

Series Fault Examples with Zbus

pu := 1 MVA := 1000kW

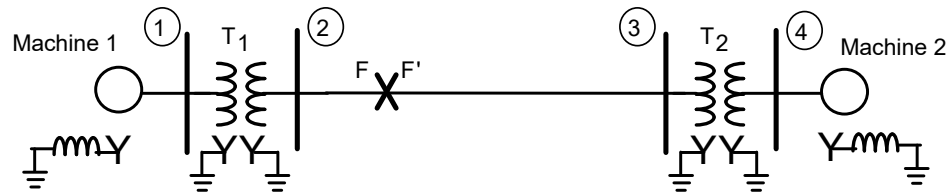
$$a := 1e^{j \cdot 120\text{deg}}$$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

Single Phase Open Examples

Example 1:

- System one-line diagram:



Machines 1 and 2: $S_{\text{Mach}} := 100\text{MVA}$ $V_{\text{machine}} := 20\text{kV}$
 $X_{\text{dMach}''} := 20\%$ $X_{1\text{Mach}} := X_{\text{dMach}''}$ $X_{2\text{Mach}} := X_{1\text{Mach}}$
 $X_{0\text{Mach}} := 4\%$ $X_{\text{nMach}} := 5\%$

Transformers T1 and T2: $S_{\text{Tran}} := 1000\text{MVA}$ $V_{\text{HV}} := 345\text{kV}$ $V_{\text{LV}} := 20\text{kV}$ $X_{\text{T}} := 8\%$

Transmission Line $X_{\text{L1}} := 15\%$ $X_{\text{L2}} := X_{\text{L1}}$ $X_{\text{L0}} := 50\%$

$S_{\text{Base}} := 100\text{MVA}$

$$V_{BLine} := 345\text{kV} \quad V_{B_mach} := V_{BLine} \cdot \left(\frac{V_{LV}}{V_{HV}} \right) \quad V_{B_mach} = 20\cdot\text{kV}$$

No change of base calculations are needed for this system.

Determine internal source voltages:

$$\text{mag}S_{pre} := 80\text{MVA} \quad \text{pf}_{pre} := 0.85 \text{ lagging} \quad \theta_{pre} := \text{acos}(\text{pf}_{pre}) \quad \theta_{pre} = 31.79\cdot\text{deg}$$

$$S_{pre} := \frac{\text{mag}S_{pre}}{S_{Base}} \cdot e^{j\cdot\theta_{pre}} \quad S_{pre} = (0.68 + 0.42i)\cdot\text{pu} \quad |S_{pre}| = 0.8\cdot\text{pu}$$

Assume bus 3 voltage is 1.0 pu at and angle of 0 degrees.

$$V_3 := 1.0$$

$$I_{load} := \left(\frac{S_{pre}}{V_3} \right) \quad I_{load} = 0.68 - 0.42i \quad |I_{load}| = 0.8\cdot\text{pu} \quad \arg(I_{load}) = -31.79\cdot\text{deg}$$

Internal voltage on the motor (since we don't know steady-state synchronous reactance, use X1):

$$E_2 := V_3 - I_{load} \cdot j(X_T + X_{1Mach}) \quad |E_2| = 0.9 \quad \phi_2 := \arg(E_2) \quad \phi_2 = -12.18\cdot\text{deg}$$

Generator internal voltage:

$$E_1 := V_3 + I_{load} \cdot (j \cdot X_{L1} + j \cdot X_T + j \cdot X_{1Mach}) \quad |E_1| = 1.22\cdot\text{pu} \quad \phi_1 := \arg(E_1) \quad \phi_1 = 13.9\cdot\text{deg}$$

Prefault voltages at each bus:

$$V_1 := V_3 + I_{\text{load}} \cdot (j \cdot X_{L1} + j \cdot X_T) \quad |V_1| = 1.11 \cdot \text{pu} \quad \arg(V_1) = 8.11 \cdot \text{deg}$$

$$V_2 := V_3 + I_{\text{load}} \cdot (j \cdot X_{L1}) \quad |V_2| = 1.07 \cdot \text{pu} \quad \arg(V_2) = 5.48 \cdot \text{deg}$$

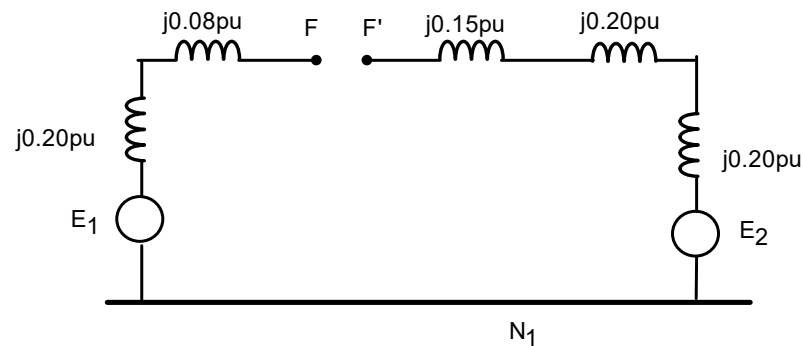
$$V_4 := V_3 - I_{\text{load}} \cdot (j \cdot X_T) \quad |V_4| = 0.97 \cdot \text{pu} \quad \arg(V_4) = -3.22 \cdot \text{deg}$$

Check result by calculating power transfer between sources and current:

$$P_{\text{trans}} := \frac{|E_1| \cdot |E_2| \cdot \sin(\phi_1 - \phi_2)}{2 \cdot X_{1\text{Mach}} + 2 \cdot X_T + X_{L1}} \quad P_{\text{trans}} - \text{Re}(S_{\text{pre}}) = 0$$

$$I_{\text{trans}} := \frac{E_1 - E_2}{j(2 \cdot X_{1\text{Mach}} + 2 \cdot X_T + X_{L1})} \quad I_{\text{trans}} - I_{\text{load}} = 0$$

- Positive sequence equivalent circuit (with phase open point indicated).



Find total impedance counterclockwise around loop from F to F'

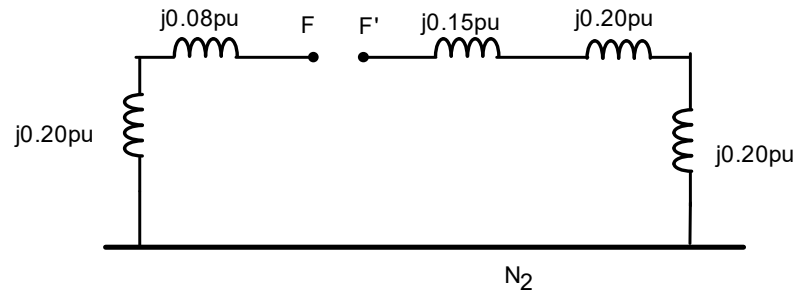
$$Z_{1\text{total}} := j \cdot (X_{1\text{Mach}} + X_T + X_{L1} + X_T + X_{1\text{Mach}})$$

$$Z_{1\text{total}} = 0.71i \cdot \text{pu}$$

$$Z_{1FF'} := Z_{1\text{total}}$$

$$V_{\text{equiv}} := E_1 - E_2$$

- Negative sequence equivalent circuit:



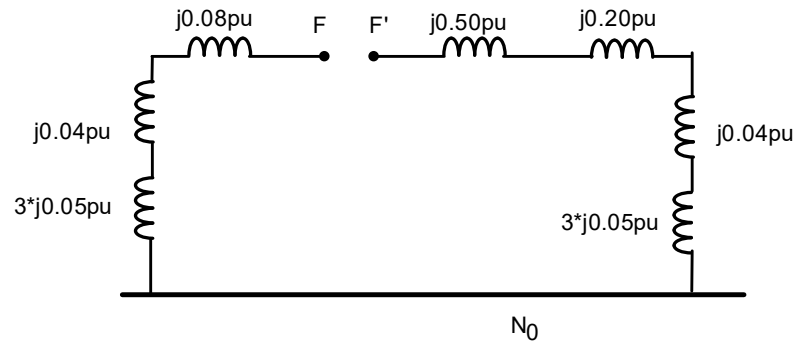
Find total impedance counterclockwise around loop from F to F'

$$Z_{2total} := j \cdot (X_{2Mach} + X_T + X_{L2} + X_T + X_{2Mach})$$

$$Z_{2total} = 0.71i \cdot pu$$

$$Z_{2FF'} := Z_{2total}$$

- Zero sequence equivalent:



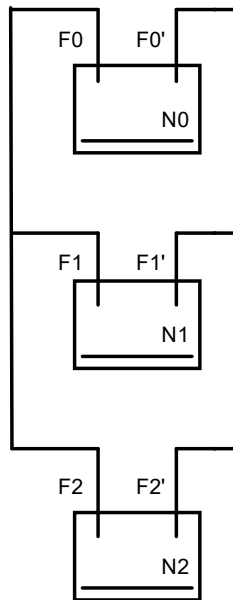
Find total impedance counterclockwise around loop from F to F'

$$Z_{0total} := j \cdot (2 \cdot X_{0Mach} + 2 \cdot X_T + X_{L0} + 2 \cdot 3 \cdot X_{nMach})$$

$$Z_{0total} = 1.04i \cdot pu$$

$$Z_{0FF'} := Z_{0total}$$

Now solve for the single phase open circuit currents and voltages:



$$I_1 := \frac{V_{\text{equiv}}}{Z_{1FF'} + \left(\frac{1}{Z_{2FF'}} + \frac{1}{Z_{0FF'}} \right)^{-1}}$$

$$I_1 = (0.43 - 0.26i) \cdot \text{pu}$$

$$|I_1| = 0.5 \cdot \text{pu} \quad \arg(I_1) = -31.79 \cdot \text{deg}$$

$$I_2 := -I_1 \cdot \left(\frac{Z_{0FF'}}{Z_{2FF'} + Z_{0FF'}} \right)$$

$$I_2 = (-0.25 + 0.16i) \cdot \text{pu}$$

$$|I_2| = 0.3 \cdot \text{pu} \quad \arg(I_2) = 148.21 \cdot \text{deg}$$

$$I_0 := -I_1 \cdot \left(\frac{Z_{2FF'}}{Z_{2FF'} + Z_{0FF'}} \right)$$

$$I_0 = (-0.17 + 0.11i) \cdot \text{pu}$$

$$|I_0| = 0.2 \cdot \text{pu} \quad \arg(I_0) = 148.21 \cdot \text{deg}$$

$$I_{\text{abc}} := A_{012} \cdot \begin{pmatrix} I_0 \\ I_1 \\ I_2 \end{pmatrix} \quad \overrightarrow{|I_{\text{abc}}|} = \begin{pmatrix} 0 \\ 0.76 \\ 0.76 \end{pmatrix} \cdot \text{pu}$$

$$\arg(I_{\text{abc}_1}) = -145.57 \cdot \text{deg}$$

$$\arg(I_{\text{abc}_2}) = 82 \cdot \text{deg}$$

Using the right hand side the sequence equivalent circuits:

$$V_{3\text{new}1} := E_2 + I_1 \cdot j \cdot (X_{1\text{Mach}} + X_T)$$

$$|V_{3\text{new}1}| = 0.96 \cdot \text{pu}$$

$$\arg(V_{3\text{new}1}) = -4.25 \cdot \text{deg}$$

$$V_{3\text{new}2} := 0 + I_2 \cdot j \cdot (X_{2\text{Mach}} + X_T)$$

$$|V_{3\text{new}2}| = 0.08 \cdot \text{pu}$$

$$\arg(V_{3\text{new}2}) = -121.79 \cdot \text{deg}$$

$$V_{3\text{new}0} := 0 + I_0 \cdot j \cdot (X_{0\text{Mach}} + X_T + 3 \cdot X_{n\text{Mach}}) \quad |V_{3\text{new}0}| = 0.05 \cdot \text{pu} \quad \arg(V_{3\text{new}0}) = -121.79 \cdot \text{deg}$$

$$V_{3\text{newABC}} := A_{012} \cdot \begin{pmatrix} V_{3\text{new}0} \\ V_{3\text{new}1} \\ V_{3\text{new}2} \end{pmatrix} \quad |V_{3\text{newABC}}| = \begin{pmatrix} 0.9 \\ 0.97 \\ 1.01 \end{pmatrix} \cdot \text{pu} \quad \arg(V_{3\text{newABC}}) = \begin{pmatrix} -12.06 \\ -119.95 \\ 118.58 \end{pmatrix} \cdot \text{deg}$$

$$\Delta V_{\text{ABC}} := 1.0 \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} - V_{3\text{newABC}} \quad |\Delta V_{\text{ABC}}| = \begin{pmatrix} 0.22 \\ 0.03 \\ 0.03 \end{pmatrix} \quad \arg(\Delta V_{\text{ABC}}) = \begin{pmatrix} 58.21 \\ -121.79 \\ -121.79 \end{pmatrix} \cdot \text{deg}$$

Solve using Zbus method. First get the Zbus matrices for the positive, negative and zero sequence networks:

$$Y_{\text{bus}1} := \begin{pmatrix} \frac{1}{j \cdot X_{1\text{Mach}}} + \frac{1}{j \cdot X_T} & \frac{-1}{j \cdot X_T} & 0 & 0 \\ \frac{-1}{j \cdot X_T} & \frac{1}{j X_T} + \frac{1}{j X_{L1}} & \frac{-1}{j \cdot X_{L1}} & 0 \\ 0 & \frac{-1}{j \cdot X_{L1}} & \frac{1}{j X_T} + \frac{1}{j X_{L1}} & \frac{-1}{j \cdot X_T} \\ 0 & 0 & \frac{-1}{j \cdot X_T} & \frac{1}{j \cdot X_{1\text{Mach}}} + \frac{1}{j \cdot X_T} \end{pmatrix} \quad Z_{\text{bus}1} := Y_{\text{bus}1}^{-1}$$

$$Z_{\text{bus}2} := Z_{\text{bus}1}$$

$$Y_{bus0} := \begin{pmatrix} \frac{1}{j \cdot X_{0Mach} + 3 \cdot j \cdot X_{nMach}} + \frac{1}{j \cdot X_T} & \frac{-1}{j \cdot X_T} & 0 & 0 \\ \frac{-1}{j \cdot X_T} & \frac{1}{j X_T} + \frac{1}{j X_{L0}} & \frac{-1}{j \cdot X_{L0}} & 0 \\ 0 & \frac{-1}{j \cdot X_{L0}} & \frac{1}{j X_T} + \frac{1}{j X_{L0}} & \frac{-1}{j \cdot X_T} \\ 0 & 0 & \frac{-1}{j \cdot X_T} & \frac{1}{j \cdot X_{0Mach} + 3 \cdot j \cdot X_{nMach}} + \frac{1}{j \cdot X_T} \end{pmatrix}$$

$$Z_{bus0} := Y_{bus0}^{-1}$$

Reset origin for matrices and vectors: ORIGIN := 1

Equivalent impedances looking into the network from the open segment. See section 12.6 in the book by Grainger and Stevenson or 6.6.4 in the Tlies, 1st edition or 7.6.4 in Tleis 2nd edition.

$$Z_{1pp'} := \frac{-(j \cdot X_{L1})^2}{Z_{bus1_{2,2}} + Z_{bus1_{3,3}} - 2 \cdot Z_{bus1_{2,3}} - j \cdot X_{L1}} \quad Z_{1pp'} = 0.71i \cdot pu \quad \text{From above: } Z_{1total} = 0.71i \cdot pu$$

- Notice that: the demonator has the Thevenin impedance at the bus on either side of the open condition, and subtracts the transfer impedance and the line impedance.

$$Z_{2pp'} := Z_{1pp'}$$

$$Z_{0pp'} := \frac{-(j \cdot X_{L0})^2}{Z_{bus0_{2,2}} + Z_{bus0_{3,3}} - 2 \cdot Z_{bus0_{2,3}} - j \cdot X_{L0}} \quad Z_{0pp'} = 1.04i \cdot pu \quad \text{Same as calculated above}$$

$$Z_{0total} = 1.04i \cdot pu$$

Find the sequence currents based on the prefault load current:

$$I_{1aopen} := I_{trans} \cdot \frac{Z_{1pp'}}{Z_{1pp'} + \left(\frac{1}{Z_{0pp'}} + \frac{1}{Z_{2pp'}} \right)^{-1}}$$

$$|I_{1aopen}| = 0.5 \cdot \text{pu} \quad \arg(I_{1aopen}) = -31.79 \cdot \text{deg}$$

same as above. $|I_1| = 0.5 \cdot \text{pu} \quad \arg(I_1) = -31.79 \cdot \text{deg}$

$$I_{2aopen} := -I_{1aopen} \cdot \left(\frac{Z_{0pp'}}{Z_{2pp'} + Z_{0pp'}} \right)$$

$$|I_{2aopen}| = 0.3 \cdot \text{pu} \quad \arg(I_{2aopen}) = 148.21 \cdot \text{deg}$$

same as above. $|I_2| = 0.3 \cdot \text{pu} \quad \arg(I_2) = 148.21 \cdot \text{deg}$

$$I_{0aopen} := -I_{1aopen} \cdot \left(\frac{Z_{2pp'}}{Z_{2pp'} + Z_{0pp'}} \right)$$

$$|I_{0aopen}| = 0.2 \cdot \text{pu} \quad \arg(I_{0aopen}) = 148.21 \cdot \text{deg}$$

same as above. $|I_0| = 0.2 \cdot \text{pu} \quad \arg(I_0) = 148.21 \cdot \text{deg}$

$$I_{ABC_Line_Aopen} := A_{012} \cdot \begin{pmatrix} I_{0aopen} \\ I_{1aopen} \\ I_{2aopen} \end{pmatrix}$$

$$\vec{|I_{ABC_Line_Aopen}|} = \begin{pmatrix} 0 \\ 0.7571 \\ 0.7571 \end{pmatrix} \cdot \text{pu}$$

$$\vec{\arg(I_{ABC_Line_Aopen})} = \begin{pmatrix} -26.57 \\ -145.57 \\ 82 \end{pmatrix} \cdot \text{deg}$$

Sequence voltages across the open circuit (based on the sequence connection for 1 phase open):

$$V_{1Aopen} := I_{1aopen} \cdot \frac{Z_{2pp'} \cdot Z_{0pp'}}{Z_{2pp'} + Z_{0pp'}}$$

$$|V_{1Aopen}| = 0.21 \cdot \text{pu} \quad \arg(V_{1Aopen}) = 58.21 \cdot \text{deg}$$

Since the positive, negative and zero sequence voltages are equal for the phase A open case:

$$V_{2Aopen} := V_{1Aopen} \quad V_{0Aopen} := V_{1Aopen}$$

Calculate change in voltage at each bus due to the open

$$\Delta V_{1A_Open} := Z_{bus1} \cdot \begin{pmatrix} 0 \\ \frac{V_{1Aopen}}{j \cdot X_{L1}} \\ -\frac{V_{1Aopen}}{j \cdot X_{L1}} \\ 0 \end{pmatrix}$$

- Positive injection for sending end of line with series fault
- Negative injection for receiving end of line with series fault

$$V_{1A_Open} := \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} + \Delta V_{1A_Open}$$

$$\overrightarrow{|V_{1A_Open}|} = \begin{pmatrix} 1.15 \\ 1.12 \\ 0.96 \\ 0.94 \end{pmatrix} \cdot \text{pu}$$

$$\overrightarrow{\arg(V_{1A_Open})} = \begin{pmatrix} 10.4 \\ 8.88 \\ -4.25 \\ -6.41 \end{pmatrix} \cdot \text{deg}$$

$$V_{2A_Open} := Z_{bus2} \cdot \begin{pmatrix} 0 \\ \frac{V_{2Aopen}}{j \cdot X_{L2}} \\ -\frac{V_{2Aopen}}{j \cdot X_{L2}} \\ 0 \end{pmatrix}$$

$$\overrightarrow{|V_{2A_Open}|} = \begin{pmatrix} 0.06 \\ 0.08 \\ 0.08 \\ 0.06 \end{pmatrix} \cdot \text{pu}$$

$$\overrightarrow{\arg(V_{2A_Open})} = \begin{pmatrix} 58.21 \\ 58.21 \\ -121.79 \\ -121.79 \end{pmatrix} \cdot \text{deg}$$

$$V_{0A_Open} := Z_{bus0} \cdot \begin{pmatrix} 0 \\ \frac{V_{0Aopen}}{j \cdot X_{L0}} \\ -\frac{V_{0Aopen}}{j \cdot X_{L0}} \\ 0 \end{pmatrix} \quad \overrightarrow{|V_{0A_Open}|} = \begin{pmatrix} 0.04 \\ 0.05 \\ 0.05 \\ 0.04 \end{pmatrix} \cdot pu \quad \overrightarrow{\arg(V_{0A_Open})} = \begin{pmatrix} 58.21 \\ 58.21 \\ -121.79 \\ -121.79 \end{pmatrix} \cdot deg$$

Calculate Trasformer T1 Current:

$$I_{T1_Aopen} := \frac{V_{1A_Open1} - V_{1A_Open2}}{j \cdot X_T} \quad |I_{T1_Aopen}| = 0.5 \cdot pu \quad \arg(I_{T1_Aopen}) = -31.79 \cdot deg$$

$$I_{T2_Aopen} := \frac{V_{2A_Open1} - V_{2A_Open2}}{j \cdot X_T} \quad |I_{T2_Aopen}| = 0.3 \cdot pu \quad \arg(I_{T2_Aopen}) = 148.21 \cdot deg$$

$$I_{T0_Aopen} := \frac{V_{0A_Open1} - V_{0A_Open2}}{j \cdot X_T} \quad |I_{T0_Aopen}| = 0.2 \cdot pu \quad \arg(I_{T0_Aopen}) = 148.21 \cdot deg$$

$$I_{ABC_T1_Aopen} := A_{012} \cdot \begin{pmatrix} I_{T0_Aopen} \\ I_{T1_Aopen} \\ I_{T2_Aopen} \end{pmatrix} \quad \overrightarrow{|I_{ABC_T1_Aopen}|} = \begin{pmatrix} 0 \\ 0.7571 \\ 0.7571 \end{pmatrix} \cdot pu \quad \overrightarrow{\arg(I_{ABC_T1_Aopen})} = \begin{pmatrix} 97.99 \\ -145.57 \\ 82 \end{pmatrix} \cdot deg$$

Same as above

- Approach can be used on any branch except the one with the series fault.

$$V_{3ABC_Aopen} := A_{012} \cdot \begin{pmatrix} V_{0A_Open3} \\ V_{1A_Open3} \\ V_{2A_Open3} \end{pmatrix}$$

$$\overrightarrow{|V_{3ABC_Aopen}|} = \begin{pmatrix} 0.9 \\ 0.97 \\ 1.01 \end{pmatrix} \cdot pu$$

$$\overrightarrow{\arg(V_{3ABC_Aopen})} = \begin{pmatrix} -12.06 \\ -119.95 \\ 118.58 \end{pmatrix} \cdot deg$$

Alternate approach to get just the change in voltage at Bus 3:

$$\Delta V_{3_1aopen} := \left(\frac{Z_{bus1_{3,2}} - Z_{bus1_{3,3}}}{j \cdot X_{L1}} \right) \cdot V_{1Aopen} \quad |\Delta V_{3_1aopen}| = 0.0835 \cdot pu \quad \arg(\Delta V_{3_1aopen}) = -121.79 \cdot deg$$

$$\Delta V_{3_2aopen} := \left(\frac{Z_{bus2_{3,2}} - Z_{bus2_{3,3}}}{j \cdot X_{L2}} \right) \cdot V_{2Aopen} \quad |\Delta V_{3_2aopen}| = 0.0835 \cdot pu \quad \arg(\Delta V_{3_2aopen}) = -121.79 \cdot deg$$

$$\Delta V_{3_0aopen} := \left(\frac{Z_{bus0_{3,2}} - Z_{bus0_{3,3}}}{j \cdot X_{L0}} \right) \cdot V_{0Aopen} \quad |\Delta V_{3_0aopen}| = 0.055 \cdot pu \quad \arg(\Delta V_{3_0aopen}) = -121.79 \cdot deg$$

$$\Delta V_{3ABC} := A_{012} \cdot \begin{pmatrix} \Delta V_{3_0aopen} \\ \Delta V_{3_1aopen} \\ \Delta V_{3_2aopen} \end{pmatrix}$$

$$\overrightarrow{|\Delta V_{3ABC}|} = \begin{pmatrix} 0.22 \\ 0.03 \\ 0.03 \end{pmatrix} \cdot pu$$

$$\overrightarrow{\arg(\Delta V_{3ABC})} = \begin{pmatrix} -121.79 \\ 58.21 \\ 58.21 \end{pmatrix} \cdot deg$$

$$V_{3ABC_aopen} := 1.0 \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} + \Delta V_{3ABC}$$

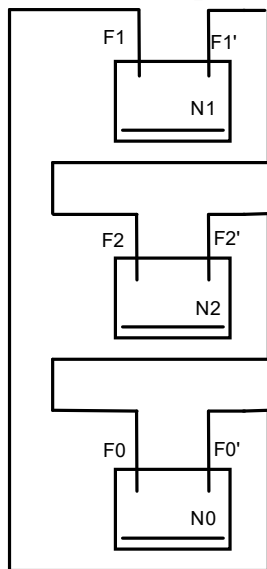
$$\overrightarrow{|V_{3ABC_aopen}|} = \begin{pmatrix} 0.9 \\ 0.97 \\ 1.01 \end{pmatrix} \cdot pu$$

$$\arg(V_{3ABC_aopen}) = \begin{pmatrix} -12.06 \\ -119.95 \\ 118.58 \end{pmatrix} \cdot deg$$

From above:

$$\overrightarrow{|V_{3\text{newABC}}|} = \begin{pmatrix} 0.9 \\ 0.97 \\ 1.01 \end{pmatrix} \cdot \text{pu} \quad \overrightarrow{\arg(V_{3\text{newABC}})} = \begin{pmatrix} -12.06 \\ -119.95 \\ 118.58 \end{pmatrix} \cdot \text{deg}$$

Now solve the two phase open circuit below for the sequence currents:



$$I_1 := \frac{V_{\text{equiv}}}{Z_{1FF'} + Z_{2FF'} + Z_{0FF'}}$$

$$I_1 = (0.2 - 0.12i) \cdot \text{pu}$$

$$|I_1| = 0.23 \cdot \text{pu} \quad \arg(I_1) = -31.79 \cdot \text{deg}$$

$$I_2 := I_1 \quad I_0 := I_1$$

$$I_{\text{abc}} := A_{012} \cdot \begin{pmatrix} I_0 \\ I_1 \\ I_2 \end{pmatrix}$$

$$\overrightarrow{|I_{\text{abc}}|} = \begin{pmatrix} 0.69 \\ 0 \\ 0 \end{pmatrix} \cdot \text{pu}$$

$$\overrightarrow{\arg(I_{\text{abc}})} = \begin{pmatrix} -31.79 \\ 81.87 \\ 81.87 \end{pmatrix} \cdot \text{deg}$$

$$V_{3\text{new1}} := E_2 + I_1 \cdot j \cdot (X_{1\text{Mach}} + X_T)$$

$$V_{3\text{new1}} = (0.92 - 0.14i) \cdot \text{pu}$$

$$V_{3\text{new2}} := 0 + I_2 \cdot j \cdot (X_{2\text{Mach}} + X_T)$$

$$V_{3\text{new2}} = (0.03 + 0.05i) \cdot \text{pu}$$

$$V_{3\text{new0}} := 0 + I_0 \cdot j \cdot (X_{0\text{Mach}} + X_T + 3 \cdot X_{n\text{Mach}})$$

$$V_{3\text{new0}} = (0.03 + 0.05i) \cdot \text{pu}$$

$$V_{3\text{newABC}} := A_{012} \cdot \begin{pmatrix} V_{3\text{new0}} \\ V_{3\text{new1}} \\ V_{3\text{new2}} \end{pmatrix} \quad \overrightarrow{|V_{3\text{newABC}}|} = \begin{pmatrix} 0.98 \\ 0.9 \\ 0.9 \end{pmatrix} \cdot \text{pu} \quad \overrightarrow{\arg(V_{3\text{newABC}})} = \begin{pmatrix} -1.6 \\ -132.16 \\ 107.93 \end{pmatrix} \cdot \text{deg}$$

$$\Delta V_{ABC} := 1.0 \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} - V_{3\text{newABC}} \quad \overrightarrow{|\Delta V_{ABC}|} = \begin{pmatrix} 0.03 \\ 0.22 \\ 0.22 \end{pmatrix} \cdot \text{pu} \quad \overrightarrow{\arg(\Delta V_{ABC})} = \begin{pmatrix} 58.21 \\ -61.27 \\ 177.7 \end{pmatrix} \cdot \text{deg}$$

Using Zbus approach

- Finding the current:

$$I_{1\text{bcopen}} := I_{\text{trans}} \cdot \left(\frac{Z_{1\text{pp}'}}{Z_{1\text{pp}'} + Z_{2\text{pp}'} + Z_{0\text{pp}'}} \right) \quad |I_{1\text{bcopen}}| = 0.23 \cdot \text{pu} \quad \arg(I_{1\text{bcopen}}) = -31.79 \cdot \text{deg}$$

$$I_{2\text{bcopen}} := I_{1\text{bcopen}} \quad I_{0\text{bcopen}} := I_{1\text{bcopen}}$$

$$I_{ABC_bcopen} := A_{012} \cdot \begin{pmatrix} I_{0\text{bcopen}} \\ I_{1\text{bcopen}} \\ I_{2\text{bcopen}} \end{pmatrix} \quad \overrightarrow{|I_{ABC_bcopen}|} = \begin{pmatrix} 0.69 \\ 0 \\ 0 \end{pmatrix} \quad \overrightarrow{\arg(I_{ABC_bcopen})} = \begin{pmatrix} -31.79 \\ 78.69 \\ 78.69 \end{pmatrix} \cdot \text{deg}$$

- Finding the voltage across the open circuit (from the sequence network connections):

$$V_{1\text{bcopen}} := I_{1\text{bcopen}} \cdot (Z_{2\text{pp}'} + Z_{0\text{pp}'}) \quad |V_{1\text{bcopen}}| = 0.4 \cdot \text{pu} \quad \arg(V_{1\text{bcopen}}) = 58.21 \cdot \text{deg}$$

$$V_{2bcopen} := I_{2bcopen} \cdot (-Z_{2pp'}) \quad |V_{2bcopen}| = 0.16 \cdot pu \quad \arg(V_{2bcopen}) = -121.79 \cdot deg$$

$$V_{0bcopen} := I_{0bcopen} \cdot (-Z_{0pp'}) \quad |V_{0bcopen}| = 0.24 \cdot pu \quad \arg(V_{0bcopen}) = -121.79 \cdot deg$$

- Alternate way for finding the voltage across the open circuit (from the sequence network connections):

$$V_{1bcopenA} := I_{trans} \cdot \left[\frac{Z_{1pp'} \cdot (Z_{2pp'} + Z_{0pp'})}{Z_{1pp'} + Z_{2pp'} + Z_{0pp'}} \right] \quad |V_{1bcopenA}| = 0.4 \cdot pu \quad \arg(V_{1bcopenA}) = 58.21 \cdot deg$$

$$V_{2bcopenA} := I_{trans} \cdot \left(\frac{-Z_{1pp'} \cdot Z_{2pp'}}{Z_{1pp'} + Z_{2pp'} + Z_{0pp'}} \right) \quad |V_{2bcopenA}| = 0.16 \cdot pu \quad \arg(V_{2bcopenA}) = -121.79 \cdot deg$$

$$V_{0bcopenA} := I_{trans} \cdot \left(\frac{-Z_{1pp'} \cdot Z_{0pp'}}{Z_{1pp'} + Z_{2pp'} + Z_{0pp'}} \right) \quad |V_{0bcopenA}| = 0.24 \cdot pu \quad \arg(V_{0bcopenA}) = -121.79 \cdot deg$$

Calculate change in voltage at each bus due to the two phase open

$$\Delta V_{1BC_Open} := Z_{bus1} \cdot \begin{pmatrix} 0 \\ \frac{V_{1bcopen}}{j \cdot X_{L1}} \\ -\frac{V_{1bcopen}}{j \cdot X_{L1}} \\ 0 \end{pmatrix} \quad V_{1BC_Open} := \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} + \Delta V_{1BC_Open}$$

$$\overrightarrow{|V_{1BC_Open}|} = \begin{pmatrix} 1.18 \\ 1.17 \\ 0.93 \\ 0.92 \end{pmatrix} \cdot \text{pu} \qquad \overrightarrow{\arg(V_{1BC_Open})} = \begin{pmatrix} 12.34 \\ 11.69 \\ -8.41 \\ -9.47 \end{pmatrix} \cdot \text{deg}$$

$$V_{2BC_Open} := Z_{bus2} \cdot \begin{pmatrix} 0 \\ \frac{V_{2bcopen}}{j \cdot X_{L2}} \\ -V_{2bcopen} \\ \frac{-V_{2bcopen}}{j \cdot X_{L2}} \\ 0 \end{pmatrix} \qquad \overrightarrow{|V_{2BC_Open}|} = \begin{pmatrix} 0.05 \\ 0.06 \\ 0.06 \\ 0.05 \end{pmatrix} \cdot \text{pu} \qquad \overrightarrow{\arg(V_{2BC_Open})} = \begin{pmatrix} -121.79 \\ -121.79 \\ 58.21 \\ 58.21 \end{pmatrix} \cdot \text{deg}$$

$$V_{0BC_Open} := Z_{bus0} \cdot \begin{pmatrix} 0 \\ \frac{V_{0bcopen}}{j \cdot X_{L0}} \\ -V_{0bcopen} \\ \frac{-V_{0bcopen}}{j \cdot X_{L0}} \\ 0 \end{pmatrix} \qquad \overrightarrow{|V_{0BC_Open}|} = \begin{pmatrix} 0.04 \\ 0.06 \\ 0.06 \\ 0.04 \end{pmatrix} \cdot \text{pu} \qquad \overrightarrow{\arg(V_{0BC_Open})} = \begin{pmatrix} -121.79 \\ -121.79 \\ 58.21 \\ 58.21 \end{pmatrix} \cdot \text{deg}$$

Calculate Transformer T1 Current:

$$I_{T1_BCopen} := \frac{V1_{BC_Open1} - V1_{BC_Open2}}{j \cdot X_T} \quad |I_{T1_BCopen}| = 0.23 \cdot \text{pu} \quad \arg(I_{T1_BCopen}) = -31.79 \cdot \text{deg}$$

$$I_{T2_BCopen} := \frac{V2_{BC_Open1} - V2_{BC_Open2}}{j \cdot X_T} \quad |I_{T2_BCopen}| = 0.23 \cdot \text{pu} \quad \arg(I_{T2_BCopen}) = -31.79 \cdot \text{deg}$$

$$I_{T0_BCopen} := \frac{V0_{BC_Open1} - V0_{BC_Open2}}{j \cdot X_T} \quad |I_{T0_BCopen}| = 0.23 \cdot \text{pu} \quad \arg(I_{T0_BCopen}) = -31.79 \cdot \text{deg}$$

$$I_{ABC_T1_BCopen} := A_{012} \cdot \begin{pmatrix} I_{T0_BCopen} \\ I_{T1_BCopen} \\ I_{T2_BCopen} \end{pmatrix} \quad \overrightarrow{|I_{ABC_T1_BCopen}|} = \begin{pmatrix} 0.6927 \\ 0 \\ 0 \end{pmatrix} \cdot \text{pu} \quad \overrightarrow{\arg(I_{ABC_T1_BCopen})} = \begin{pmatrix} -31.79 \\ 104.09 \\ -11.93 \end{pmatrix} \cdot \text{deg}$$

Same as above

- Approach can be used on any branch except the one with the series fault.

$$V_{3ABC_BCopen} := A_{012} \cdot \begin{pmatrix} V0_{BC_Open3} \\ V1_{BC_Open3} \\ V2_{BC_Open3} \end{pmatrix} \quad \overrightarrow{|V_{3ABC_BCopen}|} = \begin{pmatrix} 0.98 \\ 0.9 \\ 0.9 \end{pmatrix} \cdot \text{pu} \quad \overrightarrow{\arg(V_{3ABC_BCopen})} = \begin{pmatrix} -1.6 \\ -132.16 \\ 107.93 \end{pmatrix} \cdot \text{deg}$$

- Another approach to find change bus 3 voltage:

$$\Delta V_{3_1bcopen} := \left(\frac{Z_{bus1_{3,2}} - Z_{bus1_{3,3}}}{j \cdot X_{L1}} \right) \cdot V_{1bcopen} \quad \left| \Delta V_{3_1bcopen} \right| = 0.1593 \cdot \text{pu} \quad \arg(\Delta V_{3_1bcopen}) = -121.79 \cdot \text{deg}$$

$$\Delta V_{3_2bcopen} := \left(\frac{Z_{bus2_{3,2}} - Z_{bus2_{3,3}}}{j \cdot X_{L2}} \right) \cdot V_{2bcopen} \quad \left| \Delta V_{3_2bcopen} \right| = 0.0647 \cdot \text{pu} \quad \arg(\Delta V_{3_2bcopen}) = 58.21 \cdot \text{deg}$$

$$\Delta V_{3_0bcopen} := \left(\frac{Z_{bus0_{3,2}} - Z_{bus0_{3,3}}}{j \cdot X_{L0}} \right) \cdot V_{0bcopen} \quad \left| \Delta V_{3_0bcopen} \right| = 0.0623 \cdot \text{pu} \quad \arg(\Delta V_{3_0bcopen}) = 58.21 \cdot \text{deg}$$

$$\Delta V_{3ABC_bcopen} := A_{012} \cdot \begin{pmatrix} \Delta V_{3_0bcopen} \\ \Delta V_{3_1bcopen} \\ \Delta V_{3_2bcopen} \end{pmatrix} \quad \left| \Delta V_{3ABC_bcopen} \right| = \begin{pmatrix} 0.03 \\ 0.22 \\ 0.22 \end{pmatrix} \cdot \text{pu} \quad \arg(\Delta V_{3ABC_bcopen}) = \begin{pmatrix} -121.79 \\ 118.73 \\ -2.3 \end{pmatrix} \cdot \text{deg}$$

Then the voltage at bus 3 would be:

$$V_{3ABC_bcopen} := 1.0 \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} + \Delta V_{3ABC_bcopen} \quad \left| V_{3ABC_bcopen} \right| = \begin{pmatrix} 0.98 \\ 0.9 \\ 0.9 \end{pmatrix} \cdot \text{pu} \quad \arg(V_{3ABC_bcopen}) = \begin{pmatrix} -1.6 \\ -132.16 \\ 107.93 \end{pmatrix} \cdot \text{deg}$$

Note the plus sign instead of the minus sign as in the circuit based case above.

As a check:

$$V_{1_BUS3} := 1.0 + \Delta V_{3_1bcopen}$$

$$V_{2_BUS3} := 0 + \Delta V_{3_2bcopen}$$

$$V_{0_BUS3} := 0 + \Delta V_{3_0bcopen}$$

$$I_{0_right} := \frac{V_{0_BUS3} - 0}{\left[j \cdot (X_T + X_{0Mach} + 3 \cdot X_{nMach}) \right]} \quad |I_{0_right}| = 0.23 \cdot \text{pu} \quad \arg(I_{0_right}) = -31.79 \cdot \text{deg}$$

$$I_{2_right} := \frac{V_{2_BUS3} - 0}{\left[j \cdot (X_T + X_{2Mach}) \right]} \quad |I_{2_right}| = 0.23 \cdot \text{pu} \quad \arg(I_{2_right}) = -31.79 \cdot \text{deg}$$

$$I_{1_right} := \frac{V_{1_BUS3} - E_2}{\left[j \cdot (X_T + X_{2Mach}) \right]} \quad |I_{1_right}| = 0.23 \cdot \text{pu} \quad \arg(I_{1_right}) = -31.79 \cdot \text{deg}$$

$$I_{ABC_right} := A_{012} \cdot \begin{pmatrix} I_{0_right} \\ I_{1_right} \\ I_{2_right} \end{pmatrix} \quad \overrightarrow{|I_{ABC_right}|} = \begin{pmatrix} 0.69 \\ 0 \\ 0 \end{pmatrix} \quad \overrightarrow{\arg(I_{ABC_right})} = \begin{pmatrix} -31.79 \\ 54.25 \\ -115.98 \end{pmatrix} \cdot \text{deg}$$