

ECE 523
Symmetrical Components

Session 24

ECE 523: Homework #5

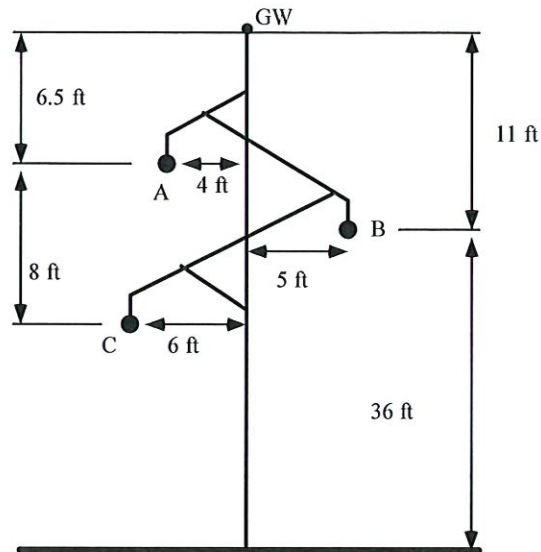
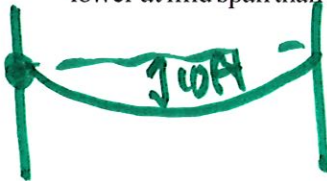
Due Session 27 (November 28)

1. Compute the per mile positive and negative sequence impedance for the line configuration of figure below where the conductor is 336,400 CM, 26/7 Strand ACSR. Ignore the ground wire for problems 1-4.

Conductor data from table:

GMR := 0.0244ft diameter := 0.721 in
 Rac := $0.278 \frac{\text{ohm}}{\text{mi}}$ at 25C and 60Hz

Assume each conductor is 10 feet lower at mid span than at tower.



2. Compute the phase impedance matrix Z_{abc} for the line described in problem 1. Assume that the line is 70 miles long and is not transposed. Ignore the ground wire. Calculate the sequence impedance matrix.

3. Compute the total impedance matrix Z_{abc} for the lines of problem 2 with the following transposition arrangements. Calculate the sequence impedance matrix for each.

<i>Fraction</i>	<i>Configuration</i>
(a) f1 = 0.20	a-b-c
f2 = 0.80	b-c-a
f3 = 0.00	c-a-b
(b) f1 = 0.30	a-b-c
f2 = 0.60	c-a-b
f3 = 0.10	c-b-a
(c) f1 = 1/3	a-b-c
f2 = 1/3	c-a-b
f3 = 1/3	b-c-a

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4 Consider the line configuration shown in the figure for problem 1. Instead of using a single conductor of 336,400 CM ACSR in each phase, with current carrying capacity of 530 amperes, suppose that each phase consists of a two-conductor bundle of two 3/0 ACSR conductors with capacity of 300 amperes/conductor. Let the two conductors of each bundle be separated by 1.0ft vertically. Assume same sag as for problem 1.

(a) Compute the 6x6 phase impedance matrix Z_{abc} for the bundled conductor configuration and reduce it to the 3x3 equivalent and compare with the previous solution (problem 2).

$$R_{ac4} := 0.560 \frac{\text{ohm}}{\text{mi}} \quad \text{from table}$$

$$GMR4 := 0.01404\text{ft} \quad \text{diameter4} := 0.502\text{in}$$

(b) Calculate geometric mean radius of the bundle and use the 3x3 matrix method. This is an approximation of the 6x6 matrix approach. Compare the results to part (a).

(c) Compute the sequence impedance matrix for part (a) and compare to problem 2.

5. Consider an untransposed line described in problem 2 with a ground wire added. Let the ground wire be 1/0 ACSR and recalculate the phase impedance matrix Z_{abc} , the sequence impedance matrix Z_{012} , and the unbalance factors. Compare with previous results from problem 2 for the same line without the ground wire. Assume phase conductors have same sag as problem 1, and that the groundwire is 7 feet lower at mid span than at the tower.

- Ground wire data:

$$R_{ac_gw} := 0.888 \frac{\text{ohm}}{\text{mi}} \quad \text{at 25C and 60Hz}$$

$$GMR_gw := 0.01113\text{ft} \quad \text{diameter_gw} := .398\text{in}$$

- Phase conductors same as in problem 2.

6. Repeat problem 5 with the transposition of problem 3, part (c).

7. Repeat problems 1-2 calculating capacitance.

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h27

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$$Z_{\text{line}} = \begin{pmatrix} 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 23.965i \\ 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 27.33i \\ 3.812 + 23.965i & 3.812 + 27.33i & 14.932 + 58.374i \end{pmatrix} \Omega$$

$$a := 1 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}}$$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix}$$

$$Z_{012} := A_{012}^{-1} \cdot Z_{\text{line}} \cdot A_{012}$$

$$Z_{012} = \begin{pmatrix} 22.5555 + 110.7903i & 0.9712 - 0.5607i & -0.9712 - 0.5607i \\ -0.9712 - 0.5607i & 11.12 + 32.1661i & -1.9424 + 1.1214i \\ 0.9712 - 0.5607i & 1.9424 + 1.1214i & 11.12 + 32.1661i \end{pmatrix} \Omega$$

angle of Z_0 differs from angle of $Z_1 = Z_2$

By comparison:

$$\rightarrow D_m := (D_{ab} \cdot D_{bc} \cdot D_{ac})^{\frac{1}{3}} \quad \text{geometric mean distance}$$

$$Z_1 := \left(R_{ac} + j \cdot 2 \cdot \pi \cdot \text{freq} \cdot \frac{\mu_0}{2 \cdot \pi} \cdot \ln \left(\frac{D_m}{D_s} \right) \right) \cdot 40 \text{mi}$$

$$Z_1 = (11.12 + 32.1661i) \Omega$$

$$Z_{012,1,1} - Z_1 = 0 \Omega$$

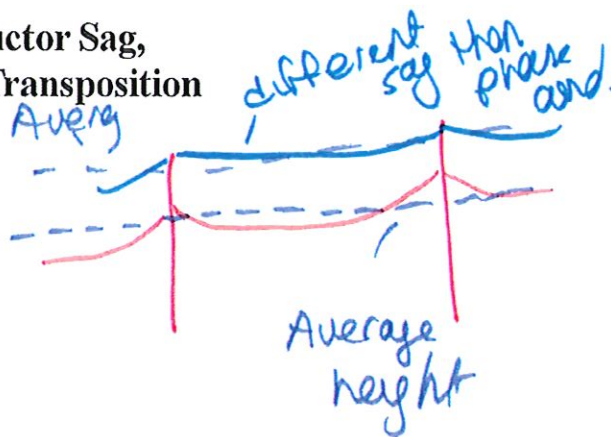
Z_{01}

Z_0

Z_1

Z_2

Representation of Conductor Sag, Rotating and Twisting for Transposition



Conductor Sag

• $H_{tower} := 77\text{ft}$

• $Mid := 33\text{ft}$ - height at mid span

$sag := H_{tower} - Mid$ $sag = 44\text{ft}$

— $H_{ave1} := H_{tower} - \frac{2}{3} \cdot sag$ $H_{ave1} = 47.66667\text{ft}$

$H_{ave2} := Mid + \frac{1}{3} \cdot sag$ $H_{ave2} = 47.66667\text{ft}$

— $H_{ave3} := \frac{H_{tower}}{3} + \frac{2}{3} \cdot Mid$ $H_{ave3} = 47.66667\text{ft}$

Rotation

$$Z_{123} := \begin{pmatrix} 100 & 12 & 13 \\ 21 & 200 & 23 \\ 31 & 32 & 300 \end{pmatrix} \quad V_{123} := \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$R_p := \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$V_{312} := R_p \cdot V_{123} \quad V_{312} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

$$V_{231} := R_p^{-1} \cdot V_{123} \quad V_{231} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

use this on impedance matrix too

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$$R_p \cdot Z_{123} = \begin{pmatrix} 31 & 32 & 300 \\ 100 & 12 & 13 \\ 21 & 200 & 23 \end{pmatrix}$$

premultiply
swaps rows

$$Z_{123} \cdot R_p^{-1} = \begin{pmatrix} 13 & 100 & 12 \\ 23 & 21 & 200 \\ 300 & 31 & 32 \end{pmatrix}$$

post multiply
swap columns

$$Z_{312} := R_p \cdot Z_{123} \cdot R_p^{-1}$$

swap rows
& cols

$$Z_{312} = \begin{pmatrix} 300 & 31 & 32 \\ 13 & 100 & 12 \\ 23 & 21 & 200 \end{pmatrix}$$

3 1 2
3
1
2

$$R_p^{-1} \cdot Z_{123} = \begin{pmatrix} 21 & 200 & 23 \\ 31 & 32 & 300 \\ 100 & 12 & 13 \end{pmatrix}$$

$$Z_{123} \cdot R_p = \begin{pmatrix} 12 & 13 & 100 \\ 200 & 23 & 21 \\ 32 & 300 & 31 \end{pmatrix}$$

$$Z_{231} := R_p^{-1} \cdot Z_{123} \cdot R_p$$

$$Z_{231} = \begin{pmatrix} 200 & 23 & 21 \\ 32 & 300 & 31 \\ 12 & 13 & 100 \end{pmatrix}$$

2 3 1
2
3
1

$$f_1 := \frac{1}{3}$$

$$f_2 := \frac{1}{3}$$

$$f_3 := \frac{1}{3}$$

$$f_1 + f_2 + f_3 = 1$$

$$Z_{eq} := f_1 \cdot Z_{123} + f_2 \cdot Z_{231} + f_3 \cdot Z_{312} \quad Z_{eq} = \begin{pmatrix} 200 & 22 & 22 \\ 22 & 200 & 22 \\ 22 & 22 & 200 \end{pmatrix}$$

- Note that it is balanced, the numbers themselves don't have meaning

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• **Transposition**

$$R_p := \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Example 1:

$$f1 := 0.2 \quad f2 := 0.3 \quad f3 := 0.5$$

$$Z_{net} = \begin{pmatrix} 14.9318 + 58.3742i & 3.8118 + 25.6473i & 3.8118 + 26.6566i \\ 3.8118 + 25.6473i & 14.9318 + 58.3742i & 3.8118 + 26.3202i \\ 3.8118 + 26.6566i & 3.8118 + 26.3202i & 14.9318 + 58.3742i \end{pmatrix} \Omega$$

$$Z_{0121} := A_{012}^{-1} \cdot Z_{net} \cdot A_{012}$$

$$Z_{0121} = \begin{pmatrix} 22.5555 + 110.7903i & -0.2914 - 0.0561i & 0.2914 - 0.0561i \\ 0.2914 - 0.0561i & 11.12 + 32.1661i & 0.5827 + 0.1121i \\ -0.2914 - 0.0561i & -0.5827 + 0.1121i & 11.12 + 32.1661i \end{pmatrix} \Omega$$

*off diagonal swapped
(E0, E1, E2)*

Example 2:

$$f13 := 0.4 \quad f23 := 0.6 \quad f33 := 0.0$$

$$Z_{net3} := f13 \cdot Z_{line} + f23 \cdot R_p^{-1} \cdot Z_{line} \cdot R_p + f33 \cdot R_p \cdot Z_{line} \cdot R_p^{-1}$$

$$Z_{net3} = \begin{pmatrix} 14.9318 + 58.3742i & 3.8118 + 27.3295i & 3.8118 + 25.9838i \\ 3.8118 + 27.3295i & 14.9318 + 58.3742i & 3.8118 + 25.3109i \\ 3.8118 + 25.9838i & 3.8118 + 25.3109i & 14.9318 + 58.3742i \end{pmatrix} \Omega$$

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Measure of imbalance

define unit: $mS := \frac{S}{1000}$

Original System

$Y_{012} := Z_{012}^{-1}$

$$Y_{012} = \begin{pmatrix} 1.7687 - 8.6713i & 0.3107 + 0.0104i & -0.1464 - 0.2743i \\ -0.1464 - 0.2743i & 9.7166 - 27.8455i & -1.9327 - 0.2845i \\ 0.3107 + 0.0104i & 0.72 + 1.816i & 9.7166 - 27.8455i \end{pmatrix} \cdot mS$$

First transposed case:

$Y_{0121} := Z_{0121}^{-1}$

$$Y_{0121} = \begin{pmatrix} 1.7647 - 8.6671i & -0.0572 - 0.0523i & 0.0739 + 0.0256i \\ 0.0739 + 0.0256i & 9.6082 - 27.7751i & 0.3356 + 0.3876i \\ -0.0572 - 0.0523i & -0.4562 - 0.2345i & 9.6082 - 27.7751i \end{pmatrix} \cdot mS$$

Second transposed case:

$Y_{0123} := Z_{0123}^{-1}$

$$Y_{0123} = \begin{pmatrix} 1.7655 - 8.668i & 0.0235 + 0.1483i & -0.1436 + 0.0535i \\ -0.1436 + 0.0535i & 9.6322 - 27.7908i & -0.0464 - 1.0228i \\ 0.0235 + 0.1483i & 1.0058 - 0.1965i & 9.6322 - 27.7908i \end{pmatrix} \cdot mS$$

calculate series
Y012 same as
(not Y012 balance)
shunt capacitance
from

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Fully Transposed Case

$$Y_{0126} := Z_{0126}^{-1}$$

$$Y_{0126} = \begin{pmatrix} 1.7645 - 8.6668i & 0 & 0 \\ 0 & 9.6002 - 27.7698i & 0 \\ 0 & 0 & 9.6002 - 27.7698i \end{pmatrix} \cdot \text{mS}$$

pos to zero
pos to zero

Imbalance factors:

$$M_{0_original} := \frac{Y_{012_{0,1}}}{Y_{012_{1,1}}} \quad |M_{0_original}| = 0.0105$$

$$M_{2_original} := \frac{Y_{012_{2,1}}}{Y_{012_{1,1}}} \quad |M_{2_original}| = 0.0662$$

$$M_{0_trans1} := \frac{Y_{0121_{0,1}}}{Y_{0121_{1,1}}} \quad |M_{0_trans1}| = 2.6354 \times 10^{-3}$$

$$M_{2_trans1} := \frac{Y_{0121_{2,1}}}{Y_{0121_{1,1}}} \quad |M_{2_trans1}| = 0.0175$$

$$M_{0_trans2} := \frac{Y_{0123_{0,1}}}{Y_{0123_{1,1}}} \quad |M_{0_trans2}| = 5.1064 \times 10^{-3}$$

$$M_{2_trans2} := \frac{Y_{0123_{2,1}}}{Y_{0123_{1,1}}} \quad |M_{2_trans2}| = 0.0348$$

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$$M_{0_trans3} := \frac{Y_{0126_{0,1}}}{Y_{0126_{1,1}}} \quad |M_{0_trans3}| = 0$$

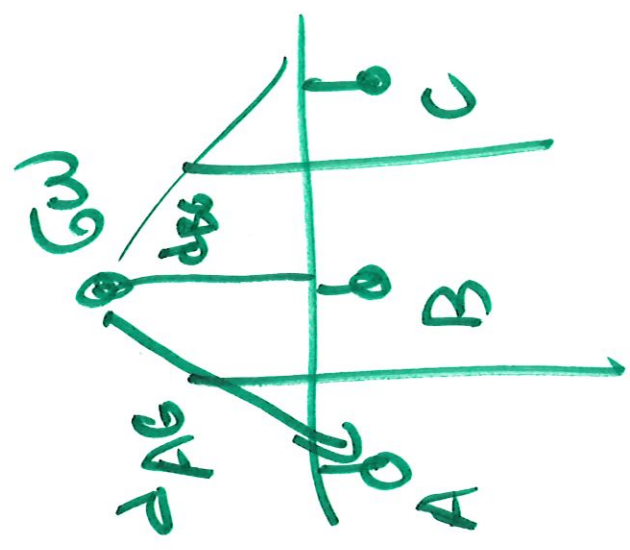
$$M_{2_trans3} := \frac{Y_{0126_{2,1}}}{Y_{0126_{1,1}}} \quad |M_{2_trans3}| = 0$$

B. Ground Wire Added *- steel*
AC Resistance from table (not available in our textbook):

$$R_{gw} := 4.0 \frac{\text{ohm}}{\text{mi}} \quad R_{selfgw} := R_{gw} + R_d =$$

$$R' := \begin{pmatrix} R_{self} & R_d & R_d & R_d \\ R_d & R_{self} & R_d & R_d \\ R_d & R_d & R_{self} & R_d \\ R_d & R_d & R_d & R_{selfgw} \end{pmatrix} \quad \text{Ground}$$

$$R' = \begin{pmatrix} 0.3733 & 0.0953 & 0.0953 & 0.0953 \\ 0.0953 & 0.3733 & 0.0953 & 0.0953 \\ 0.0953 & 0.0953 & 0.3733 & 0.0953 \\ 0.0953 & 0.0953 & 0.0953 & 4.0953 \end{pmatrix} \frac{\text{ohm}}{\text{mi}}$$



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Calculate GMR from conductor diameter

$$\text{dia} := 0.528 \text{ in}$$

$$\text{GMR} := e^{-\frac{1}{4}} \cdot \frac{\text{dia}}{2} \quad \text{GMR} = 0.01713 \cdot \text{ft}$$

$D_{sgw} := 0.01 \text{ ft}$ From table (not available in the appendices in Anderson)

$$D_{agw} := \sqrt{(10 \text{ ft})^2 + (15 \text{ ft})^2} \quad D_{cgw} := D_{agw} \quad D_{bgw} := 15 \text{ ft}$$

$$L' := \frac{\mu_0}{2 \cdot \pi} \cdot \begin{pmatrix} \ln\left(\frac{D_e}{D_s}\right) & \ln\left(\frac{D_e}{D_{ab}}\right) & \ln\left(\frac{D_e}{D_{ac}}\right) & \ln\left(\frac{D_e}{D_{agw}}\right) \\ \ln\left(\frac{D_e}{D_{ab}}\right) & \ln\left(\frac{D_e}{D_s}\right) & \ln\left(\frac{D_e}{D_{bc}}\right) & \ln\left(\frac{D_e}{D_{bgw}}\right) \\ \ln\left(\frac{D_e}{D_{ac}}\right) & \ln\left(\frac{D_e}{D_{bc}}\right) & \ln\left(\frac{D_e}{D_s}\right) & \ln\left(\frac{D_e}{D_{cgw}}\right) \\ \ln\left(\frac{D_e}{D_{agw}}\right) & \ln\left(\frac{D_e}{D_{bgw}}\right) & \ln\left(\frac{D_e}{D_{cgw}}\right) & \ln\left(\frac{D_e}{D_{sgw}}\right) \end{pmatrix}$$

A B C Gw

$$L' = \begin{pmatrix} 3.8711 & 1.8123 & 1.5892 & 1.6227 \\ 1.8123 & 3.8711 & 1.8123 & 1.6818 \\ 1.5892 & 1.8123 & 3.8711 & 1.6227 \\ 1.6227 & 1.6818 & 1.6227 & 4.0357 \end{pmatrix} \frac{\text{mH}}{\text{mi}}$$

$$Z'_{gw} := R' + j \cdot 2 \cdot \pi \cdot \text{freq} \cdot L'$$

$$Z'_{gw} = \begin{pmatrix} 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.599i & 0.095 + 0.612i \\ 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.634i \\ 0.095 + 0.599i & 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.612i \\ 0.095 + 0.612i & 0.095 + 0.634i & 0.095 + 0.612i & 4.095 + 1.521i \end{pmatrix} \frac{\text{ohm}}{\text{mi}}$$

$$\begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \\ \cancel{V_{Sw-G}} \end{bmatrix} = \begin{bmatrix} Z_A & & & \\ & Z_B & & \\ & & Z_C & \\ & & & Z_D \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_{Gw} \end{bmatrix}$$

$\left. \begin{matrix} Z_A & & & \\ & Z_B & & \\ & & Z_C & \\ & & & Z_D \end{matrix} \right\} I_{ABC}$

$$\underline{V_{ABC}} = [Z_A] \cdot \underline{I_{ABC}} + [Z_B] I_{Gw}$$

$$0 = V_{GwG} = [Z_C] (\underline{I_{ABC}}) + Z_D I_{Gw}$$

$$- [Z_D] I_{Gw} = [Z_C] \underline{I_{ABC}}$$

$$I_{Gw} = - [Z_D]^{-1} [Z_C] \underline{I_{ABC}} \rightarrow \text{substitute for } I_{Gw}$$

$$\underline{V}_{ABC} = [Z_A] \underline{I}_{ABC} + [Z_B] [-[Z_D][Z_C]^{-1}] \underline{I}_{ABC}$$

$$= \underbrace{[Z_A] - [Z_B][Z_D]^{-1}[Z_C]}_{[Z_{ABC\phi}]} \underline{I}_{ABC}$$

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If length = 40 miles: $Z_{linegw} := Z_{gw} \cdot 40 \text{mi}$

$$Z_{linegw} = \begin{pmatrix} 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 23.965i & 3.812 + 24.469i \\ 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 25.362i \\ 3.812 + 23.965i & 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 24.469i \\ 3.812 + 24.469i & 3.812 + 25.362i & 3.812 + 24.469i & 163.842 + 60.857i \end{pmatrix} \Omega$$

Reduce to 3x3
we have connection
ONLY have connection
A, B, C terminated
at towers
- GW is surrounded
even
- continuous

Reduce to equivalent 3x3 matrix by removing 4th row and column

Kron Reduct

$Z_a := \text{submatrix}(Z_{linegw}, 0, 2, 0, 2)$ $Z_b := \text{submatrix}(Z_{linegw}, 0, 2, 3, 3)$

$Z_c := \text{submatrix}(Z_{linegw}, 3, 3, 0, 2)$ $Z_d := \text{submatrix}(Z_{linegw}, 3, 3, 3, 3)$

$Z_{abceq} := Z_a - Z_b \cdot Z_d^{-1} \cdot Z_c$

$$Z_{abceq} = \begin{pmatrix} 17.6939 + 56.2093i & 6.6842 + 25.1028i & 6.5739 + 21.8003i \\ 6.6842 + 25.1028i & 17.9189 + 56.0842i & 6.6842 + 25.1028i \\ 6.5739 + 21.8003i & 6.6842 + 25.1028i & 17.6939 + 56.2093i \end{pmatrix} \Omega$$

$Z_{012GW} := A_{012}^{-1} Z_{abceq} A_{012}$

$$Z_{012GW} = \begin{pmatrix} 31.0638 + 104.1716i & 0.8614 - 0.6264i & -0.9731 - 0.4328i \\ -0.9731 - 0.4328i & 11.1214 + 32.1656i & -1.9436 + 1.1205i \\ 0.8614 - 0.6264i & 1.9421 + 1.1229i & 11.1214 + 32.1656i \end{pmatrix} \Omega$$

small change in Z_b & Z_c

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same as original 343 600
w/ 0

For comparison, without the groundwire:

$$Z_{012old} := A_{012}^{-1} \cdot Z_a \cdot A_{012}$$

$$Z_{012old} = \begin{pmatrix} 22.5555 + 110.7903i & 0.9712 - 0.5607i & -0.9712 - 0.5607i \\ -0.9712 - 0.5607i & 11.12 + 32.1661i & -1.9424 + 1.1214i \\ 0.9712 - 0.5607i & 1.9424 + 1.1214i & 11.12 + 32.1661i \end{pmatrix} \Omega$$

As a check:

$$\overrightarrow{|Z_{012GW} - Z_{012old}|} = \begin{pmatrix} 10.7796 & 0.128 & 0.128 \\ 0.128 & 1.5191 \times 10^{-3} & 1.5191 \times 10^{-3} \\ 0.128 & 1.5191 \times 10^{-3} & 1.5191 \times 10^{-3} \end{pmatrix} \Omega$$

Unbalance Factors:

$$Y_{012GW} := Z_{012GW}^{-1}$$

$$M_{0gw} := \frac{Y_{012GW_{0,1}}}{Y_{012GW_{1,1}}} \quad |M_{0gw}| = 0.0104$$

Compare to case without GW:

$$|M_{0_original}| = 0.0105 \quad \text{Small change}$$

$$\left| \frac{|M_{0_original}| - |M_{0gw}|}{|M_{0_original}|} \right| = 1.2224\%$$

zero
square
unchanged

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$$M2_{gw} := \frac{Y012_{GW_{2,1}}}{Y012_{GW_{1,1}}}$$

$$|M2_{gw}| = 0.0662$$

3 } neg self
imbalance step up 6W
factor same

Compare to case without GW:

$$|M2_{original}| = 0.0662 \quad \text{No change}$$

$$\left| \frac{|M2_{original}| - |M2_{gw}|}{|M2_{original}|} \right| = 1.4306 \times 10^{-3} \cdot \%$$

C. Example with Two Conductor Bundles (no ground wire). Conductors are 795 kCMIL ACSR, 18 inches apart

AC Resistance from table $R_{acbund} := 0.117 \frac{\text{ohm}}{\text{mi}}$ at 25 C and $\text{freq} := 60\text{Hz}$

$$R_{selfbund} := R_{acbund} + R_d \quad R_{selfbund} = 0.2123 \cdot \frac{\text{ohm}}{\text{mi}}$$

$$R' := \begin{pmatrix} R_{selfbund} & R_d & R_d & R_d & R_d & R_d \\ R_d & R_{selfbund} & R_d & R_d & R_d & R_d \\ R_d & R_d & R_{selfbund} & R_d & R_d & R_d \\ R_d & R_d & R_d & R_{selfbund} & R_d & R_d \\ R_d & R_d & R_d & R_d & R_{selfbund} & R_d \\ R_d & R_d & R_d & R_d & R_d & R_{selfbund} \end{pmatrix}$$