ECE 523
Symmetrical Components

Session 25

L25 1/21 ~~L24 15/15~~

$$M2_{gw} := \frac{Y012GW_{2,1}}{Y012GW_{1,1}} \qquad |M2_{gw}| = 0.0662$$

neg seq imbalance factor stays same w/ GW

Compare to case without GW:

$$|M_{2\_original}| = 0.0662 \qquad \text{No change}$$

$$\left| \frac{|M_{2\_original}| - |M2_{gw}|}{|M_{2\_original}|} \right| = 1.4306 \times 10^{-3} \cdot \%$$

C. Example with Two Conductor Bundles (no ground wire). Conductors are 795 kCMIL ACSR, 18 inches apart

AC Resistance from table $\quad Rac_{bund} := 0.117 \frac{ohm}{mi}$ at 25 C and $\quad freq := 60Hz$

$$Rself_{bund} := Rac_{bund} + R_d \qquad Rself_{bund} = 0.2123 \cdot \frac{ohm}{mi}$$

$$R' := \begin{pmatrix} Rself_{bund} & R_d & R_d & R_d & R_d & R_d \\ R_d & Rself_{bund} & R_d & R_d & R_d & R_d \\ R_d & R_d & Rself_{bund} & R_d & R_d & R_d \\ R_d & R_d & R_d & Rself_{bund} & R_d & R_d \\ R_d & R_d & R_d & R_d & Rself_{bund} & R_d \\ R_d & R_d & R_d & R_d & R_d & Rself_{bund} \end{pmatrix}$$

L25 2/21

Conductor GMR from table:

Ds := 0.0375ft

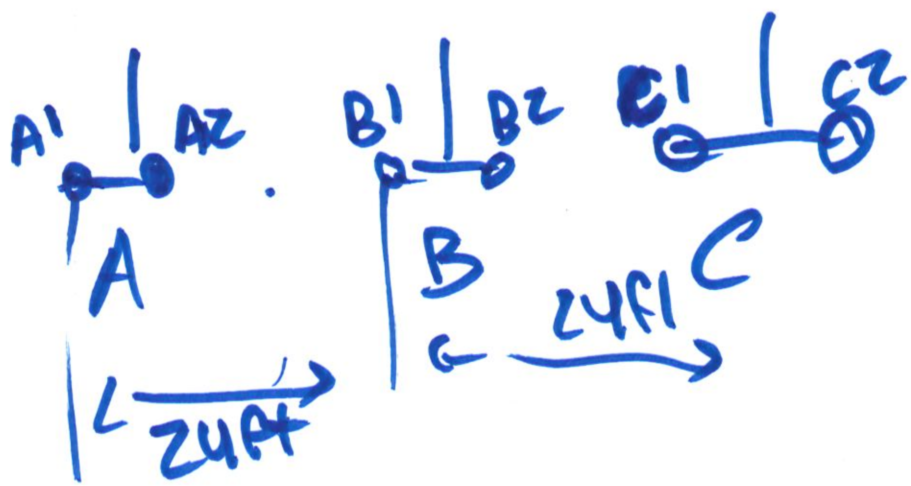
Spacing:

Within the bundle:

Da1a2 := 1.5ft	Db1b2 := 1.5ft	Dc1c2 := 1.5ft

Between Phases

Da1b1 := 24ft	Da1b2 := Da1b1 + Db1b2	Da1b2 = 25.5·ft
Da2b1 := Da1b1 − Da1a2	Da2b1 = 22.5·ft	Da2b2 := 24ft
Da1c1 := 48ft	Da1c2 := Da1c1 + Dc1c2	Da1c2 = 49.5·ft
Da2c1 := Da1c1 − Da1a2	Da2c1 = 46.5·ft	Da2c2 := 48ft
Db1c1 := 24ft	Db1c2 := Db1c1 + Dc1c2	Db1c2 = 25.5·ft
Db2c1 := Db1c1 − Db1b2	Db2c1 = 22.5·ft	Db2c2 := 24ft



L25 3/21

a1, b1, c1

a1→b2
a1-c2 etc

$$L' := \frac{\mu_0}{2\cdot\pi}\cdot\begin{pmatrix} \ln\left(\frac{D_e}{Ds}\right) & \ln\left(\frac{D_e}{Da1b1}\right) & \ln\left(\frac{D_e}{Da1c1}\right) & \ln\left(\frac{D_e}{Da1a2}\right) & \ln\left(\frac{D_e}{Da1b2}\right) & \ln\left(\frac{D_e}{Da1c2}\right) \\ \ln\left(\frac{D_e}{Da1b1}\right) & \ln\left(\frac{D_e}{Ds}\right) & \ln\left(\frac{D_e}{Db1c1}\right) & \ln\left(\frac{D_e}{Da2b1}\right) & \ln\left(\frac{D_e}{Db1b2}\right) & \ln\left(\frac{D_e}{Db1c2}\right) \\ \ln\left(\frac{D_e}{Da1c1}\right) & \ln\left(\frac{D_e}{Db1c1}\right) & \ln\left(\frac{D_e}{Ds}\right) & \ln\left(\frac{D_e}{Da2c1}\right) & \ln\left(\frac{D_e}{Db2c1}\right) & \ln\left(\frac{D_e}{Dc1c2}\right) \\ \ln\left(\frac{D_e}{Da1a2}\right) & \ln\left(\frac{D_e}{Da2b1}\right) & \ln\left(\frac{D_e}{Da2c1}\right) & \ln\left(\frac{D_e}{Ds}\right) & \ln\left(\frac{D_e}{Da2b2}\right) & \ln\left(\frac{D_e}{Da2c2}\right) \\ \ln\left(\frac{D_e}{Da1b2}\right) & \ln\left(\frac{D_e}{Db1b2}\right) & \ln\left(\frac{D_e}{Db2c1}\right) & \ln\left(\frac{D_e}{Da2b2}\right) & \ln\left(\frac{D_e}{Ds}\right) & \ln\left(\frac{D_e}{Db2c2}\right) \\ \ln\left(\frac{D_e}{Da1c2}\right) & \ln\left(\frac{D_e}{Db1c2}\right) & \ln\left(\frac{D_e}{Dc1c2}\right) & \ln\left(\frac{D_e}{Da2c2}\right) & \ln\left(\frac{D_e}{Db2c2}\right) & \ln\left(\frac{D_e}{Ds}\right) \end{pmatrix}$$

1-2

2-2

$$Z' := R' + j\cdot 2\cdot\pi\cdot freq\cdot L'$$

$$Z' = \begin{pmatrix} 0.212+1.361i & 0.095+0.577i & 0.095+0.493i & 0.095+0.913i & 0.095+0.57i & 0.095+0.489i \\ 0.095+0.577i & 0.212+1.361i & 0.095+0.577i & 0.095+0.585i & 0.095+0.913i & 0.095+0.57i \\ 0.095+0.493i & 0.095+0.577i & 0.212+1.361i & 0.095+0.497i & 0.095+0.585i & 0.095+0.913i \\ 0.095+0.913i & 0.095+0.585i & 0.095+0.497i & 0.212+1.361i & 0.095+0.577i & 0.095+0.493i \\ 0.095+0.57i & 0.095+0.913i & 0.095+0.585i & 0.095+0.577i & 0.212+1.361i & 0.095+0.577i \\ 0.095+0.489i & 0.095+0.57i & 0.095+0.913i & 0.095+0.493i & 0.095+0.577i & 0.212+1.361i \end{pmatrix}\cdot\frac{ohm}{mi}$$

L25 4/21

If length = 40 miles: $Zline := Z' \cdot 40mi$

Za Zb

$$Zline = \left(\begin{array}{ccc|ccc} 8.492+54.442i & 3.812+23.08i & 3.812+19.716i & 3.812+36.537i & 3.812+22.786i & 3.812+19.567i \\ 3.812+23.08i & 8.492+54.442i & 3.812+23.08i & 3.812+23.394i & 3.812+36.537i & 3.812+22.786i \\ 3.812+19.716i & 3.812+23.08i & 8.492+54.442i & 3.812+19.87i & 3.812+23.394i & 3.812+36.537i \\ \hline 3.812+36.537i & 3.812+23.394i & 3.812+19.87i & 8.492+54.442i & 3.812+23.08i & 3.812+19.716i \\ 3.812+22.786i & 3.812+36.537i & 3.812+23.394i & 3.812+23.08i & 8.492+54.442i & 3.812+23.08i \\ 3.812+19.567i & 3.812+22.786i & 3.812+36.537i & 3.812+19.716i & 3.812+23.08i & 8.492+54.442i \end{array}\right)\Omega$$

Zc Zd

3X3 equivalent

$Za := \mathrm{submatrix}(Z', 0, 2, 0, 2)$

$$Za = \begin{pmatrix} 0.2123+1.3611i & 0.0953+0.577i & 0.0953+0.4929i \\ 0.0953+0.577i & 0.2123+1.3611i & 0.0953+0.577i \\ 0.0953+0.4929i & 0.0953+0.577i & 0.2123+1.3611i \end{pmatrix} \cdot \frac{ohm}{mi}$$

$Zb := \mathrm{submatrix}(Z', 0, 2, 3, 5)$

$$Zb = \begin{pmatrix} 0.0953+0.9134i & 0.0953+0.5697i & 0.0953+0.4892i \\ 0.0953+0.5848i & 0.0953+0.9134i & 0.0953+0.5697i \\ 0.0953+0.4968i & 0.0953+0.5848i & 0.0953+0.9134i \end{pmatrix} \cdot \frac{ohm}{mi}$$

$Zc := \mathrm{submatrix}(Z', 3, 5, 0, 2)$

$$Zc = \begin{pmatrix} 0.0953+0.9134i & 0.0953+0.5848i & 0.0953+0.4968i \\ 0.0953+0.5697i & 0.0953+0.9134i & 0.0953+0.5848i \\ 0.0953+0.4892i & 0.0953+0.5697i & 0.0953+0.9134i \end{pmatrix} \cdot \frac{ohm}{mi}$$

L25 5/21

$$\begin{bmatrix} V_{A1} \\ V_{B1} \\ V_{C1} \\ V_{A2} \\ V_{B2} \\ V_{C2} \end{bmatrix} = \left[\begin{array}{c|c} Z_A & Z_B \\ \hline Z_C & Z_D \end{array}\right] \begin{bmatrix} I_{A1} \\ I_{B1} \\ I_{C1} \\ I_{A2} \\ I_{B2} \\ I_{C2} \end{bmatrix}$$

Rearrange so

$$\begin{bmatrix} V_{A1} \\ V_{B1} \\ V_{C1} \\ V_{A1}-V_{A2} \\ V_{B1}-V_{B2} \\ V_{C1}-V_{C2} \end{bmatrix} = \left[\begin{array}{c|c} Z_A & Z_{Bnew} \\ \hline Z_{Cnew} & Z_{Dnew} \end{array}\right] \begin{bmatrix} I_{A1} \\ I_{B1} \\ I_{C1} \\ I_{A2new} \\ I_{B2new} \\ I_{C2new} \end{bmatrix}$$

$V_{A1}-V_{A2} = 0$, $V_{B1}-V_{B2} = 0$, $V_{C1}-V_{C2} = 0$

$I_{A1} \approx I_{A2}$

$I_{A2new} = I_A + I_{A2}$

$I_{Bnew} = I_{B1} + I_{B2}$

LZS 6/21

$Zd := \text{submatrix}(Z', 3, 5, 3, 5)$

$$Zd = \begin{pmatrix} 0.2123 + 1.3611i & 0.0953 + 0.577i & 0.0953 + 0.4929i \\ 0.0953 + 0.577i & 0.2123 + 1.3611i & 0.0953 + 0.577i \\ 0.0953 + 0.4929i & 0.0953 + 0.577i & 0.2123 + 1.3611i \end{pmatrix} \cdot \frac{\text{ohm}}{\text{mi}}$$

- Modify matrix by performing: Va'b'c'-Vabc and Iabc + Ia'b'c'

$Zbnew := Zb - Za$

$Zcnew := Zc - Za$

$Zdnew := Za - Zb - Zc + Zd$ From textbook

$$Zdnew = \begin{pmatrix} 0.234 + 0.8952i & -4.7492i \times 10^{-4} & -1.1856i \times 10^{-4} \\ -4.7492i \times 10^{-4} & 0.234 + 0.8952i & -4.7492i \times 10^{-4} \\ -1.1856i \times 10^{-4} & -4.7492i \times 10^{-4} & 0.234 + 0.8952i \end{pmatrix} \cdot \frac{\text{ohm}}{\text{mi}}$$

$Zequiv := Za - Zbnew \cdot Zdnew^{-1} \cdot Zcnew$ - same as we had with GW

$$Zequiv = \begin{pmatrix} 0.1538 + 1.1372i & 0.0953 + 0.5771i & 0.0953 + 0.493i \\ 0.0953 + 0.5771i & 0.1538 + 1.1371i & 0.0953 + 0.5771i \\ 0.0953 + 0.493i & 0.0953 + 0.5771i & 0.1538 + 1.1372i \end{pmatrix} \cdot \frac{\text{ohm}}{\text{mi}}$$

LZS 7/21

$Z_{line\_bund} := Zequiv \cdot 40mi$

$$Z_{line\_bund} = \begin{pmatrix} 6.1526 + 45.4868i & 3.8121 + 23.0838i & 3.8112 + 19.7196i \\ 3.8121 + 23.0838i & 6.1531 + 45.485i & 3.8121 + 23.0838i \\ 3.8112 + 19.7196i & 3.8121 + 23.0838i & 6.1526 + 45.4868i \end{pmatrix} \Omega$$

effective GMR is larger

$L' \propto \ln\left(\frac{Dist}{Rad}\right)$

$Z012_{bund} := A_{012}^{-1} \cdot Z_{line\_bund} \cdot A_{012}$

$$Z012_{bund} = \begin{pmatrix} \underset{Z_0}{13.7764 + 89.411i} & 0.9704 - 0.5608i & -0.9709 - 0.56i \\ -0.9709 - 0.56i & \overset{Z_1}{2.3409 + 23.5238i} & -1.9426 + 1.1221i \\ 0.9704 - 0.5608i & 1.9431 + 1.1213i & \underset{Z_2}{2.3409 + 23.5238i} \end{pmatrix} \cdot ohm$$

- single conductor about same total ampacity

Compare to:

$$Z_{012} = \begin{pmatrix} 22.5555 + 110.7903i & 0.9712 - 0.5607i & -0.9712 - 0.5607i \\ -0.9712 - 0.5607i & 11.12 + 32.1661i & -1.9424 + 1.1214i \\ 0.9712 - 0.5607i & 1.9424 + 1.1214i & 11.12 + 32.1661i \end{pmatrix} \Omega$$

- Note that the off diagonal terms are almost identical, but the diagonal terms are smaller, due to the bundling.
- Compare positive sequence resistance to ac resistance of conductors.

$Rac_{bund} \cdot 40mi = 4.68\,\Omega$ — one conductor from Bundle

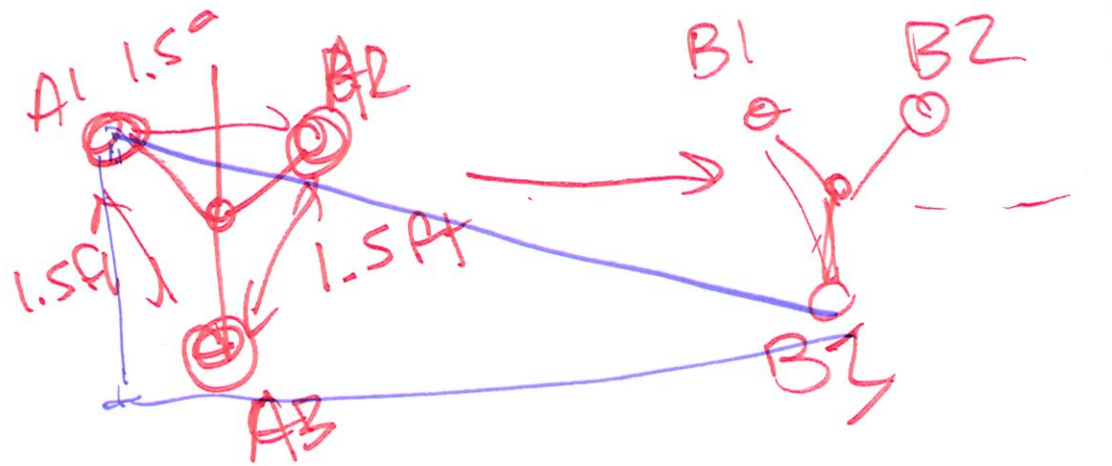
$Re\left(Z012_{bund_{1,1}}\right) = 2.3409\,\Omega$

L25 8/21

Conductor GMR from table:

$Ds := 0.0375ft$

Spacing:



Within the bundle:

$Da1a2 := 1.5ft$ $Db1b2 := 1.5ft$ $Dc1c2 := 1.5ft$

$Da1a3 := 1.5ft$ $Db1b3 := 1.5ft$ $Dc1c3 := 1.5ft$

$Da2a3 := 1.5ft$ $Db2b3 := 1.5ft$ $Dc2c3 := 1.5ft$

Between Phases $vert := \sqrt{18^2 - 9^2}\,in$ $vert = 1.299\cdot ft$

$Da1b1 := 24ft$ $Da1b2 := Da1b1 + Db1b2$ $Da1b2 = 25.5\cdot ft$

$Da1b3 := \sqrt{24.75^2 + 1.299^2}\,ft$ $Da1b3 = 24.7841\cdot ft$

$Da2b1 := Da1b1 - Da1a2$ $Da2b1 = 22.5\cdot ft$ $Da2b2 := 24ft$

$Da2b3 := \sqrt{23.25^2 + 1.299^2}\,ft$ $Da2b3 = 23.2863\cdot ft$

$Da3b1 := Da2b3$ $Da3b2 := Da1b3$ $Da3b3 := 24ft$

$Da1c1 := 48ft$ $Da1c2 := Da1c1 + Dc1c2$ $Da1c2 = 49.5\cdot ft$

$Da1c3 := \sqrt{48.75^2 + 1.299^2}\,ft$ $Da1c3 = 48.7673\cdot ft$

LZS 9/21

$$Za := \mathrm{submatrix}(Z',0,2,0,2) \qquad Za = \begin{pmatrix} 0.2123+1.3611i & 0.0953+0.577i & 0.0953+0.4929i \\ 0.0953+0.577i & 0.2123+1.3611i & 0.0953+0.577i \\ 0.0953+0.4929i & 0.0953+0.577i & 0.2123+1.3611i \end{pmatrix} \cdot \frac{\mathrm{ohm}}{\mathrm{mi}}$$

$$Zb1 := \mathrm{submatrix}(Z',0,2,3,5) \qquad Zb1 = \begin{pmatrix} 0.0953+0.9134i & 0.0953+0.5697i & 0.0953+0.4892i \\ 0.0953+0.5848i & 0.0953+0.9134i & 0.0953+0.5697i \\ 0.0953+0.4968i & 0.0953+0.5848i & 0.0953+0.9134i \end{pmatrix} \cdot \frac{\mathrm{ohm}}{\mathrm{mi}}$$

$$Zb2 := \mathrm{submatrix}(Z',0,2,6,8) \qquad Zb2 = \begin{pmatrix} 0.0953+0.9134i & 0.0953+0.5731i & 0.0953+0.491i \\ 0.0953+0.5807i & 0.0953+0.9134i & 0.0953+0.5731i \\ 0.0953+0.4948i & 0.0953+0.5807i & 0.0953+0.9134i \end{pmatrix} \cdot \frac{\mathrm{ohm}}{\mathrm{mi}}$$

$$Zc1 := \mathrm{submatrix}(Z',3,5,0,2) \qquad Zc1 = \begin{pmatrix} 0.0953+0.9134i & 0.0953+0.5848i & 0.0953+0.4968i \\ 0.0953+0.5697i & 0.0953+0.9134i & 0.0953+0.5848i \\ 0.0953+0.4892i & 0.0953+0.5697i & 0.0953+0.9134i \end{pmatrix} \cdot \frac{\mathrm{ohm}}{\mathrm{mi}}$$

$$Zc2 := \mathrm{submatrix}(Z',6,8,0,2) \qquad Zc2 = \begin{pmatrix} 0.0953+0.9134i & 0.0953+0.5807i & 0.0953+0.4948i \\ 0.0953+0.5731i & 0.0953+0.9134i & 0.0953+0.5807i \\ 0.0953+0.491i & 0.0953+0.5731i & 0.0953+0.9134i \end{pmatrix} \cdot \frac{\mathrm{ohm}}{\mathrm{mi}}$$

$$Zd := \mathrm{submatrix}(Z',3,8,3,8)$$

6x6

LZS 10/21

$$Zd = \begin{pmatrix} 0.2123 + 1.3611i & 0.0953 + 0.577i & 0.0953 + 0.4929i & 0.0953 + 0.9134i & 0.0953 + 0.5807i & 0.0953 + 0.4948i \\ 0.0953 + 0.577i & 0.2123 + 1.3611i & 0.0953 + 0.577i & 0.0953 + 0.5731i & 0.0953 + 0.9134i & 0.0953 + 0.5807i \\ 0.0953 + 0.4929i & 0.0953 + 0.577i & 0.2123 + 1.3611i & 0.0953 + 0.491i & 0.0953 + 0.5731i & 0.0953 + 0.9134i \\ 0.0953 + 0.9134i & 0.0953 + 0.5731i & 0.0953 + 0.491i & 0.2123 + 1.3611i & 0.0953 + 0.577i & 0.0953 + 0.4929i \\ 0.0953 + 0.5807i & 0.0953 + 0.9134i & 0.0953 + 0.5731i & 0.0953 + 0.577i & 0.2123 + 1.3611i & 0.0953 + 0.577i \\ 0.0953 + 0.4948i & 0.0953 + 0.5807i & 0.0953 + 0.9134i & 0.0953 + 0.4929i & 0.0953 + 0.577i & 0.2123 + 1.3611i \end{pmatrix} \cdot \frac{ohm}{mi}$$

- join matrices

$Zbnew := augment(Zb1 - Za, Zb2 - Za)$

$Zcnew := stack(Zc1 - Za, Zc2 - Za)$

6 row, 3 column

$$Zbnew^{T} - Zcnew = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \frac{ohm}{mi}$$

$$Zcnew = \begin{pmatrix} -0.117 - 0.4476i & 7.8312i \times 10^{-3} & 3.8524i \times 10^{-3} \\ -7.3563i \times 10^{-3} & -0.117 - 0.4476i & 7.8312i \times 10^{-3} \\ -3.7339i \times 10^{-3} & -7.3563i \times 10^{-3} & -0.117 - 0.4476i \\ -0.117 - 0.4476i & 3.6633i \times 10^{-3} & 1.8651i \times 10^{-3} \\ -3.9008i \times 10^{-3} & -0.117 - 0.4476i & 3.6633i \times 10^{-3} \\ -1.9244i \times 10^{-3} & -3.9008i \times 10^{-3} & -0.117 - 0.4476i \end{pmatrix} \cdot \frac{ohm}{mi}$$

$Zd1 := Za - Zb1 - Zc1 + submatrix(Zd, 0, 2, 0, 2)$

$Zd2 := Za - Zb2 - Zc1 + submatrix(Zd, 0, 2, 3, 5)$

$Zd3 := Za - Zb1 - Zc2 + submatrix(Zd, 3, 5, 0, 2)$

$Zd4 := Za - Zb2 - Zc2 + submatrix(Zd, 3, 5, 3, 5)$

L25 11/21

$ZdA := \text{augment}(Zd1, Zd2)$ $\qquad$ $ZdB := \text{augment}(Zd3, Zd4)$

$$ZdA = \begin{pmatrix} 0.234+0.8952i & -4.7492i\times10^{-4} & -1.1856i\times10^{-4} & 0.117+0.4476i & -2.6708i\times10^{-4} & -6.2981i\times10^{-5} \\ -4.7492i\times10^{-4} & 0.234+0.8952i & -4.7492i\times10^{-4} & -2.0784i\times10^{-4} & 0.117+0.4476i & -2.6708i\times10^{-4} \\ -1.1856i\times10^{-4} & -4.7492i\times10^{-4} & 0.234+0.8952i & -5.5575i\times10^{-5} & -2.0784i\times10^{-4} & 0.117+0.4476i \end{pmatrix}\cdot\frac{ohm}{mi}$$

$$ZdB = \begin{pmatrix} 0.117+0.4476i & -2.0784i\times10^{-4} & -5.5575i\times10^{-5} & 0.234+0.8952i & 2.3744i\times10^{-4} & 5.9273i\times10^{-5} \\ -2.6708i\times10^{-4} & 0.117+0.4476i & -2.0784i\times10^{-4} & 2.3744i\times10^{-4} & 0.234+0.8952i & 2.3744i\times10^{-4} \\ -6.2981i\times10^{-5} & -2.6708i\times10^{-4} & 0.117+0.4476i & 5.9273i\times10^{-5} & 2.3744i\times10^{-4} & 0.234+0.8952i \end{pmatrix}\cdot\frac{ohm}{mi}$$

$Zdnew := \text{stack}(ZdA, ZdB)$

6x6

$$Zdnew = \begin{pmatrix} 0.234+0.8952i & -4.7492i\times10^{-4} & -1.1856i\times10^{-4} & 0.117+0.4476i & -2.6708i\times10^{-4} & -6.2981i\times10^{-5} \\ -4.7492i\times10^{-4} & 0.234+0.8952i & -4.7492i\times10^{-4} & -2.0784i\times10^{-4} & 0.117+0.4476i & -2.6708i\times10^{-4} \\ -1.1856i\times10^{-4} & -4.7492i\times10^{-4} & 0.234+0.8952i & -5.5575i\times10^{-5} & -2.0784i\times10^{-4} & 0.117+0.4476i \\ 0.117+0.4476i & -2.0784i\times10^{-4} & -5.5575i\times10^{-5} & 0.234+0.8952i & 2.3744i\times10^{-4} & 5.9273i\times10^{-5} \\ -2.6708i\times10^{-4} & 0.117+0.4476i & -2.0784i\times10^{-4} & 2.3744i\times10^{-4} & 0.234+0.8952i & 2.3744i\times10^{-4} \\ -6.2981i\times10^{-5} & -2.6708i\times10^{-4} & 0.117+0.4476i & 5.9273i\times10^{-5} & 2.3744i\times10^{-4} & 0.234+0.8952i \end{pmatrix}\cdot\frac{ohm}{mi}$$

L25 12/21

$$\underset{3\times3}{\text{Zequiv}} := \text{Za} - \underset{3\times6}{\text{Zbnew}}\cdot\underset{6\times6}{\text{Zdnew}}^{-1}\cdot\underset{6\times3}{\text{Zcnew}}$$

$$\text{Zequiv} = \begin{pmatrix} 0.1343 + 1.0626i & 0.0953 + 0.577i & 0.0953 + 0.493i \\ 0.0953 + 0.577i & 0.1343 + 1.0625i & 0.0953 + 0.577i \\ 0.0953 + 0.493i & 0.0953 + 0.577i & 0.1343 + 1.0626i \end{pmatrix} \cdot \frac{\text{ohm}}{\text{mi}}$$

$$\text{GMR}_{\text{Bundle}} = \sqrt[n]{D_s \cdot d_{12} \cdot d_{13} \cdots d_{1n}}$$

Now if we had instead used the approximations of using the GMR of the bundle

conductor GMR

$$\text{Rsbundle} := (\text{Ds}\cdot\text{Da1a2}\cdot\text{Da1a3})^{\frac{1}{3}}$$

Geometric mean radius of Bundle

a1 a2 a3 da1a2 da1a3

$$L_{\text{perlengthnew}} := \frac{\mu_0}{2\cdot\pi}\cdot\begin{pmatrix} \ln\left(\frac{D_e}{\text{Rsbundle}}\right) & \ln\left(\frac{D_e}{\text{Da1b1}}\right) & \ln\left(\frac{D_e}{\text{Da1c1}}\right) \\ \ln\left(\frac{D_e}{\text{Da1b1}}\right) & \ln\left(\frac{D_e}{\text{Rsbundle}}\right) & \ln\left(\frac{D_e}{\text{Db1c1}}\right) \\ \ln\left(\frac{D_e}{\text{Da1c1}}\right) & \ln\left(\frac{D_e}{\text{Db1c1}}\right) & \ln\left(\frac{D_e}{\text{Rsbundle}}\right) \end{pmatrix}$$

The resistance matrix must also be modified since there are now parallel conductors

\# of conductors in bundle

$$R_{\text{perlengthnew}} := \begin{pmatrix} \frac{\text{Rac}_{\text{bund3}}}{3} + R_d & R_d & R_d \\ R_d & \frac{\text{Rac}_{\text{bund3}}}{3} + R_d & R_d \\ R_d & R_d & \frac{\text{Rac}_{\text{bund3}}}{3} + R_d \end{pmatrix}$$

$$Z_{perlengthnew} := R_{perlengthnew} + j\cdot 2\cdot\pi\cdot freq\cdot L_{perlengthnew}$$

$$Z_{perlengthnew} = \begin{pmatrix} 0.1343 + 1.0626i & 0.0953 + 0.577i & 0.0953 + 0.4929i \\ 0.0953 + 0.577i & 0.1343 + 1.0626i & 0.0953 + 0.577i \\ 0.0953 + 0.4929i & 0.0953 + 0.577i & 0.1343 + 1.0626i \end{pmatrix}\cdot\frac{ohm}{mi}$$

Error between these methods: - simple approx

$$Error := Z_{perlengthnew} - Zequiv$$

- 9x9 → 3x3

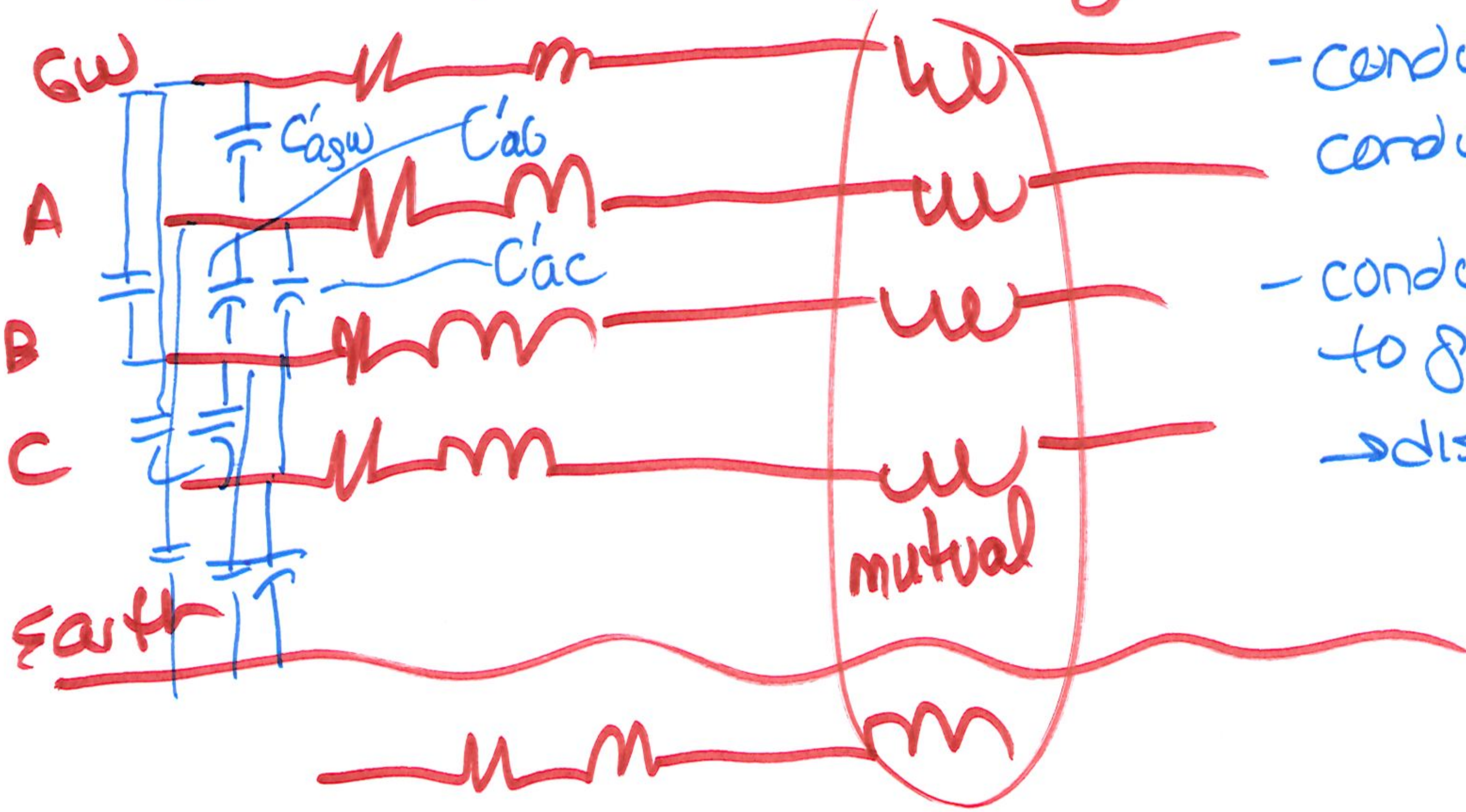
$$Error = \begin{pmatrix} -1.967\times10^{-5} + 7.5221i\times10^{-5} & -7.8588\times10^{-6} + 3.0065i\times10^{-5} & 1.5731\times10^{-5} - 6.0149i\times10^{-5} \\ -7.8588\times10^{-6} + 3.0065i\times10^{-5} & -3.1441\times10^{-5} + 1.203i\times10^{-4} & -7.8588\times10^{-6} + 3.0065i\times10^{-5} \\ 1.5731\times10^{-5} - 6.0149i\times10^{-5} & -7.8588\times10^{-6} + 3.0065i\times10^{-5} & -1.967\times10^{-5} + 7.5221i\times10^{-5} \end{pmatrix}\cdot\frac{ohm}{mi}$$

$$\overrightarrow{\left|\frac{Error}{\left|Zequiv\right|}\right|} = \begin{pmatrix} 7.2594\times10^{-3} & 5.3138\times10^{-3} & 0.0124 \\ 5.3138\times10^{-3} & 0.0116 & 5.3138\times10^{-3} \\ 0.0124 & 5.3138\times10^{-3} & 7.2594\times10^{-3} \end{pmatrix}\cdot\%$$

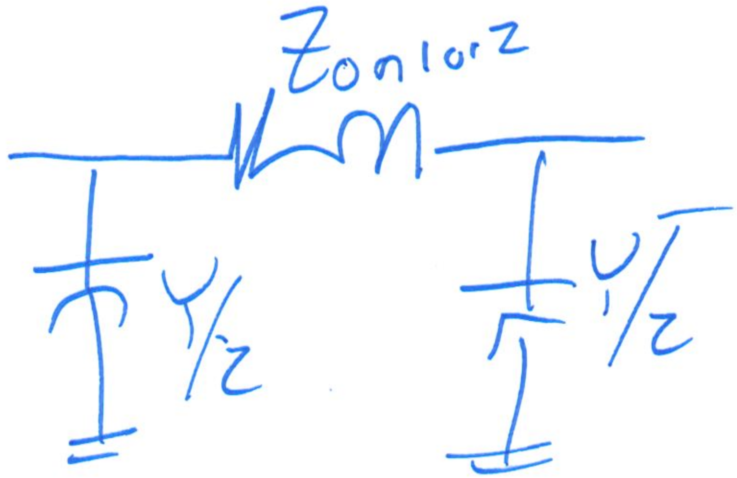
equivalent GMR of Bundle is accurate

- We calculated series impedance parameters

Now
- Shunt capacitance per length



- conductor to conductor
- conductor to ground
→ distributed capacitance



Y_0 or Y_1 or Y_2

$$Y_1 = y_1/\text{length} \cdot \text{length}$$

$$Y = G + j\omega C \quad \text{mhos } (\mho) \text{ or Siemens } (S)$$

small
-usually
neglected

related to loss due to corona to current on leakage on insulators

- common formula for pos + negative sequence

$$C' = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D_m}{r}\right)}$$

ε_0 – permittivity of free space 8.854×10^{-12} F/m

C': F/m

In air $\varepsilon_r = 1$

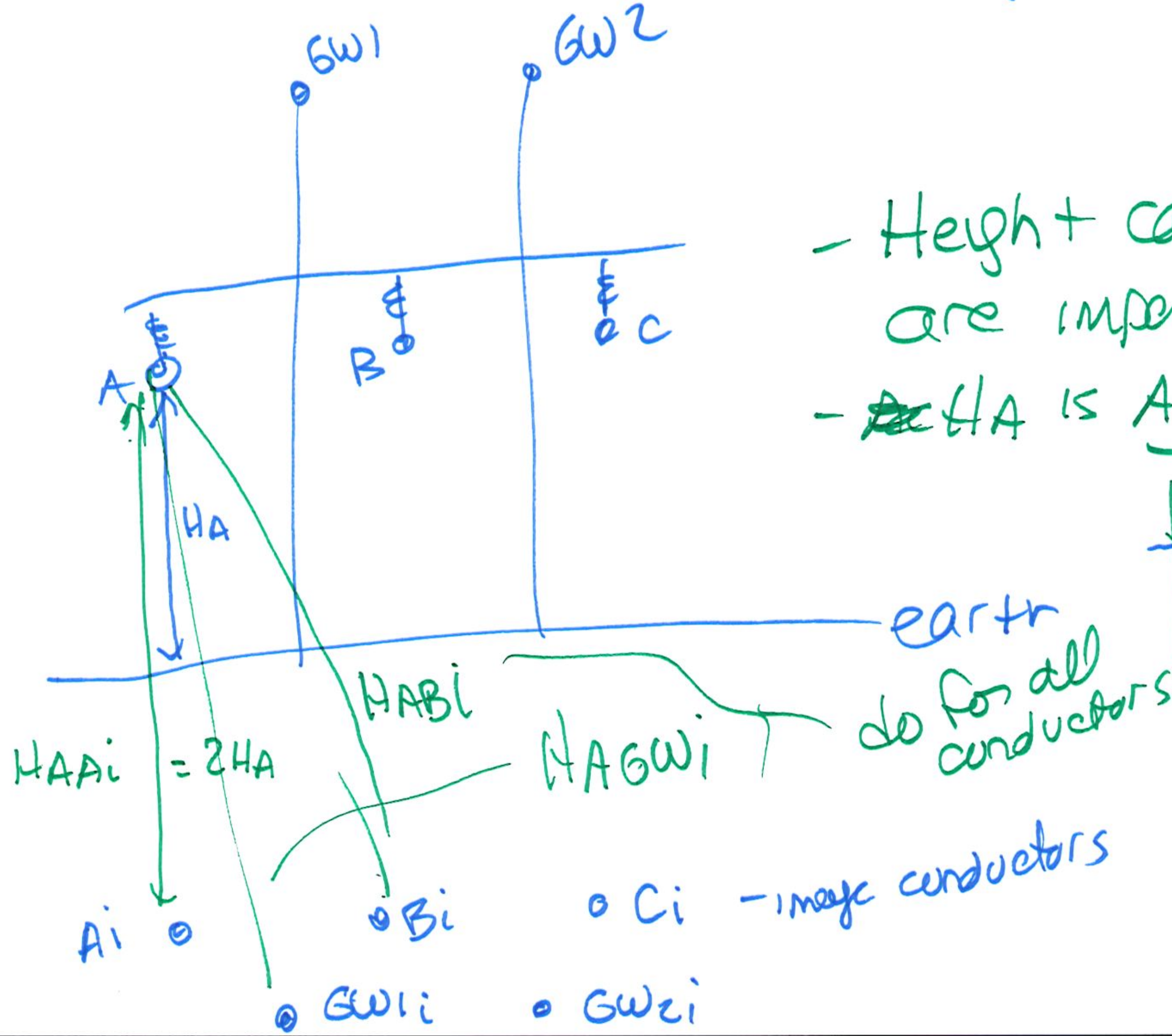
D_m = Geometric Dist Between phases $\sqrt[3]{\quad}$

r = conductor radius (not same as GMR)

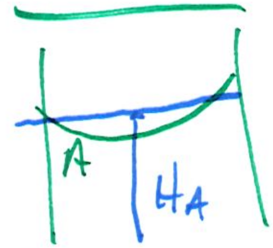
Bundle – $GMR_c = \sqrt[n]{r \cdot d_{12} \cdot d_{13} \cdots d_{1n}}$

Matrix approach - sets C_0

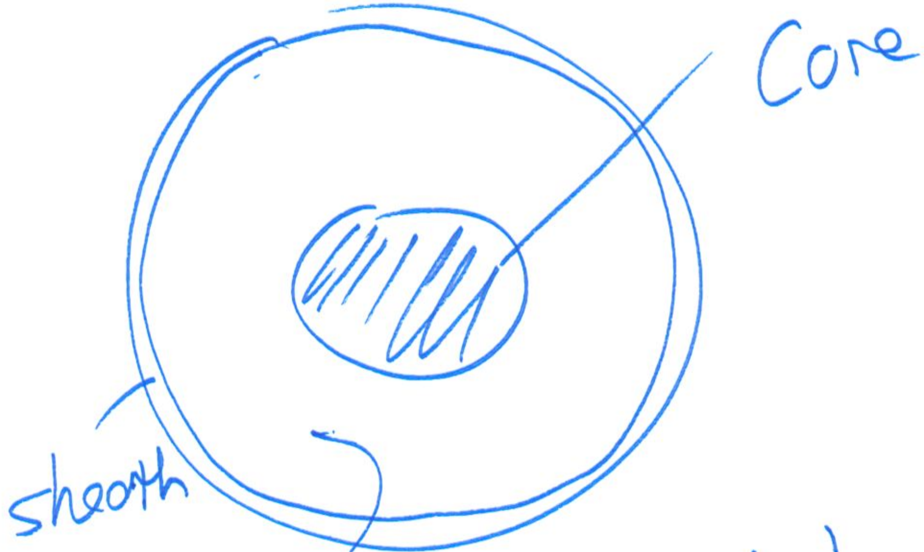
Method of Images



- Height calculations are impacted by sag
- ~~A~~ HA is AVERAGE height



with a cable



insulator (dielectric)

ϵ_r

relative permittivity of 3–5

$$C' = \frac{2\pi \epsilon_r \epsilon_0}{\ln\left(\frac{D}{r}\right)}$$

distance to sheath

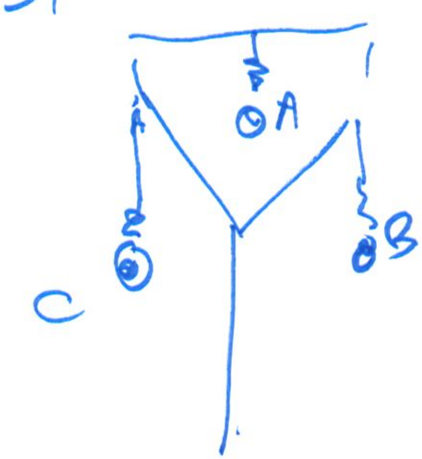
method vector potentials

A B C

$$P = \frac{1}{2\pi\varepsilon_0} \cdot \begin{bmatrix} \ln\left(\frac{H_{aai}}{r}\right) & \ln\left(\frac{H_{ab'}}{D_{ab}}\right) & \ln\left(\frac{H_{aci}}{D_{ac}}\right) \\ & \ln\left(\frac{H_{bbi}}{r}\right) & \\ & & \ln\left(\frac{H_{cci}}{r}\right) \end{bmatrix}$$

$2\pi\varepsilon_r\varepsilon_0$

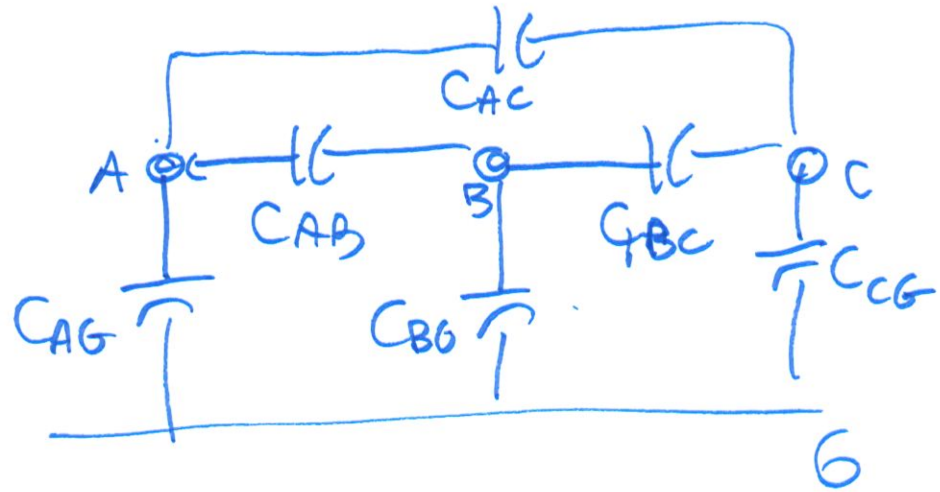
$\varepsilon_r = 1$



Ai Bi Ci

P has units $\frac{m}{F}$

$C' = P^{-1}$



$q = CV$

$$\begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix} = \begin{bmatrix} C_{aa} & -C_{AB} & -C_{AC} \\ -C_{BA} & C_{BB} & -C_{BC} \\ -C_{CA} & -C_{CB} & +C_{CC} \end{bmatrix} \begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix}$$

$$q_a = (V_{AG} - V_{BG})C_{AB} + (V_{AG} - V_{CG})C_{AC} + V_{AG}C_{AG}$$

$$= V_{AG}\underbrace{(C_{AG} + C_{AB} + C_{AC})}_{C_{aa}} - V_{BG}C_{AB} - V_{CG} \cdot C_{AC}$$

$C_{ABC} = C' \cdot \text{length}$

$C_{012} = A_{012}^{-1} C_{ABC} \cdot A_{012}$

sequence capacitance