University of Idaho

ECE 523
Symmetrical Components

Session 23

University of Idaho

Natural log terms

$$\ln\left(\frac{R_f}{D_{AD}}\right)^2 \Rightarrow -\ln\left(\left(\frac{R_f}{D_{AD}}\right)^2\right) = +\ln\left(\frac{D_{AD}}{R_f}\right)^2$$

$$\ln\frac{R_f}{D_{SA}} - 2\ln\frac{R_f}{D_{AD}} + \ln\frac{R_f}{D_{SD}}$$

$$= \ln\left(\frac{R_f}{D_{SA}}\right) + \ln\left(\left(\frac{D_{AD}}{R_f}\right)^2\right) + \ln\left(\frac{R_f}{D_{SD}}\right)$$

collect inside natural log

$$= \ln\left(\frac{\cancel{R_f}}{D_{SA}} \cdot \left(\frac{D_{AD}}{\cancel{R_f}}\right)^2 \cdot \frac{\cancel{R_f}}{\cancel{D_{SD}}}\right)$$

$$= \ln\left(\frac{D_{AD}^2}{D_{SA} \cdot \underbrace{\cancel{D_{SD}}}_{1}}\right) = \ln\frac{D_{AD}^2}{D_{SA}}$$

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$$\frac{D_{AD}^2}{D_{SA}} \equiv \underline{D_e = 2160\sqrt{\frac{\rho}{f}}\ \text{ft}}$$

ρ = earth resistivity

$$D_e = 685.85\sqrt{\frac{\rho}{f}}\ \text{m}$$

Earth Resistivity

- Best option is to take measurements

→ can change seasonally
  - soil moisture content

→ not necessarily the same for length of a line

- impacts zero sequence impedance

<u>Typical values</u>: $\rho \rightarrow$ units ohms·meters

sea water: ρ is range of $0.01 - 1.0\ \Omega \cdot m$

swampy ground: $\rho \Rightarrow 10 - 100\ \Omega \cdot m$

Average Damp Earth: $100\ \Omega \cdot m$ ← people often use this if they don't a number

Dry Earth: $1000\ \Omega \cdot m$

sandstone: $10^9\ \Omega \cdot m$

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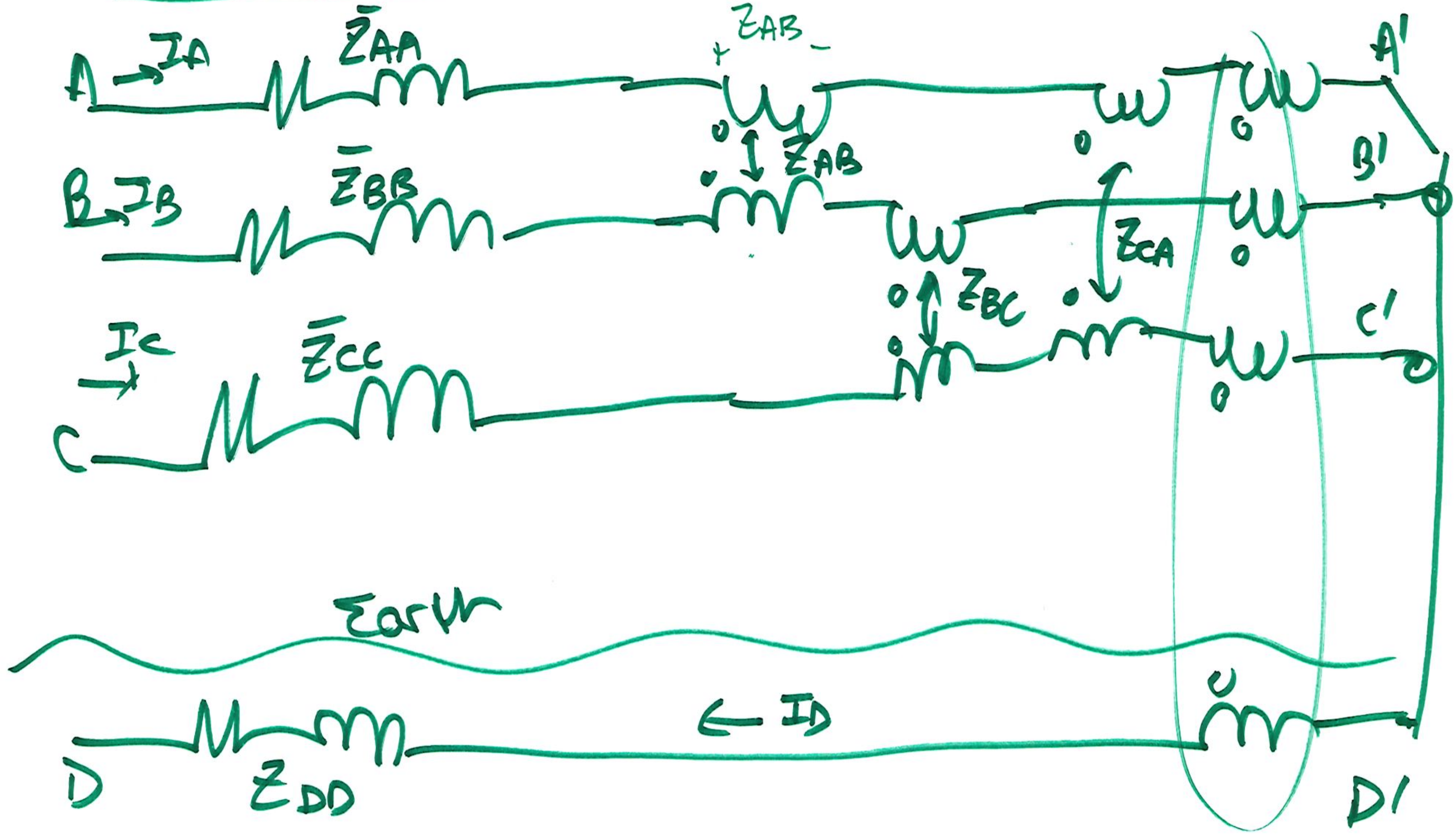
3 phase line



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$$\left.\begin{matrix} Z_{AA} \\ Z_{BB} \\ Z_{CC} \end{matrix}\right\}$$ we will come back - like Z_{AA} earlier

$$\bar{Z}_{AA} = \underbrace{R'_{AC}}_{\text{per length}} + \frac{\mu_0}{2\pi} \ln\left(\frac{R_f}{D_s}\right)$$

dist to arbitrary ref

GMR of phase conductor

same for $\bar{Z}_{BB}$ & $\bar{Z}_{CC}$ since R_f is big

- voltage drop on phase A due to I_B

$$Z_{AB_{eff}} = \frac{\bar{V}_A}{\bar{I}_B} = \bar{Z}_{AB} - \bar{Z}_{AD} - \bar{Z}_{BD} + \bar{Z}_{DD}$$

$$\bar{Z}_{AB} = j\omega \frac{\mu_0}{2\pi} \ln\left(\frac{R_f}{D_{AB}}\right) \quad \Omega/\text{distance}$$

$$\bar{Z}_{AD} = j\omega \frac{\mu_0}{2\pi} \ln \frac{R_f}{D_{AD}}$$

$$\bar{Z}_{BD} = j\omega \frac{\mu_0}{2\pi} \ln \frac{R_f}{D_{BD}}$$

$$\bar{Z}_{DD} = r_d + j\omega \frac{\mu_0}{2\pi} \ln \frac{R_f}{D_{sd}}$$

- GMR of earth return path ≈ 1

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At power frequency, the earth return current is fairly deep

$$D_{AD} \approx D_{BD} \approx D_{CD}$$

so $Z_{AD} = Z_{BD} = Z_{CD}$

$$Z_{AB_{eff}} = \underbrace{Z_{AB} - 2 Z_{AD} + Z'_{DD}}_{\text{using simplification from last time}}$$

$$Z_{AB_{eff}} = r_d + j\omega \frac{\mu_0}{2\pi} \ln\left(\frac{D_e}{D_{AB}}\right)$$

average distance A→B

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similar for $Z_{BC\,eff}$ & $Z_{AC\,eff}$

$$\underset{D_{BC}}{\uparrow} \qquad \underset{D_{AC}}{\uparrow}$$

$$Z_{AA\,eff} = Z_{AA} - 2Z_{AD} + Z_{DD}$$

$$= (R_{AC} + r_d) + j\omega \cdot \frac{\mu_0}{2\pi} \frac{D_e}{\underbrace{D_{SA}}_{\text{GMR of phase conductor}}}$$

usually

$$Z_{AA\,eff} = Z_{BB\,eff} = Z_{CC\,eff}$$

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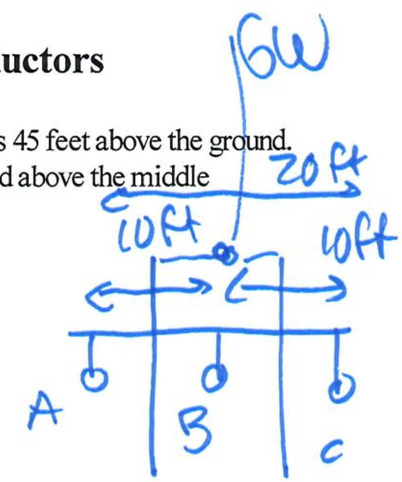
EE 523: Line Constants: Transposition, Ground Wire and Bundled Conductors

diameter

A three-phase, 69 kV line has flat spacing with 10 feet between adjacent conductors. The average height of the conductors is 45 feet above the ground. The phase conductors are 19 strand, 4/0, hard-drawn copper conductors. There is a 0.375 in steel ground wire located 15 feed above the middle conductor. The line is 40 miles long. Assume an earth resistivity of 100 ohm-m and an operating temperature of 25 C



A. Find the sequence impedance with the ground wire neglected.

AC Resistance from table:

$R_{ac} := 0.278\dfrac{ohm}{mi}$ at 25 C and $freq := 60Hz$ From Table B.4 - Anderson Book

$\mu_0 := 4\cdot\pi\cdot 10^{-7}\dfrac{H}{m}$ $\rho := 100ohm\cdot m$ Conductor GMR from Table B.4 in Anderson $D_s := 0.01668ft$

$$CarsonsResistConst := 9.869\times 10^{-7}\frac{ohm}{m\cdot Hz}$$

$R_d := CarsonsResistConst\cdot freq$ $R_d = 0.0953\cdot\dfrac{ohm}{mi}$

$R_{self} := R_{ac} + R_d$ $R_{self} = 0.3733\cdot\dfrac{ohm}{mi}$

$$R_{perlength} := \begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ & \begin{pmatrix} R_{self} & R_d & R_d \\ R_d & R_{self} & R_d \\ R_d & R_d & R_{self} \end{pmatrix} \begin{array}{c} A \\ B \\ C \end{array} \end{array}$$

$$R_{perlength} = \begin{pmatrix} 0.3733 & 0.0953 & 0.0953 \\ 0.0953 & 0.3733 & 0.0953 \\ 0.0953 & 0.0953 & 0.3733 \end{pmatrix}\cdot\frac{ohm}{mi}$$

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$$D_{e\_const} := 2160 \cdot \frac{ft \cdot Hz^{0.5}}{(ohm \cdot m)^{0.5}}$$

$$D_e := D_{e\_const} \cdot \sqrt{\frac{\rho}{freq}} \qquad D_e = 2.7885 \times 10^3 \cdot ft$$

$$D_{ab} := 10ft \qquad D_{ac} := 20ft \qquad D_{bc} := 10ft$$

$$L_{perlength} := \frac{\mu_0}{2 \cdot \pi} \cdot \begin{array}{cc} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{pmatrix} \ln\left(\frac{D_e}{D_s}\right) & \ln\left(\frac{D_e}{D_{ab}}\right) & \ln\left(\frac{D_e}{D_{ac}}\right) \\ \ln\left(\frac{D_e}{D_{ab}}\right) & \ln\left(\frac{D_e}{D_s}\right) & \ln\left(\frac{D_e}{D_{bc}}\right) \\ \ln\left(\frac{D_e}{D_{ac}}\right) & \ln\left(\frac{D_e}{D_{bc}}\right) & \ln\left(\frac{D_e}{D_s}\right) \end{pmatrix} & \begin{array}{c} A \\ B \\ C \end{array} \end{array}$$

$$L_{perlength} = \begin{pmatrix} 3.8711 & 1.8123 & 1.5892 \\ 1.8123 & 3.8711 & 1.8123 \\ 1.5892 & 1.8123 & 3.8711 \end{pmatrix} \cdot \frac{mH}{mi}$$

$$Z' := R_{perlength} + j \cdot 2 \cdot \pi \cdot freq \cdot L_{perlength}$$

$$Z' = \begin{pmatrix} 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.599i \\ 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.683i \\ 0.095 + 0.599i & 0.095 + 0.683i & 0.373 + 1.459i \end{pmatrix} \cdot \frac{ohm}{mi}$$

If length = 40 miles: $\quad Z_{line} := Z' \cdot 40mi$

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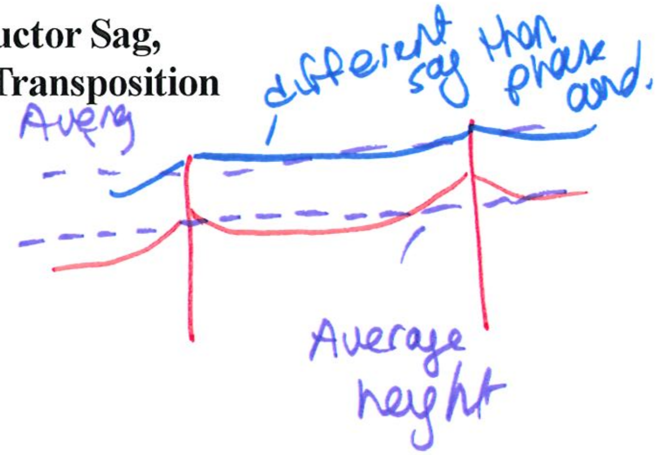
Representation of Conductor Sag, Rotating and Twisting for Transposition



Conductor Sag

- $\text{Htower} := 77\text{ft}$
- $\text{Mid} := 33\text{ft}$ - height at mid span

$\text{sag} := \text{Htower} - \text{Mid}$ $\qquad$ $\text{sag} = 44 \cdot \text{ft}$

$H_{ave1} := \text{Htower} - \frac{2}{3} \cdot \text{sag}$ $\qquad$ $H_{ave1} = 47.66667 \cdot \text{ft}$

$H_{ave2} := \text{Mid} + \frac{1}{3} \cdot \text{sag}$ $\qquad$ $H_{ave2} = 47.66667 \cdot \text{ft}$

$H_{ave3} := \frac{\text{Htower}}{3} + \frac{2}{3} \cdot \text{Mid}$ $\qquad$ $H_{ave3} = 47.66667 \cdot \text{ft}$

Rotation

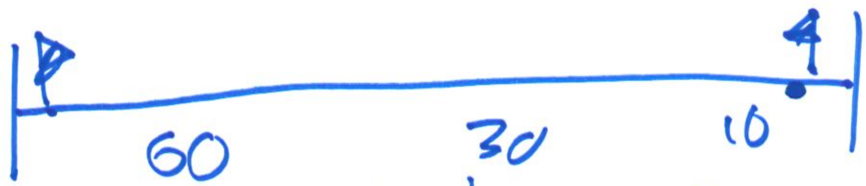
$$Z_{123} := \begin{pmatrix} 100 & 12 & 13 \\ 21 & 200 & 23 \\ 31 & 32 & 300 \end{pmatrix} \qquad V_{123} := \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

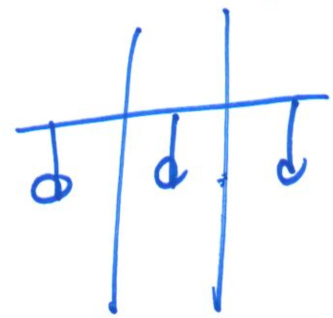
$$R_p := \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

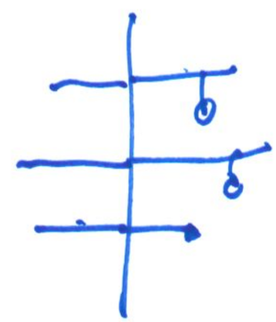
$$V_{312} := R_p \cdot V_{123} \qquad V_{312} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

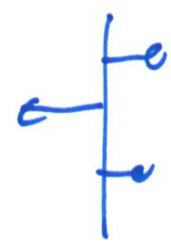
$$V_{231} := R_p^{-1} \cdot V_{123} \qquad V_{231} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

100 km



Z_{ABC_1} 60 ~~km~~ km has  $\}$ calculate Z_{ABC} for 60 km

Z_{ABC_2} 30 km has  $\}$ calculate Z_{ABC} and mult by 30 km

Z_{ABC_3} 10 km  $\}$ calculate Z_{ABC} & multiply by length

$Z_{ABC_1} + Z_{ABC_2} + Z_{ABC_3} \rightarrow$ afterwards put in sequence domain

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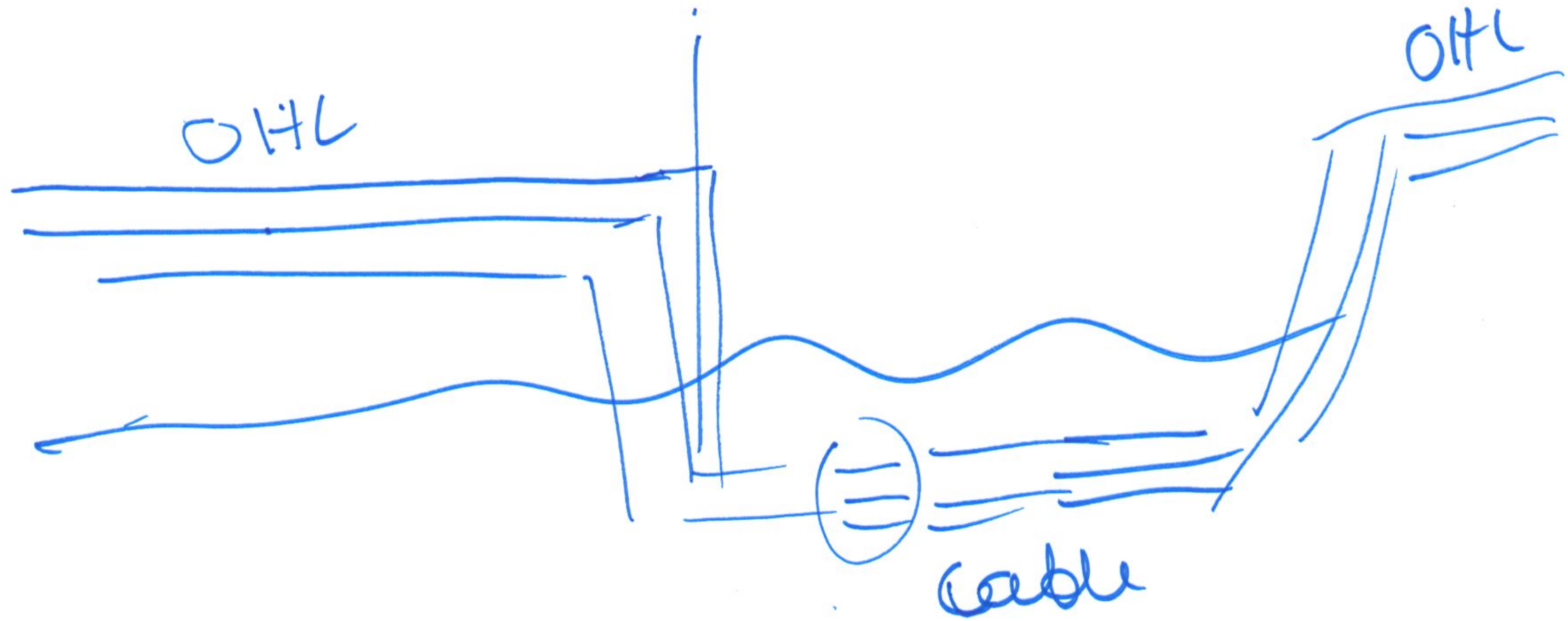
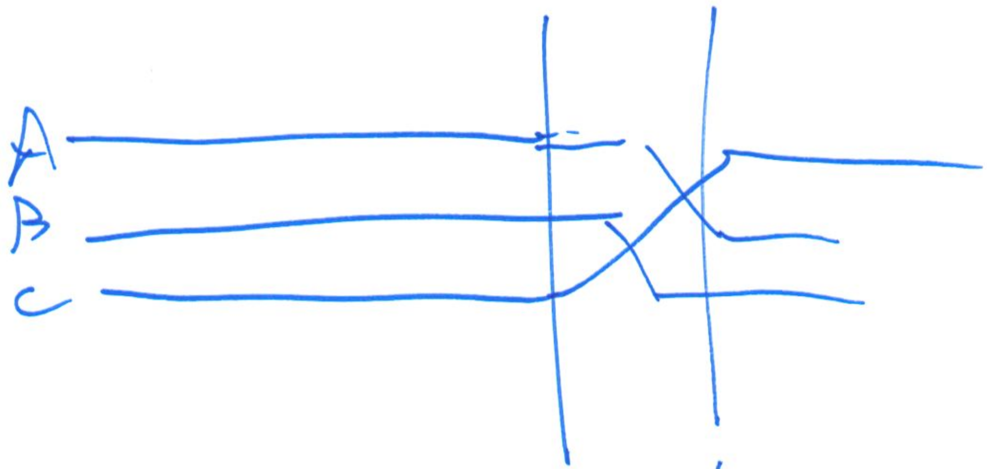
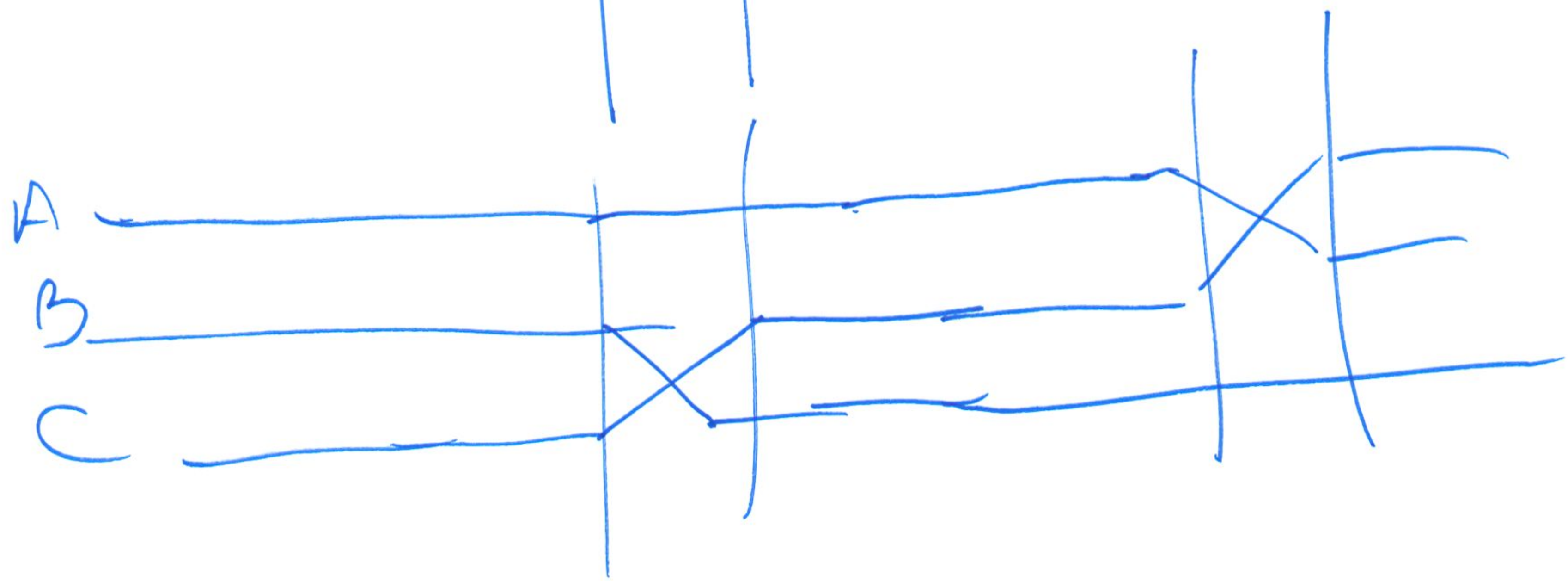



A
B
C


A
B
C