ECE 523

SYMMETRICAL COMPONENTS

SESSION no. 19
Transmission lines/cables models

Steady-state/quasi-steady state

Short
\[ R \quad jX \]
Overhead lines
\[ \approx 50 \text{ km or less} \]
Underground/underwater
\[ \approx 5 \text{ km or less} \]
Values with cable type

Medium
\[ R \quad jX \]

Long
\[ R' \quad jX' \]
Correction factors for wave effects

Overhead lines
\[ T \quad 50 \text{ km - 150 km} \]
Underground/underwater
\[ S \quad 5 \text{ km - cables} \]
\[ R_{\text{length}} = R_{\text{A c}} \frac{1}{\text{length}} \]

- correct for skin effect
- stranded

\[ R_{\text{dc}} \rightarrow R_{\text{A c}} \text{ - look up from tables} \]

\[ L' = \frac{N_0}{2\pi} \ln \left( \frac{D_m}{R_6} \right) \]

\[ D_m = \sqrt[3]{D_{\text{PA}} - D_{\text{Ac}} - D_{\text{BC}}} \]

- geometric mean distance

\[ R_6 = r' = \text{radius} \cdot e^{-N/4} \]

- geometric mean radius

\[ \Rightarrow \text{better to get from tables} \]
If bundled conductor

\[ R_b = \sqrt{D_5 \cdot R_{12} \cdot R_{13} \cdot R_{1-n}} \]

Conductor GMR from table

\[ Z_{\text{series}} = (R_{\text{length}} + j\omega L) \cdot \text{length} \]

\[ Z_1 = Z_2 \]

But no information on \( Z_0 \)
\[ C' = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D}{2R_6}\right)} \]

Geometric mean radius for capacitance calculations

- Single conductor \[ R_6^c = \frac{C}{\text{radius of conductor}} \]
- Bundle \[ R_6^c = \sqrt{r_1 d_1 \cdot d_2 \cdot d_3 \cdots d_n} \]

Positive sequence (negative too)
conduction to ground

- corona
- leakage current at insulators
Calculating Z_{series}

\[ Z' = R' + jX' \]  \( \text{Ohm per length} \)

\( R' \rightarrow \text{ac resistance at a specific frequency} \)

\[ jX' = j\omega L' \]

POS sequence short cut equations for POS sequence accurate if: C' \( \rightarrow \) transposed, no ground wires

L' \( \rightarrow \) transposed, no ground wires
We want an approach that accounts for ground wires and calculates zero sequence.
\[
\begin{bmatrix}
V_A - V_{A'} \\
V_D - V_D'
\end{bmatrix}
= 
\begin{bmatrix}
Z_{AA} & Z_{AD} \\
Z_{BA} & Z_{DD}
\end{bmatrix}
\begin{bmatrix}
I_A \\
-I_A
\end{bmatrix}
\]

\[Z_{AD} = Z_{DA}\]

\[V_A = V_{AG}\]
\[V_A' = V_{A'G}\]
\[V_A' = V_D'\]

\[V_D = V_{DG} = 0\]
\[V_D' = V_{D'G}\]
\[ V_A - V_{A'} = Z_{AA} I_A - Z_{AD} I_A \] (1)

\[ 0 - V_{D'} = Z_{AD} I_A - Z_{DD} I_A \] (2)

Equation (1) - (2)

\[ V_A - V_{A'} - (0 - V_{D'}) = V_{A} \]

\[ V_A = I_A \left[ Z_{AA} - 2Z_{AD} + Z_{DD} \right] \]
\[ Z_{DD} = (r_d + j \omega L_{self-earth}) \cdot \text{Length} \]

\[ r_d = 1.588 \times 10^{-3} \cdot \text{frequency} \ \text{mile} \]

\[ L_{self-earth} = \frac{N_0}{2\pi} \left( \frac{R_f}{D_{sd}} \right) \]

\[ Z_{AD} = j \omega \frac{N_0}{2\pi} \ln \left( \frac{R_f}{D_{AD}} \right) \]

- Row 1: \[ R_f \] distance to an arbitrary reference.
- Row 2: \[ GMR \] of earth return path.
- Row 3: Distance of phase conductor above earth return path.
Carson's Line Model

1 conductor and earth return

\[ Z_{AA} \]

\[ Z_{AD} \]

\[ Z_{DD} \]

Ground potential: \( V_{DG} = 0 \)

\( V_{DD} \neq 0 \)
\[ Z_{AA} = R_{Ac} + jw \frac{N_0}{2\pi} \left[ \frac{R_c}{D_sA} \right] \]

6MR of phase conductor

or

\[ (\ln \frac{Z_{s1}}{D_sA} - 1) \]

5 = length
9 conductor path

\[ \frac{V_A}{I_A} = Z_{A_{\text{collective}}} = Z_{AA} - 2Z_{AD} + Z_{DD} \]

\[ = \left[ (R_{Ac} + r_d) + jw \frac{N_0}{2\pi} \left[ \ln \frac{R_c}{D_sA} - 2\ln \frac{R_c}{D_{AD}} + \ln \frac{R_c}{D_{SD}} \right] \right] \]

* length
Natural log terms...

\[
\ln\left(\frac{R_f}{D_{SA}}\right) - 2\ln\left(\frac{R_f}{D_{AD}}\right) + \ln\left(\frac{R_f}{D_{SD}}\right)
\]

\[= \ln\left[\frac{R_f}{D_{SA}} \cdot \left(\frac{D_{AD}}{R_f}\right)^2 \cdot \frac{R_f}{D_{SD}}\right]
\]

\[= \ln\left(\frac{D_{AD}^2}{D_{SA} \cdot D_{SD}}\right)
\]

\[\text{Define } D_e = \left\{ \begin{array}{l}
2160 \cdot \sqrt{\frac{P}{C}} \text{ (ft)} \\
685.85 \sqrt{\frac{P}{C}} \text{ (m)}
\end{array} \right.
\]

\[\ln\left(\frac{D_e}{D_{SA}}\right) \frac{D_{AD}^2}{D_{SD}}
\]
\[ Z_{AA\, \text{effective}} = \left[ (R_{Ac} + R_d) + j \omega \frac{m_0}{2 \pi} \ln \left( \frac{Dc}{Dsa} \right) \right] \times \text{length} \]

Similar for

\[ Z_{BB} + Z_{cc} \] for 300 case

\[
\begin{bmatrix}
V_A \\
V_B \\
V_c
\end{bmatrix} =
\begin{bmatrix}
Z_{AA \, \text{un}} & Z_{AB \, \text{un}} & Z_{AC \, \text{un}} \\
Z_{BA \, \text{un}} & Z_{BB \, \text{un}} & Z_{BC \, \text{un}} \\
Z_{CA \, \text{un}} & Z_{CB \, \text{un}} & Z_{CC \, \text{un}}
\end{bmatrix}
\begin{bmatrix}
I_A \\
I_B \\
I_c
\end{bmatrix}
\]
$A_0^{12} Z_{ABC} A_{012} = Z_{012}$

series $E$ for short to medium

- need to correction factors
  for long