 0.42i \qquad |I_{load}| = 0.8 \cdot pu \qquad arg(I_{load}) = -31.79 \cdot deg$

Internal voltage on the motor (since we don't know steady-state synchronous reactance, use X1):

$E_2 := V_3 - I_{load} \cdot j(X_T + X_{1Mach}) \qquad |E_2| = 0.9 \qquad \phi_2 := arg(E_2) \qquad \phi_2 = -12.18 \cdot deg$

Generator internal voltage:

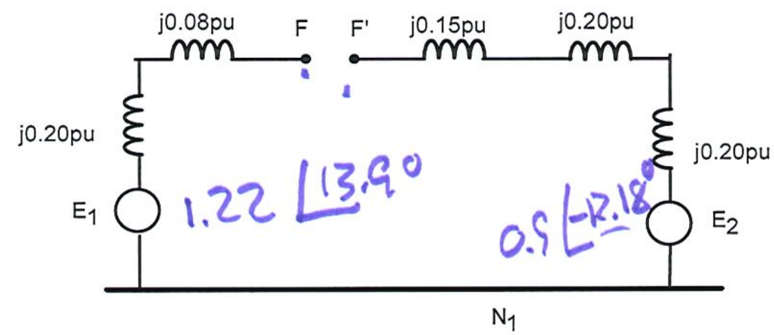
$E_1 := V_3 + I_{load} \cdot (j \cdot X_{L1} + j \cdot X_T + j \cdot X_{1Mach}) \qquad |E_1| = 1.22 \cdot pu \qquad \phi_1 := arg(E_1) \qquad \phi_1 = 13.9 \cdot deg$

Check result by calculating power transfer between sources and current:

$$P_{trans} := \frac{|E_1| \cdot |E_2| \cdot \sin(\phi_1 - \phi_2)}{2 \cdot X_{1Mach} + 2 \cdot X_T + X_{L1}} \qquad P_{trans} - \mathrm{Re}(S_{pre}) = 0$$

$$I_{trans} := \frac{E_1 - E_2}{j(2 \cdot X_{1Mach} + 2 \cdot X_T + X_{L1})} \qquad I_{trans} - I_{load} = 0$$

- Positive sequence equivalent circuit (with phase open point indicated).



Find total impedance counterclockwise around loop from F to F'

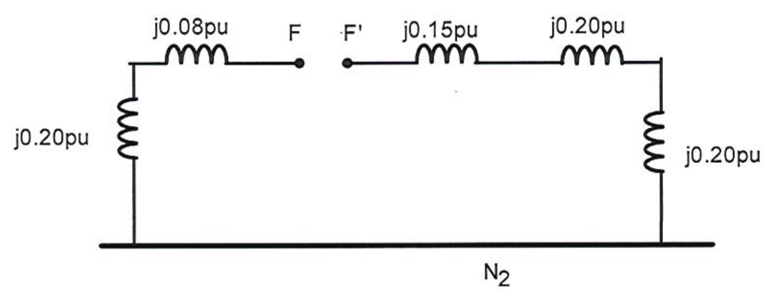
$$Z_{1total} := j \cdot (X_{1Mach} + X_T + X_{L1} + X_T + X_{1Mach})$$

$$Z_{1total} = 0.71i \cdot pu$$

$$Z_{1FF'} := Z_{1total}$$

$$V_{equiv} := E_1 - E_2 = V_{SE}$$

- Negative sequence equivalent circuit:



Find total impedance counterclockwise around loop from F to F'

$$Z_{2total} := j \cdot (X_{2Mach} + X_T + X_{L2} + X_T + X_{2Mach})$$

$$Z_{2total} = 0.71i \cdot pu$$

$$Z_{2FF'} := Z_{2total}$$

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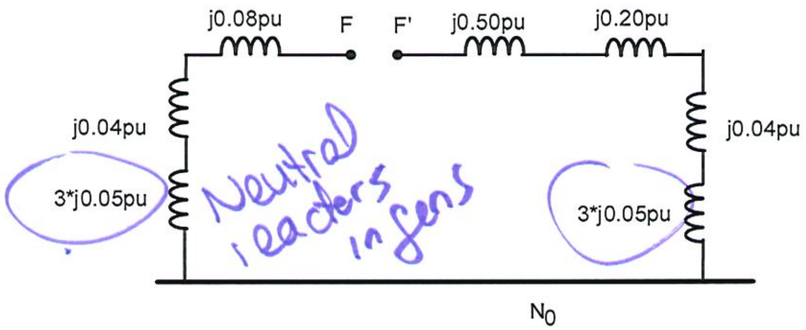
- Zero sequence equivalent:



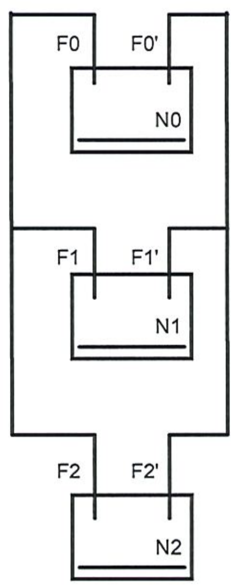
Find total impedance counterclockwise around loop from F to F'

$$Z_{0total} := j\cdot\left(2\cdot X_{0Mach} + 2\cdot X_T + X_{L0} + 2\cdot 3\cdot X_{nMach}\right)$$

$$Z_{0total} = 1.04i\cdot pu$$

$$Z_{0FF'} := Z_{0total}$$

Now solve for the single phase open circuit currents and voltages:



$$I_1 := \frac{V_{equiv}}{Z_{1FF'} + \left(\frac{1}{Z_{2FF'}} + \frac{1}{Z_{0FF'}}\right)^{-1}}$$

$$I_1 = (0.43 - 0.26i)\cdot pu$$

$$|I_1| = 0.5\cdot pu \qquad \arg(I_1) = -31.79\cdot deg$$

$$I_2 := -I_1\cdot\left(\frac{Z_{0FF'}}{Z_{2FF'} + Z_{0FF'}}\right)$$

$$I_2 = (-0.25 + 0.16i)\cdot pu$$

$$|I_2| = 0.3\cdot pu \qquad \arg(I_2) = 148.21\cdot deg$$

$$I_0 := -I_1\cdot\left(\frac{Z_{2FF'}}{Z_{2FF'} + Z_{0FF'}}\right)$$

$$I_0 = (-0.17 + 0.11i)\cdot pu$$

$$|I_0| = 0.2\cdot pu \qquad \arg(I_0) = 148.21\cdot deg$$

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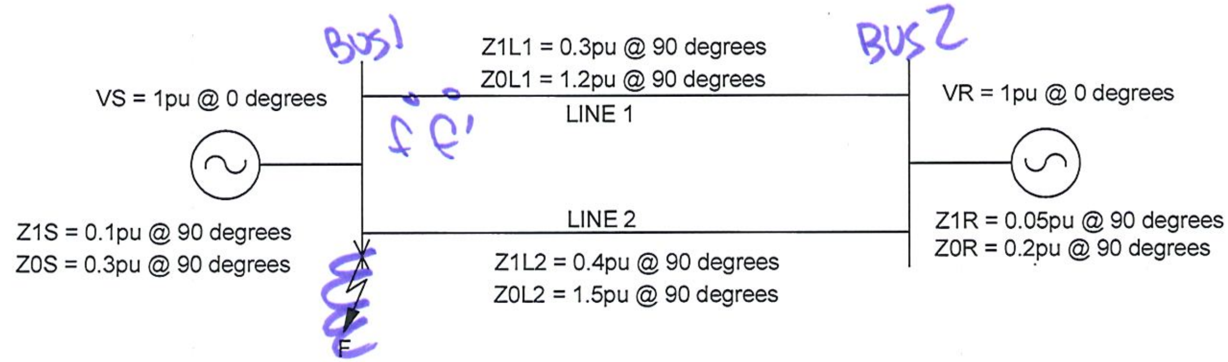
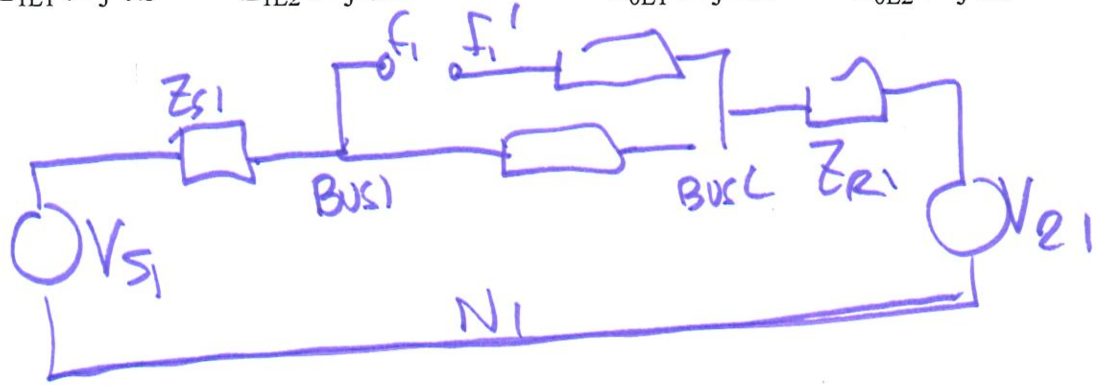
Example 2 For the system shown below, develop the sequence connection diagram for a single-phase open on Line 1.

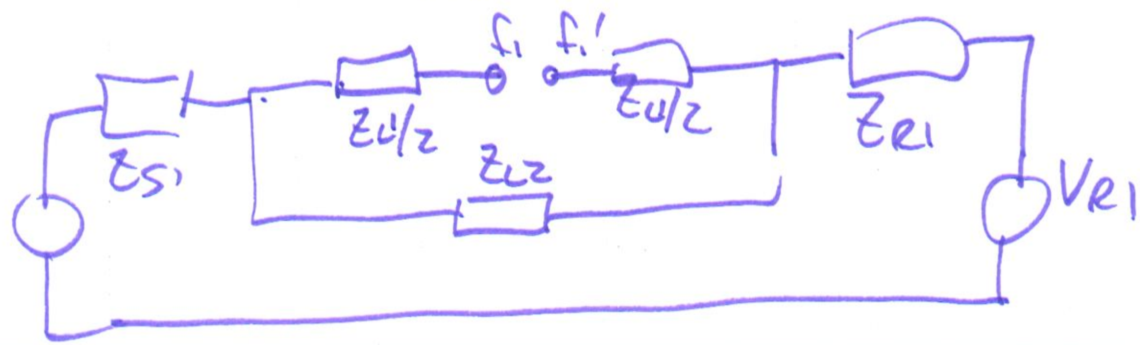


$Z_{1S} := j \cdot 0.1pu$ $\quad Z_{1R} := j \cdot 0.05pu$ $\quad Z_{0S} := j \cdot 0.3pu$ $\quad Z_{0R} := j \cdot 0.2pu$

$Z_{1L1} := j \cdot 0.3$ $\quad Z_{1L2} := j \cdot 0.4$ $\quad Z_{0L1} := j \cdot 1.2$ $\quad Z_{0L2} := j \cdot 1.5$

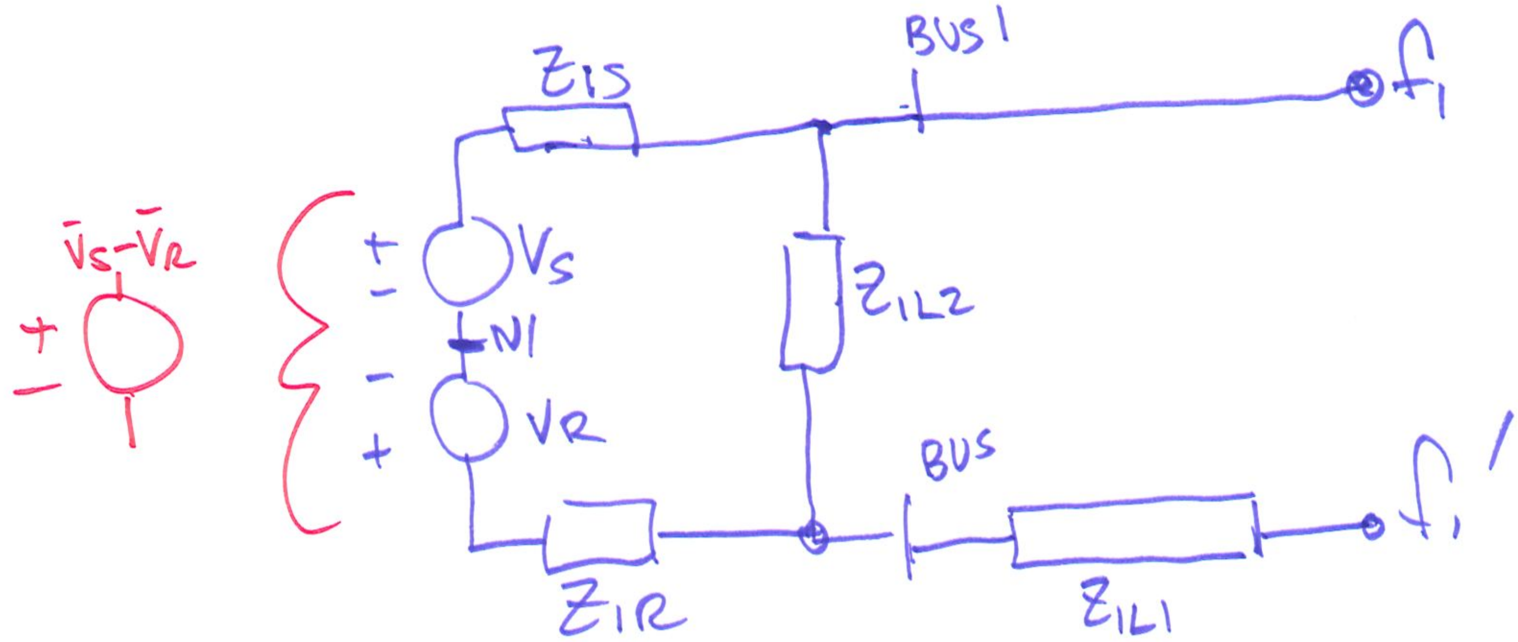


alternate

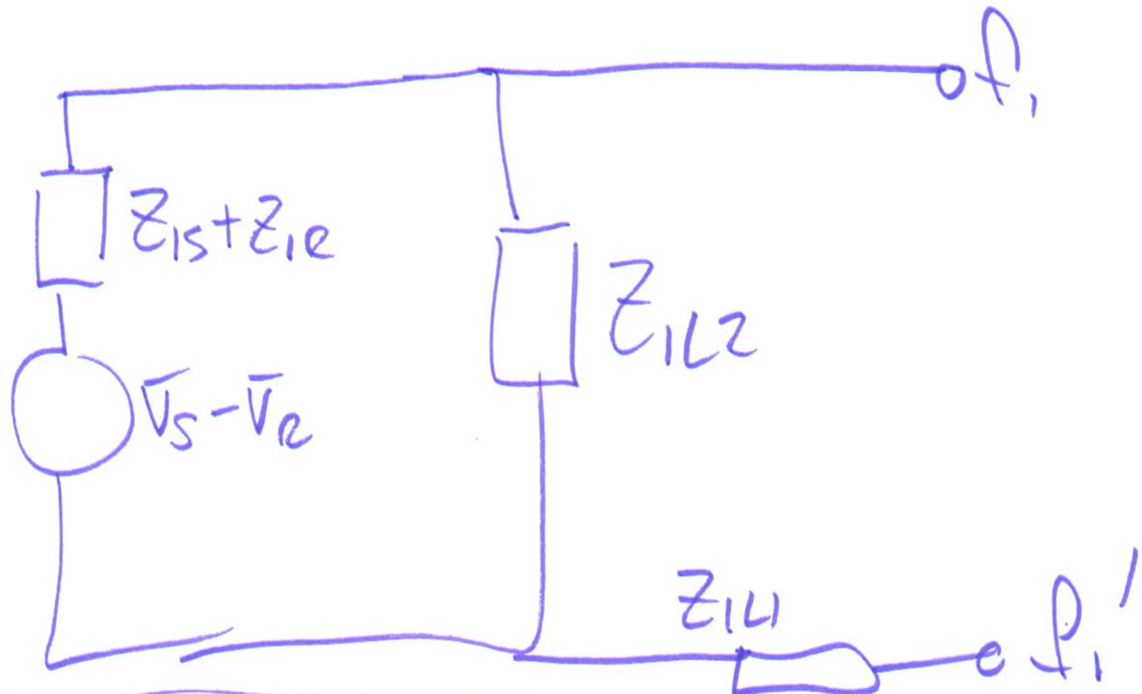


finding V_{TH} & Z_{TH} with parallel paths

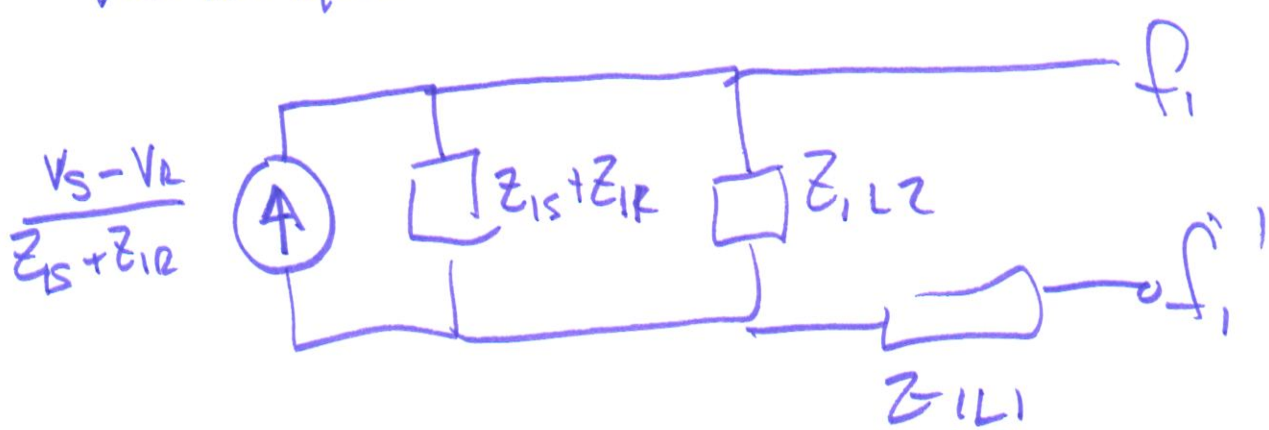
Redraw diagram:

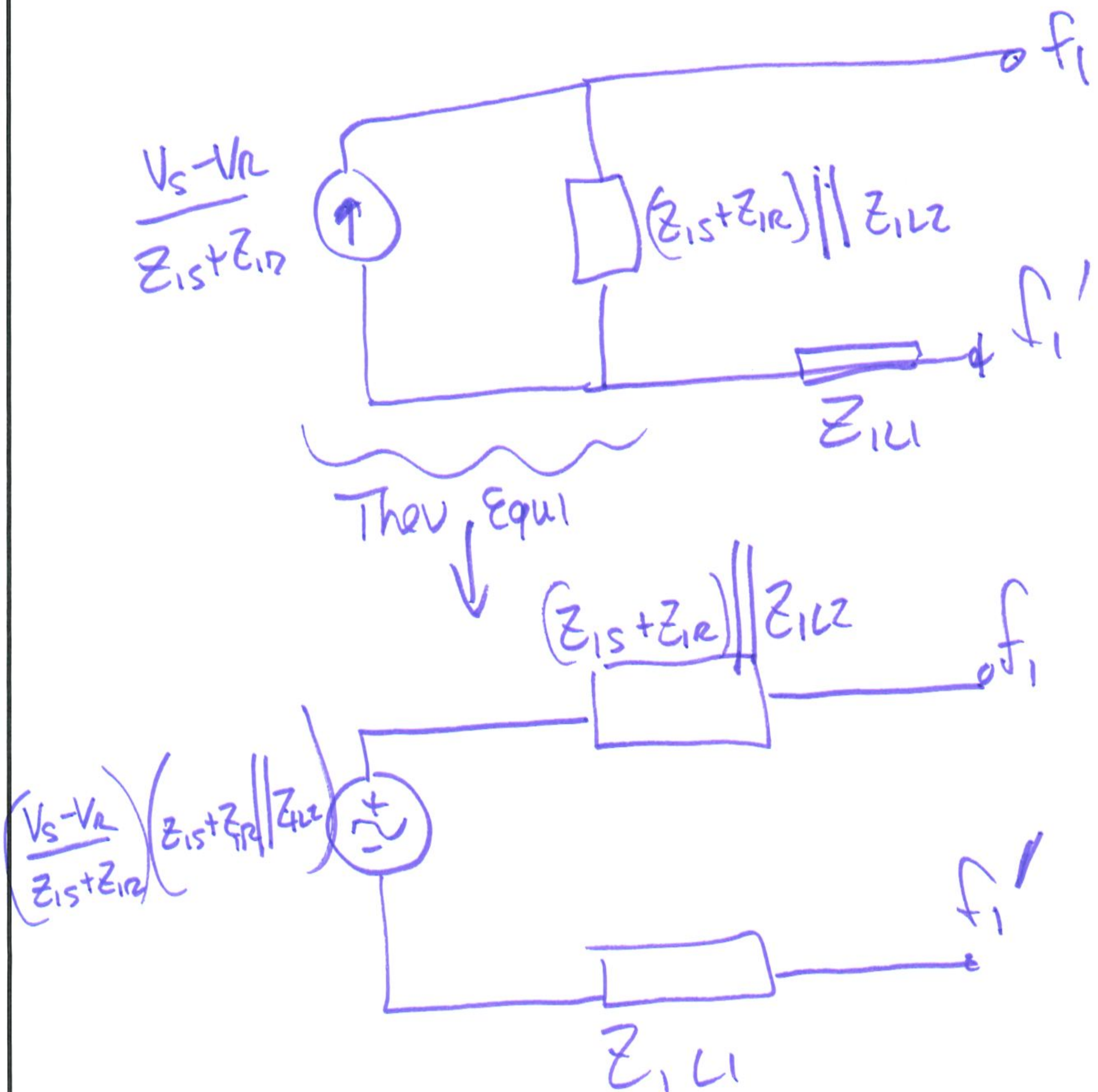


Redraw



Norton equiv





$\frac{V_S - V_R}{Z_{1S} + Z_{1R}}$
$(Z_{1S} + Z_{1R}) \parallel Z_{1L2}$
f_1
f_1'
Z_{1L1}
Thev Equi
$(Z_{1S} + Z_{1R}) \parallel Z_{1L2}$
$\left(\frac{V_S - V_R}{Z_{1S} + Z_{1R}}\right)\left((Z_{1S} + Z_{1R}) \parallel Z_{1L2}\right)$
f_1
f_1'
Z_{1L1}

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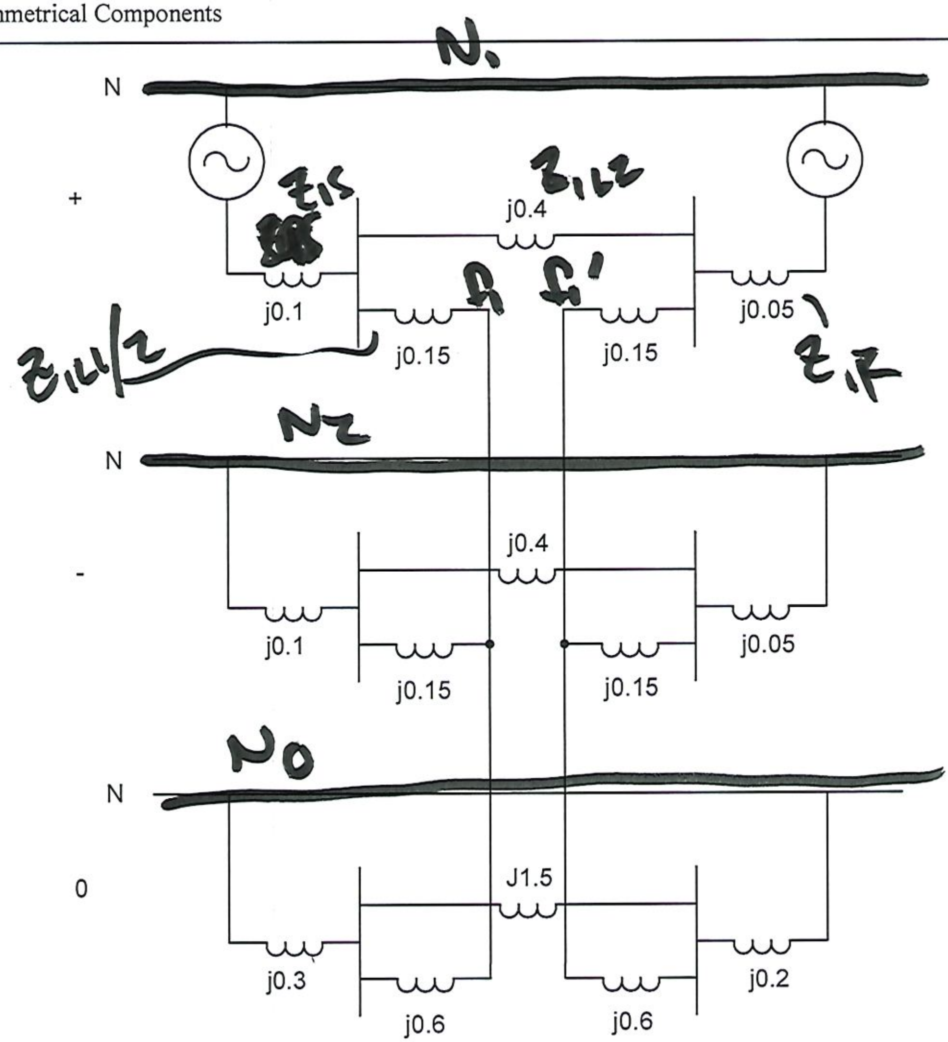

$$Z_{1equiv} := \frac{Z_{1L1}}{2} + \left(\frac{1}{Z_{1S} + Z_{1R}} + \frac{1}{Z_{1L2}}\right)^{-1} + \frac{Z_{1L1}}{2}$$

$Z_{1equiv} = 0.41i \cdot pu$ $\quad (Z_{1S} + Z_{1R}) \| Z_{1L2}$

$$Z_{2equiv} := Z_{1equiv}$$

$$Z_{0equiv} := Z_{0L1} + \left(\frac{1}{Z_{0S} + Z_{0R}} + \frac{1}{Z_{0L2}}\right)^{-1}$$

$Z_{0equiv} = 1.58i \cdot pu$

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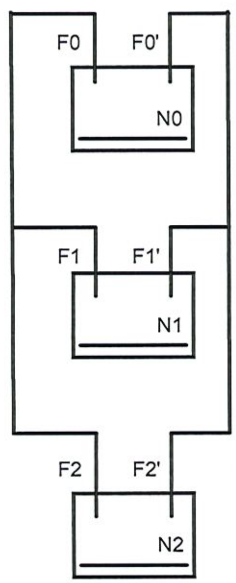
Phase A open analysis:



Equivalent voltage source for phase A open analysis:

$V_{se} := V_S - V_R \qquad V_{se} = (0.06 - 0.34i)\cdot pu$

Norton Equivalent Current: $I_{se} := \dfrac{V_{se}}{Z_{1R} + Z_{1S}} \qquad I_{se} = (-2.28 - 0.4i)\cdot pu$

Equivalent Parallel Impedance: $Z_{eq} := \left(\dfrac{1}{Z_{1L2}} + \dfrac{1}{Z_{1S} + Z_{1R}}\right)^{-1} \qquad Z_{eq} = 0.11i\cdot pu$

Convert back to Thevenin Equivalent Voltage

$V_f := Z_{eq}\cdot I_{se} \qquad |V_f| = 0.25\cdot pu \qquad \arg(V_f) = -80\cdot deg$

Two Port Thevenin voltage

Positive sequence current in line 1:

$$I_{1L1\_open} := \frac{V_f}{Z_{1equiv} + \left(\dfrac{1}{Z_{2equiv}} + \dfrac{1}{Z_{0equiv}}\right)^{-1}} \qquad |I_{1L1\_open}| = 0.34\cdot pu \qquad \arg(I_{1L1\_open}) = -170\cdot deg$$

Negative sequence current in line 1 (current divider on the line 1 current)

$$I_{2L1\_open} := -I_{1L1\_open}\cdot\frac{Z_{0equiv}}{Z_{2equiv} + Z_{0equiv}} \qquad |I_{2L1\_open}| = 0.27\cdot pu \qquad \arg(I_{2L1\_open}) = 10\cdot deg$$

Series unbalance

Assuming one phase different than other two

$\bar{Z}_A$ $\quad\quad Z_A \neq \bar{Z}_B = \bar{Z}_C$

$\bar{Z}_B$

$\bar{Z}_C = \bar{Z}_B$

Phasor Domain

$$\begin{bmatrix} V_{AGf} - V_{AGf'} \\ V_{BGf} - V_{BGf'} \\ V_{CGf} - V_{CGf'} \end{bmatrix} = \begin{bmatrix} V_{AA'} \\ V_{BB'} \\ V_{CC'} \end{bmatrix} = \begin{bmatrix} Z_A & 0 & 0 \\ 0 & Z_B & 0 \\ 0 & 0 & Z_C \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

$\Downarrow \quad \Downarrow \quad \Downarrow$

symmetrical components ..

$$\begin{bmatrix} V_{AA'} \\ V_{BB'} \\ V_{CC'} \end{bmatrix} = \left[A_{012}\right] \begin{bmatrix} V_{0ff'} \\ V_{1ff'} \\ V_{2ff'} \end{bmatrix}$$

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \left[A_{012}\right] \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$V_{0ff'} = V_{0f} - V_{0f'}$

$$\underbrace{\left[A_{012}\right]^{-1}\left[A_{012}\right]}_{[I]} \begin{bmatrix} V_{0ff'} \\ V_{1ff'} \\ V_{2ff'} \end{bmatrix} = \left[A_{012}\right]^{-1} \begin{bmatrix} Z_A & 0 & 0 \\ 0 & Z_B & 0 \\ 0 & 0 & Z_C \end{bmatrix} \left[A_{012}\right] \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

Because $\bar{Z}_A \neq \bar{Z}_B = \bar{Z}_C$

Impedance matrix

$$\begin{bmatrix} V_{AA'0} \\ V_{AA'1} \\ V_{AA'2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} Z_A + 2Z_B & Z_A - Z_B & Z_A - Z_B \\ Z_A - Z_B & Z_A + 2Z_B & Z_A - Z_B \\ Z_A - Z_B & Z_A - Z_B & Z_A + 2Z_B \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

Now figure out how sequence networks connected ...

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- Now substitute for $V_{aa'0}$ with

$$V_{aa'0} - I_0 \cdot Z_B = V_{aa'1} - I_1 \cdot Z_B$$

becomes: $$V_{aa'0} = V_{aa'1} - I_1 \cdot Z_B + I_0 \cdot Z_B$$

from (1)

- Resulting in:

$$V_{aa'1} - I_1 \cdot Z_B + I_0 \cdot Z_B + V_{aa'1} = \frac{1}{3} \cdot \left[(I_0 + I_1) \cdot (2 \cdot Z_A + Z_B) + 2 \cdot I_2 \cdot (Z_A - Z_B) \right]$$

- Rearrange terms over several steps::

$$2 \cdot V_{aa'1} - I_1 \cdot Z_B = \frac{1}{3} \cdot \left[(I_0 + I_1) \cdot (2 \cdot Z_A + Z_B) + 2 \cdot I_2 \cdot (Z_A - Z_B) - 3 \cdot I_0 \cdot Z_B \right]$$

$$2 \cdot V_{aa'1} - I_1 \cdot Z_B = \frac{1}{3} \cdot \left[I_0 \cdot (2 \cdot Z_A + Z_B - 3 \cdot Z_B) + I_1 \cdot (2 \cdot Z_A + Z_B) + 2 \cdot I_2 \cdot (Z_A - Z_B) \right]$$

$$2 \cdot V_{aa'1} - I_1 \cdot Z_B = \frac{1}{3} \cdot \left[2I_0 \cdot (Z_A - Z_B) + I_1 \cdot (2 \cdot Z_A + Z_B) + 2 \cdot I_2 \cdot (Z_A - Z_B) \right]$$

$$2 \cdot V_{aa'1} - 2I_1 \cdot Z_B = \frac{1}{3} \cdot \left[2I_0 \cdot (Z_A - Z_B) + I_1 \cdot (2 \cdot Z_A + Z_B) + 2 \cdot I_2 \cdot (Z_A - Z_B) \right] - I_1 \cdot Z_B$$

$$2 \cdot V_{aa'1} - 2I_1 \cdot Z_B = \frac{1}{3} \cdot \left[2I_0 \cdot (Z_A - Z_B) + I_1 \cdot (2 \cdot Z_A + Z_B - 3Z_B) + 2 \cdot I_2 \cdot (Z_A - Z_B) \right]$$

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Series Unbalance Derivation

$$\begin{bmatrix} V_{AA'0} \\ V_{AA'1} \\ V_{AA'2} \end{bmatrix} = \begin{bmatrix} Z_{012} \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

First step:

$$V_{aa'0} - V_{aa'1} = \frac{1}{3}\cdot\left[I_0\cdot\left[\left(Z_A + 2\cdot Z_B\right) - \left(Z_A - Z_B\right)\right] + I_1\cdot\left[\left(Z_A - Z_B\right) - \left(Z_A + 2\cdot Z_B\right)\right] + I_2\cdot\left[\left(Z_A - Z_B\right) - \left(Z_A - Z_B\right)\right]\right]$$

- Simplifying

$$V_{aa'0} - V_{aa'1} = \frac{1}{3}\cdot\left[Z_B\cdot\left(3\cdot I_0 - 3\cdot I_1\right)\right] = Z_B\cdot\left(I_0 - I_1\right)$$

- Rearrange terms so all zero sequence terms on one side and positive sequence on the other:

$$V_{aa'0} - I_0\cdot Z_B = V_{aa'1} - I_1\cdot Z_B$$

- Similarly, using V0 - V2 we get:

$$\boxed{V_{aa'0} - I_0\cdot Z_B = V_{aa'2} - I_2\cdot Z_B = V_{aa'1} - I_1\cdot Z_B} \quad (1)$$

Second Step:

$$V_{aa'0} + V_{aa'1} = \frac{1}{3}\cdot\left[I_0\cdot\left[\left(Z_A + 2\cdot Z_B\right) + \left(Z_A - Z_B\right)\right] + I_1\cdot\left[\left(Z_A - Z_B\right) + \left(Z_A + 2\cdot Z_B\right)\right] + I_2\cdot\left[\left(Z_A - Z_B\right) + \left(Z_A - Z_B\right)\right]\right]$$

- Simplifying

$$V_{aa'0} + V_{aa'1} = \frac{1}{3}\cdot\left[\left(I_0 + I_1\right)\cdot\left(2\cdot Z_A + Z_B\right) + 2\cdot I_2\cdot\left(Z_A - Z_B\right)\right]$$

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$$2 \cdot V_{aa'1} - 2I_1 \cdot Z_B = \frac{1}{3} \cdot \left[2I_0 \cdot (Z_A - Z_B) + 2I_1 \cdot (Z_A - Z_B) + 2 \cdot I_2 \cdot (Z_A - Z_B)\right]$$

- Divide everything by 2:

$$V_{aa'1} - I_1 \cdot Z_B = \frac{1}{3} \cdot \left[I_0 \cdot (Z_A - Z_B) + I_1 \cdot (Z_A - Z_B) + I_2 \cdot (Z_A - Z_B)\right]$$

Finally we get:

$$(V_{AA'0} - I_0 Z_B) = \boxed{(V_{aa'1} - I_1 \cdot Z_B) = \frac{1}{3} \cdot \left[(Z_A - Z_B) \cdot (I_0 + I_1 + I_2)\right]} \quad 2$$

$V_{AA'2} - I_2 Z_B =$

4 circuits in parallel...