

Lecture 2

Define units: MVA := 1000kW MW := MVA pu := 1
 kVA := kW kVAR := kW

1. Entering Phasors in Polar Notation in Mathcad 15

- Step 1: Go to: "Help Menu" -- "Quick Sheets"
- Step 2: Choose "Extra Math Symbols" from the list
- Step 3: Scroll down the page and copy the angle symbol to your Mathcad sheet and use it to define a function as shown below defining the complex phasor in terms of magnitude and angle, where the arguments of the function are the magnitude and angle for a complex number

$$\angle(\text{magnitude}, \text{angle}) := \text{magnitude} \cdot \cos(\text{angle}) + j \cdot \text{magnitude} \cdot \sin(\text{angle})$$

- Step 4: When you want to use this new function to enter a phasor, start typing your expression, before entering the phasor itself, go to the Evaluation Toolbar and select to enter your information as an infix number (select "x^{fy}" from the evaluation toolbar. This will allow you to enter your function in the order you normally use for entering phasors, instead of the normal Mathcad order. You should see three placeholders in your Mathcad sheet
 - Enter the magnitude of the phasor in the first placeholder
 - Copy the angle symbol into the middle one
 - Enter the angle of the phasor in the third placeholder

$$V_a := 1 \angle (45\text{deg}) \quad V_a = 0.707 + 0.707i$$

- A lot of people find it easiest to enter this once and then just copy it into other sheets.

2. The "a" operator for three phase systems

$$a := 1 \angle (120\text{deg}) \quad a = -0.5 + 0.866i$$

$$a^2 = -0.5 - 0.866i \quad \bar{a} = -0.5 - 0.866i$$

$$a^3 = 1$$

$$1 + a + a^2 = 0$$

$$1 - a = 1.5 - 0.866i \quad \sqrt{3} \angle (-30\text{deg}) = 1.5 - 0.866i$$

$$1 - a^2 = 1.5 + 0.866i \quad \sqrt{3} \angle (30\text{deg}) = 1.5 + 0.866i$$

$$a - 1 = -1.5 + 0.866i \quad \sqrt{3} \angle (150\text{deg}) = -1.5 + 0.866i$$

$$a^2 - 1 = -1.5 - 0.866i \quad \sqrt{3} \angle (-150\text{deg}) = -1.5 - 0.866i$$

$$a^2 - a = -1.732i \quad \sqrt{3} \angle (-90\text{deg}) = -1.732i$$

$$a^2 + a = -1 \quad 1 \angle (180\text{deg}) = -1$$

- Normal balanced three phase set:

$$V_{AG} := 1 \angle (0\text{deg})$$

$$V_{BG} := 1 \angle (-120\text{deg}) \quad V_{BG} = -0.5 - 0.866i$$

$$V_{CG} := 1 \angle (120\text{deg}) \quad V_{CG} = -0.5 + 0.866i$$

$$V_{ABC} := V_{AG} \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} \quad V_{ABC} = \begin{pmatrix} 1 \\ -0.5 - 0.866i \\ -0.5 + 0.866i \end{pmatrix} \quad \overrightarrow{|V_{ABC}|} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \overrightarrow{\arg(V_{ABC})} = \begin{pmatrix} 0 \\ -120 \\ 120 \end{pmatrix} \cdot \text{deg}$$

3. Some matrix operations

- Expanding a matrix: If you have a 3x3 matrix and want to add a row and column while at the stage of entering the matrix initially, make the cell where you want to add rows and columns the active cell
- Select the add matrix/vector tool from the Matrix tool bar (or type CTRL-M) and enter the number of rows and columns you want to add (note you can add 0 rows and 1 (or more) column or vice versa. The selected number of rows appears below the active cell and the selected number of columns appears to the right of the cell.

$$A_1 := \begin{pmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 2 & 2 \\ 4 & 5 & 6 & 3 \\ 5 & 55 & 54 & 4 \end{pmatrix}$$

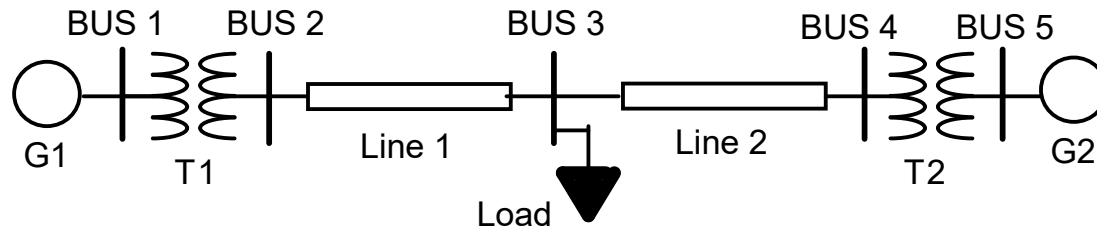
- Use the submatrix command if you want to pull out part of a matrix
- Pay attention to the setting for the internal ORIGIN variable (or reassign it if you prefer).

ORIGIN := 1

B := submatrix(A₁, 1, 3, 1, 2)

$$B = \begin{pmatrix} 1 & 2 \\ 1 & 3 \\ 4 & 5 \end{pmatrix}$$

Per Unit Example: Sketch a per unit impedance diagram for the system shown below. Use a 100MVA impedance base, and the generator 1 rated voltage as your reference voltage base. Use pi models for the lines.



- G1: 50MVA, 13.8kV
- G2: 20MVA, 14.4kV
- T1: 40MVA, Δ -Y, 13.2:161kV, X = 10%
- T2: 25MVA, Y- Δ , 161kV:13.2kV, X = 10%
- Load: 45MVA, 0.8pf lagging (Y connected, parallel impedances)
- Line 1: 100 mile, $Z = 0.28 + j0.73$ ohm/mi, $Y = 5.9 \cdot 10^{-6}$ mho/mi
- Line 2: 75 mile, $Z = 0.28 + j0.73$ ohm/mi, $Y = 5.9 \cdot 10^{-6}$ mho/mi

Define Base Quantities:

Section I (left of T1)

$$S_B := 100\text{MVA}$$

$$V_{B1} := 13.8\text{kV}$$

Line to line voltage

$$Z_{B1} := \frac{V_{B1}^2}{S_B}$$

$$Z_{B1} = 1.904 \Omega$$

$$I_{B1} := \frac{S_B}{\sqrt{3} \cdot V_{B1}}$$

$$I_{B1} = 4183.698 \text{ A}$$

Section II (between T1 and T2)

$$V_{B2} := V_{B1} \cdot \left(\frac{161 \text{ kV}}{13.2 \text{ kV}} \right) \quad V_{B2} = 168.318 \cdot \text{kV}$$

$$Z_{B2} := \frac{V_{B2}^2}{S_B} \quad Z_{B2} = 283.31 \Omega$$

$$I_{B2} := \frac{S_B}{\sqrt{3} \cdot V_{B2}} \quad I_{B2} = 343.011 \text{ A}$$

Section II (right of T2)

$$V_{B3} := V_{B2} \cdot \left(\frac{13.2 \text{ kV}}{161 \text{ kV}} \right) \quad V_{B3} = 13.8 \cdot \text{kV}$$

$$Z_{B3} := \frac{V_{B3}^2}{S_B} \quad Z_{B3} = 1.904 \Omega$$

$$I_{B3} := \frac{S_B}{\sqrt{3} \cdot V_{B3}} \quad I_{B3} = 4183.698 \text{ A}$$

Transmission Line Models:

Line 1: Length1 := 100mi

$$Z_{\text{line1}} := \left[(0.28 + j \cdot 0.73) \frac{\text{ohm}}{\text{mi}} \right] \cdot \text{Length1} \quad Z_{\text{line1}} = (28 + 73i) \cdot \text{ohm}$$

$$Y_{\text{line1}} := j \cdot 5.9 \cdot 10^{-6} \frac{\text{mho}}{\text{mi}} \cdot \text{Length1} \quad Y_{\text{line1}} = 5.9i \times 10^{-4} \cdot \text{mho} \quad \frac{Y_{\text{line1}}}{2} = 2.95i \times 10^{-4} \cdot \text{mho}$$

Line 1 is in section II, so use Zbase2

$$Z_{\text{line1pu}} := \frac{Z_{\text{line1}}}{Z_{B2}} \quad Z_{\text{line1pu}} = (0.099 + 0.258i) \cdot \text{pu}$$

Note that Ybase is 1/Zbase:

$$Y_{\text{line1pu}} := Y_{\text{line1}} \cdot Z_{B2} \quad Y_{\text{line1pu}} = 0.167i \cdot \text{pu}$$

We actually need $Y/2$ for the pi model:

$$Y_{\text{line1pi}} := \frac{Y_{\text{line1pu}}}{2} \quad Y_{\text{line1pi}} = 0.084i \cdot \text{pu}$$

Line 2:

$$\text{length2} := 75\text{mi}$$

$$Z_{\text{line2}} := \left[(0.28 + j \cdot 0.73) \frac{\text{ohm}}{\text{mi}} \right] \cdot \text{length2} \quad Z_{\text{line2}} = (21 + 54.75i) \cdot \text{ohm}$$

$$Y_{\text{line2}} := j \cdot 5.9 \cdot 10^{-6} \frac{\text{mho}}{\text{mi}} \cdot \text{length2} \quad Y_{\text{line2}} = 4.425i \times 10^{-4} \cdot \text{mho} \quad \frac{Y_{\text{line2}}}{2} = 2.212i \times 10^{-4} \cdot \text{mho}$$

Line 2 is also in section II, so use Z_{base2}

$$Z_{\text{line2pu}} := \frac{Z_{\text{line2}}}{Z_{\text{B2}}} \quad Z_{\text{line2pu}} = (0.074 + 0.193i) \cdot \text{pu}$$

$$Y_{\text{line2pu}} := Y_{\text{line2}} \cdot Z_{\text{B2}} \quad Y_{\text{line2pu}} = 0.125i \cdot \text{pu}$$

We actually need $Y/2$ for the pi model:

$$Y_{\text{line2pi}} := \frac{Y_{\text{line2pu}}}{2} \quad Y_{\text{line2pi}} = 0.0627i \cdot \text{pu}$$

Transformer Model Calculations

Transformer 1:

$$S_{\text{T1}} := 40\text{MVA} \quad V_{\text{T1Low}} := 13.2\text{kV} \quad V_{\text{T1hi}} := 161\text{kV} \quad X_{\text{T1}} := 0.10\text{pu}$$

Impedance change of base calculation

$$X_{T1\text{new}} := X_{T1} \cdot \left(\frac{V_{T1\text{Low}}}{V_{B1}} \right)^2 \cdot \left(\frac{S_B}{S_{T1}} \right) \quad X_{T1\text{new}} = 0.229 \cdot \text{pu}$$

Transformer 2:

$$S_{T2} := 25\text{MVA} \quad V_{T2\text{Low}} := 13.2\text{kV} \quad V_{T2\text{hi}} := 161\text{kV} \quad X_{T2} := 0.10\text{pu}$$

Impedance change of base calculation

$$X_{T2\text{new}} := X_{T2} \cdot \left(\frac{V_{T2\text{Low}}}{V_{B3}} \right)^2 \cdot \left(\frac{S_B}{S_{T2}} \right) \quad X_{T2\text{new}} = 0.366 \cdot \text{pu}$$

Load Model

$$\text{mag}S_{\text{load}} := 45\text{MVA} \quad \text{pload} := 0.8 \quad \text{lagging} \quad V_{\text{loadrated}} := 161\text{kV} \quad \phi_{\text{load}} := \text{acos}(\text{pload})$$

$$\phi_{\text{load}} = 36.87 \cdot \text{deg}$$

$$S_{\text{load}} := \text{mag}S_{\text{load}} \cdot e^{j \cdot \phi_{\text{load}}} \quad S_{\text{load}} = (36 + 27i) \cdot \text{MVA}$$

Since the load is wye connected with parallel impedances:

$$R_{\text{load}} := \frac{(|V_{\text{loadrated}}|)^2}{\text{Re}(S_{\text{load}})} \quad R_{\text{load}} = 720.028 \Omega \quad X_{\text{load}} := \frac{(|V_{\text{loadrated}}|)^2}{\text{Im}(S_{\text{load}})} \quad X_{\text{load}} = 960.037 \Omega$$

As a check:

$$Z_{\text{equivload}} := \left(\frac{1}{R_{\text{load}}} + \frac{1}{j \cdot X_{\text{load}}} \right)^{-1} \quad |Z_{\text{equivload}}| = 576.022 \Omega \quad \arg(Z_{\text{equivload}}) = 36.87 \cdot \text{deg}$$

$$Z_{\text{equivload}} = (460.818 + 345.613i) \Omega$$

$$S_{\text{check}} := \frac{(|V_{\text{loadrated}}|)^2}{Z_{\text{equivload}}} \quad S_{\text{check}} = (36 + 27i) \cdot \text{MVA} \quad \text{note complex conjugate in equation}$$

Convert to per unit (load in section II):

$$R_{\text{loadpu}} := \frac{R_{\text{load}}}{Z_{B2}} \quad R_{\text{loadpu}} = 2.541 \cdot \text{pu} \quad X_{\text{loadpu}} := \frac{X_{\text{load}}}{Z_{B2}} \quad X_{\text{loadpu}} = 3.389 \cdot \text{pu}$$

Per Unit Equivalent Circuit:

