Overhead Line Capacitance Calculations

A. Start with the same line used for the series impedance calculations

- Conductor radius:

$dia := 0.528in \qquad r := \dfrac{dia}{2}$

- Space between phase conductors

$Dab := 10ft \qquad Dac := 20ft \qquad Dbc := 10ft$

- Height calculations and distance to image conductors

$H_{tower} := 45ft \qquad Sag := 15ft$

$Ha := H_{tower} - \dfrac{2}{3}\cdot Sag \qquad Ha = 35\cdot ft \qquad Hb := Ha \qquad Hc := Ha$

$Haai := 2\cdot Ha \qquad Haai = 70\cdot ft$

$Hbbi := 2\cdot Hb \qquad Hcci := 2\cdot Hc$

$Habi := \sqrt{(2\cdot Ha)^2 + Dab^2} \qquad Habi = 70.71\cdot ft$

$Haci := \sqrt{(2\cdot Ha)^2 + Dac^2} \qquad Haci = 72.8\cdot ft$

$Hbci := \sqrt{(2\cdot Hc)^2 + Dbc^2} \qquad Hbci = 70.71\cdot ft$

$$P := \frac{1}{(2\cdot\pi\cdot\varepsilon_0)}\begin{pmatrix} \ln\left(\frac{Haai}{r}\right) & \ln\left(\frac{Habi}{Dab}\right) & \ln\left(\frac{Haci}{Dac}\right) \\ \ln\left(\frac{Habi}{Dab}\right) & \ln\left(\frac{Hbbi}{r}\right) & \ln\left(\frac{Hbci}{Dbc}\right) \\ \ln\left(\frac{Haci}{Dac}\right) & \ln\left(\frac{Hbci}{Dbc}\right) & \ln\left(\frac{Hcci}{r}\right) \end{pmatrix} \qquad P = \begin{pmatrix} 0.14 & 0.04 & 0.02 \\ 0.04 & 0.14 & 0.04 \\ 0.02 & 0.04 & 0.14 \end{pmatrix}\cdot\frac{m}{pF}$$

P is matrix of "Potential Coefficients"

$C' := P^{-1}$ $\qquad$ $C' = \begin{pmatrix} 11.93 & -2.58 & -1.29 \\ -2.58 & 12.35 & -2.58 \\ -1.29 & -2.58 & 11.93 \end{pmatrix} \cdot \frac{nF}{mi}$

$Length := 40mi$

$C_{untran} := C' \cdot Length$ $\qquad$ $C_{untran} = \begin{pmatrix} 0.48 & -0.1 & -0.05 \\ -0.1 & 0.49 & -0.1 \\ -0.05 & -0.1 & 0.48 \end{pmatrix} \cdot \mu F$

Transformation Matrix:

$a := 1 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}}$ $\qquad$ $A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$

$C_{012} := A_{012}{}^{-1} \cdot C_{untran} \cdot A_{012}$

$$C_{012} = \begin{pmatrix} 0.31 & 0.01 + 0.01i & 0.01 - 0.01i \\ 0.01 - 0.01i & 0.57 & -0.02 - 0.03i \\ 0.01 + 0.01i & -0.02 + 0.03i & 0.57 \end{pmatrix} \cdot \mu F$$

- Note the complex capacitance values in the off diagonal terms.

$C_0 := C_{012_{0,0}}$ $\qquad$ $C_0 = 310.93 \cdot nF$

$C_1 := C_{012_{1,1}}$ $\qquad$ $C_1 = 568.93 \cdot nF$ $\qquad$ $C_{012_{1,1}} - C_{012_{2,2}} = 0 \cdot nF$ $\qquad$ So $C2 = C1$

- If we use the per phase capacitance formulas:

$Dm := (Dab \cdot Dac \cdot Dbc)^{\frac{1}{3}}$ $\qquad$ $Dm = 12.6 \cdot ft$

$c_{phase} := \frac{2 \cdot \pi \cdot \varepsilon_0}{\ln\left(\frac{Dm}{r}\right)}$ $\qquad$ $c_{phase} = 8.761 \cdot \frac{pF}{m}$ $\qquad$ $c_{phase} \cdot Length = 563.95 \cdot nF$

Note that the per phase capacitance per length doesn't quite match the positive and negative sequence capacitance, as we discussed earlier, this is because the conductor height is left out. Modify the expression:

$$GMH := \left(\frac{Habi \cdot Haci \cdot Hbci}{Haai \cdot Hbbi \cdot Hcci}\right)^{\frac{1}{3}} \qquad GMH = 1.02$$

$$c_{phase\_cor} := \frac{2 \cdot \pi \cdot \varepsilon_0}{\ln\left(\frac{Dm}{r}\right) - \ln(GMH)}$$

$$c_{phase\_cor} = 8.79 \cdot \frac{pF}{m} \qquad c_{phase\_cor} \cdot Length = 565.71 \cdot nF \qquad C_1 = 568.93 \cdot nF$$

This is a lot closer, but still off a little bit.

- There are several formulas that can be used for the zero sequence capacitance.

$$Habc := (Habi \cdot Haci \cdot Hbci)^{\frac{1}{3}}$$

$$Haa := (Ha \cdot Hb \cdot Hc)^{\frac{1}{3}}$$

- From Siemens:

$$C\_zero\_siemens := \frac{2 \cdot \pi \cdot \varepsilon_0}{\ln\left(\frac{2 \cdot Haa \cdot Habc^2}{r \cdot Dm^2}\right)} \qquad C\_zero\_siemens = 4.82 \cdot \frac{pF}{m}$$

$$C0\_S := 40mi \cdot C\_zero\_siemens$$

$$C0\_S = 310.482 \cdot nF$$

Or from the Westinghouse T&D manual

$$C\_zero\_west := \frac{2 \cdot \pi \cdot \varepsilon_0}{\ln\left[\frac{(2 \cdot Haa)^3}{r \cdot Dm^2}\right]} \qquad C\_zero\_west = 4.84 \cdot \frac{pF}{m}$$

$C0\_W := 40mi \cdot C\_zero\_west \qquad C0\_W = 311.552 \cdot nF \qquad C_0 = 310.93 \cdot nF$

Transposed Line Examples:

$$Rp := \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Transpose Case 1:

$f1 := 0.2 \qquad f2 := 0.3 \qquad f3 := 0.5$

$$C_{net} := f1 \cdot C_{untran} + f2 \cdot Rp^{-1} \cdot C_{untran} \cdot Rp + f3 \cdot Rp \cdot C_{untran} \cdot Rp^{-1}$$

$$C_{net} = \begin{pmatrix} 0.48 & -0.08 & -0.09 \\ -0.08 & 0.48 & -0.09 \\ -0.09 & -0.09 & 0.49 \end{pmatrix} \cdot \mu F$$

$$C_{0121} := A_{012}^{-1} \cdot C_{net} \cdot A_{012}$$

$$C_{0121} = \begin{pmatrix} 0.31 & 5.84 \times 10^{-4} - 0i & 5.84 \times 10^{-4} + 0i \\ 5.84 \times 10^{-4} + 0i & 0.57 & -0 + 0.01i \\ 5.84 \times 10^{-4} - 0i & -0 - 0.01i & 0.57 \end{pmatrix} \cdot \mu F$$

Transposition Case 2:

$f13 := 0.4 \qquad f23 := 0.6 \qquad f33 := 0.0$

$$C_{net3} := f13 \cdot C_{untran} + f23 \cdot Rp^{-1} \cdot C_{untran} \cdot Rp + f33 \cdot Rp \cdot C_{untran} \cdot Rp^{-1}$$

$$C_{net3} = \begin{pmatrix} 0.49 & -0.1 & -0.08 \\ -0.1 & 0.48 & -0.07 \\ -0.08 & -0.07 & 0.48 \end{pmatrix} \cdot \mu F$$

$C_{0123} := A_{012}^{-1} \cdot C_{net3} \cdot A_{012}$

$$C_{0123} = \begin{pmatrix} 0.31 & -0+0i & -0-0i \\ -0-0i & 0.57 & 0.02-0.01i \\ -0+0i & 0.02+0.01i & 0.57 \end{pmatrix} \cdot \mu F$$

Transposition Case 3:

$f16 := \frac{1}{3}$ $\quad f26 := \frac{1}{3}$ $\quad f36 := \frac{1}{3}$

$$C_{net6} := f16 \cdot C_{untran} + f26 \cdot Rp^{-1} \cdot C_{untran} \cdot Rp + f36 \cdot Rp \cdot C_{untran} \cdot Rp^{-1}$$

$$C_{net6} = \begin{pmatrix} 0.48 & -0.09 & -0.09 \\ -0.09 & 0.48 & -0.09 \\ -0.09 & -0.09 & 0.48 \end{pmatrix} \cdot \mu F$$

$C_{0126} := A_{012}^{-1} \cdot C_{net6} \cdot A_{012}$

$$C_{0126} = \begin{pmatrix} 0.3109 & 0 & 0 \\ 0 & 0.5689 & 0 \\ 0 & 0 & 0.5689 \end{pmatrix} \cdot \mu F$$

Line with two conductor bundles

Conductor data from table:

$dia2 := 1.108in$ $\quad rad2 := \frac{dia2}{2}$ $\quad rad2 = 0.05 \cdot ft$

Spacing:

Within the bundle:

$Da1a2 := 1.5ft$ $\quad Db1b2 := 1.5ft$ $\quad Dc1c2 := 1.5ft$

Between Phases

$Da1b1 := 24ft$ $\quad Da1b2 := Da1b1 + Db1b2$ $\quad Da1b2 = 25.5 \cdot ft$

$Da2b1 := Da1b1 - Da1a2$ $\quad Da2b1 = 22.5 \cdot ft$ $\quad Da2b2 := 24ft$

$Da1c1 := 48ft$ $\quad Da1c2 := Da1c1 + Dc1c2$ $\quad Da1c2 = 49.5 \cdot ft$

$Da2c1 := Da1c1 - Da1a2$ $\quad Da2c1 = 46.5 \cdot ft$ $\quad Da2c2 := 48ft$

$Db1c1 := 24ft$ $\quad Db1c2 := Db1c1 + Dc1c2$ $\quad Db1c2 = 25.5 \cdot ft$

$Db2c1 := Db1c1 - Db1b2$ $\quad Db2c1 = 22.5 \cdot ft$ $\quad Db2c2 := 24ft$

Height Calculations (assume same height as above). Use symmetry, so don't compute all permutations.

$Ha1a1i := 2 \cdot Ha$ $\quad Hb1b1i := 2 \cdot Hb$ $\quad Hc1c1i := 2 \cdot Hc$

$Ha2a2i := 2 \cdot Ha$ $\quad Hb2b2i := 2 \cdot Hb$ $\quad Hc2c2i := 2 \cdot Hc$

$Ha1a2i := \sqrt{(2 \cdot Ha)^2 + Da1a2^2}$ $\quad Hb1b2i := \sqrt{(2 \cdot Hb)^2 + Db1b2^2}$

$Ha1b1i := \sqrt{(2 \cdot Ha)^2 + Da1b1^2}$ $\quad Ha1b2i := \sqrt{(2 \cdot Ha)^2 + Da1b2^2}$

$Ha1c1i := \sqrt{(2 \cdot Ha)^2 + Da1c1^2}$ $\quad Ha1c2i := \sqrt{(2 \cdot Ha)^2 + Da1c2^2}$

$Hb1c1i := \sqrt{(2 \cdot Hb)^2 + Db1c1^2}$ $\quad Hb1c2i := \sqrt{(2 \cdot Hb)^2 + Db1c2^2}$

$Hc1c2i := \sqrt{(2 \cdot Hc)^2 + Dc1c2^2}$ $\quad Ha2b2i := \sqrt{(2 \cdot Ha)^2 + Da2b2^2}$

$Ha2b1i := \sqrt{(2 \cdot Ha)^2 + Da2b1^2}$ $\quad Ha2c2i := \sqrt{(2 \cdot Ha)^2 + Da2c2^2}$

$Ha2c1i := \sqrt{(2 \cdot Ha)^2 + Da2c1^2}$ $\quad Hb2c1i := \sqrt{(2 \cdot Hb)^2 + Db2c1^2}$

$Hb2c2i := \sqrt{(2 \cdot Hb)^2 + Db2c2^2}$

$$P_u := \frac{1}{2\cdot\pi\cdot\varepsilon_0}\begin{bmatrix} \ln\left[\frac{(Ha1a1i)}{rad2}\right] & \ln\left(\frac{Ha1b1i}{Da1b1}\right) & \ln\left(\frac{Ha1c1i}{Da1c1}\right) & \ln\left(\frac{Ha1a2i}{Da1a2}\right) & \ln\left(\frac{Ha1b2i}{Da1b2}\right) & \ln\left(\frac{Ha1c2i}{Da1c2}\right) \\ \ln\left(\frac{Ha1b1i}{Da1b1}\right) & \ln\left[\frac{(Hb1b1i)}{rad2}\right] & \ln\left(\frac{Hb1c1i}{Db1c1}\right) & \ln\left(\frac{Ha2b1i}{Da2b1}\right) & \ln\left(\frac{Hb1b2i}{Db1b2}\right) & \ln\left(\frac{Hb1c2i}{Db1c2}\right) \\ \ln\left(\frac{Ha1c1i}{Da1c1}\right) & \ln\left(\frac{Hb1c1i}{Db1c1}\right) & \ln\left[\frac{(Hc1c1i)}{rad2}\right] & \ln\left(\frac{Ha2c1i}{Da2c1}\right) & \ln\left(\frac{Hb2c1i}{Db2c1}\right) & \ln\left(\frac{Hc1c2i}{Dc1c2}\right) \\ \ln\left(\frac{Ha1a2i}{Da1a2}\right) & \ln\left(\frac{Ha2b1i}{Da2b1}\right) & \ln\left(\frac{Ha2c1i}{Da2c1}\right) & \ln\left[\frac{(Ha2a2i)}{rad2}\right] & \ln\left(\frac{Ha2b2i}{Da2b2}\right) & \ln\left(\frac{Ha2c2i}{Da2c2}\right) \\ \ln\left(\frac{Ha1b2i}{Da1b2}\right) & \ln\left(\frac{Hb1b2i}{Db1b2}\right) & \ln\left(\frac{Hb2c1i}{Db2c1}\right) & \ln\left(\frac{Ha2b2i}{Da2b2}\right) & \ln\left[\frac{(Hb2b2i)}{rad2}\right] & \ln\left(\frac{Hb2c2i}{Db2c2}\right) \\ \ln\left(\frac{Ha1c2i}{Da1c2}\right) & \ln\left(\frac{Hb1c2i}{Db1c2}\right) & \ln\left(\frac{Hc1c2i}{Dc1c2}\right) & \ln\left(\frac{Ha2c2i}{Da2c2}\right) & \ln\left(\frac{Hb2c2i}{Db2c2}\right) & \ln\left[\frac{(Hc2c2i)}{rad2}\right] \end{bmatrix}$$

$$P_u = \begin{pmatrix} 0.13 & 0.02 & 0.01 & 0.07 & 0.02 & 0.01 \\ 0.02 & 0.13 & 0.02 & 0.02 & 0.07 & 0.02 \\ 0.01 & 0.02 & 0.13 & 0.01 & 0.02 & 0.07 \\ 0.07 & 0.02 & 0.01 & 0.13 & 0.02 & 0.01 \\ 0.02 & 0.07 & 0.02 & 0.02 & 0.13 & 0.02 \\ 0.01 & 0.02 & 0.07 & 0.01 & 0.02 & 0.13 \end{pmatrix}\cdot\frac{m}{pF}$$

$$C_{bund} := P_u^{-1}$$

$$C_{bund} = \begin{pmatrix} 10.58 & -0.49 & -0.17 & -5.38 & -0.41 & -0.15 \\ -0.49 & 10.68 & -0.48 & -0.58 & -5.3 & -0.41 \\ -0.17 & -0.48 & 10.62 & -0.18 & -0.58 & -5.38 \\ -5.38 & -0.58 & -0.18 & 10.62 & -0.48 & -0.17 \\ -0.41 & -5.3 & -0.58 & -0.48 & 10.68 & -0.49 \\ -0.15 & -0.41 & -5.38 & -0.17 & -0.49 & 10.58 \end{pmatrix}\cdot\frac{pF}{m}$$

If length = 40 miles:

$C_{line\_bu} := C_{bund} \cdot 40mi$

$$C_{line\_bu} = \begin{pmatrix} 0.68 & -0.03 & -0.01 & -0.35 & -0.03 & -0.01 \\ -0.03 & 0.69 & -0.03 & -0.04 & -0.34 & -0.03 \\ -0.01 & -0.03 & 0.68 & -0.01 & -0.04 & -0.35 \\ -0.35 & -0.04 & -0.01 & 0.68 & -0.03 & -0.01 \\ -0.03 & -0.34 & -0.04 & -0.03 & 0.69 & -0.03 \\ -0.01 & -0.03 & -0.35 & -0.01 & -0.03 & 0.68 \end{pmatrix} \cdot \mu F$$

Matrix Reduction (note this is done to the P matrix:

$Pa := \text{submatrix}(P_u, 0, 2, 0, 2)$

$$Pa = \begin{pmatrix} 0.13 & 0.02 & 0.01 \\ 0.02 & 0.13 & 0.02 \\ 0.01 & 0.02 & 0.13 \end{pmatrix} \cdot \frac{m}{pF}$$

$Pb := \text{submatrix}(P_u, 0, 2, 3, 5)$

$$Pb = \begin{pmatrix} 0.07 & 0.02 & 0.01 \\ 0.02 & 0.07 & 0.02 \\ 0.01 & 0.02 & 0.07 \end{pmatrix} \cdot \frac{m}{pF}$$

$Pc := \text{submatrix}(P_u, 3, 5, 0, 2)$

$$Pc = \begin{pmatrix} 0.07 & 0.02 & 0.01 \\ 0.02 & 0.07 & 0.02 \\ 0.01 & 0.02 & 0.07 \end{pmatrix} \cdot \frac{m}{pF}$$

$Pd := \text{submatrix}(P_u, 3, 5, 3, 5)$

$$Pd = \begin{pmatrix} 0.13 & 0.02 & 0.01 \\ 0.02 & 0.13 & 0.02 \\ 0.01 & 0.02 & 0.13 \end{pmatrix} \cdot \frac{m}{pF}$$

Modify matrix by performing: Qa'b'c'-Qabc and Vabc + Va'b'c' similar to what was done for series Zabc

$Pbnew := Pb - Pa$ $\quad$ $Pcnew := Pc - Pa$ $\quad$ $Pdnew := Pa - Pb - Pc + Pd$

Reduce to equivalent 3x3 matrix

$Pequiv := Pa - Pbnew \cdot Pdnew^{-1} \cdot Pcnew$

$$\text{Pequiv} = \begin{pmatrix} 0.1 & 0.02 & 0.01 \\ 0.02 & 0.1 & 0.02 \\ 0.01 & 0.02 & 0.1 \end{pmatrix} \cdot \frac{\text{m}}{\text{pF}} \qquad \text{Cequiv} := \text{Pequiv}^{-1}$$

$$\text{Cequiv} = \begin{pmatrix} 10.43 & -1.97 & -0.67 \\ -1.97 & 10.76 & -1.97 \\ -0.67 & -1.97 & 10.43 \end{pmatrix} \cdot \frac{\text{pF}}{\text{m}} \qquad \text{Cequiv\_len} := \text{Cequiv} \cdot \text{Length}$$

$$C_{0128} := A_{012}^{-1} \cdot \text{Cequiv\_len} \cdot A_{012}$$

$$C_{0128} = \begin{pmatrix} 0.48 & 0.01 + 0.02i & 0.01 - 0.02i \\ 0.01 - 0.02i & 0.78 & -0.03 - 0.05i \\ 0.01 + 0.02i & -0.03 + 0.05i & 0.78 \end{pmatrix} \cdot \mu\text{F}$$

Note that the original C012 was:

$$C_{012} = \begin{pmatrix} 0.31 & 0.01 + 0.01i & 0.01 - 0.01i \\ 0.01 - 0.01i & 0.57 & -0.02 - 0.03i \\ 0.01 + 0.01i & -0.02 + 0.03i & 0.57 \end{pmatrix} \cdot \mu\text{F}$$

Compare what the bundling has done to the sequence capacitances

Now if we had instead used the approximations of using the GMR of the bundle

$$\text{Rsbundle} := \sqrt{\text{rad2} \cdot \text{Da1a2}}$$

$$\text{Pequivbund} := \frac{1}{2 \cdot \pi \cdot \varepsilon_0} \cdot \begin{pmatrix} \ln\left(\frac{\text{Haai}}{\text{Rsbundle}}\right) & \ln\left(\frac{\text{Habi}}{\text{Da1b1}}\right) & \ln\left(\frac{\text{Haci}}{\text{Da1c1}}\right) \\ \ln\left(\frac{\text{Habi}}{\text{Da1b1}}\right) & \ln\left(\frac{\text{Hbbi}}{\text{Rsbundle}}\right) & \ln\left(\frac{\text{Hbci}}{\text{Db1c1}}\right) \\ \ln\left(\frac{\text{Haci}}{\text{Da1c1}}\right) & \ln\left(\frac{\text{Hbci}}{\text{Db1c1}}\right) & \ln\left(\frac{\text{Hcci}}{\text{Rsbundle}}\right) \end{pmatrix}$$

$$\text{Cequivbund} := \text{Pequivbund}^{-1} \cdot \text{Length}$$

$$\text{Cequivbund} = \begin{pmatrix} 667.35 & -124.16 & -25.76 \\ -124.16 & 689.46 & -124.16 \\ -25.76 & -124.16 & 667.35 \end{pmatrix} \cdot \text{nF}$$

Error between these methods:

$$\text{Error} := \text{Cequiv\_len} - \text{Cequivbund} \qquad \text{Error} = \begin{pmatrix} 4.098 & -2.685 & -17.278 \\ -2.685 & 3.243 & -2.685 \\ -17.278 & -2.685 & 4.098 \end{pmatrix} \cdot \text{nF}$$

Or calculating the sequence capacitance

$$C_{0128e} := A_{012}^{-1} \cdot \text{Cequivbund} \cdot A_{012}$$

$$C_{0128e} = \begin{pmatrix} 492 & 12.72 + 22.03i & 12.72 - 22.03i \\ 12.72 - 22.03i & 766.08 & -36.49 - 63.2i \\ 12.72 + 22.03i & -36.49 + 63.2i & 766.08 \end{pmatrix} \cdot \text{nF}$$

$$\text{Err} := C_{0128} - C_{0128e}$$

$$\text{Err} = \begin{pmatrix} -11.285 & -2.29 - 3.966i & -2.29 + 3.966i \\ -2.29 + 3.966i & 11.362 & 5.007 + 8.672i \\ -2.29 - 3.966i & 5.007 - 8.672i & 11.362 \end{pmatrix} \cdot \text{nF}$$

This is comparable to what we saw for the impedance

Now consider a static wire case:

- Consider two GW options

$dia_{gw\_375in} := 0.385in$ $\qquad$ $dia_{gw1F} := 0.346in$

$Dagw := \sqrt{(10ft)^2 + (15ft)^2}$ $\qquad$ $Dcgw := Dagw$ $\qquad$ $Dbgw := 15ft$

$Hgw_{tower} := 60ft$ $\qquad$ $Sag_{gw} := 15ft$

- Note that sag for ground wire is often different than for phase conductors. This will impact distance between phase conductors and ground wire

$Hgw := Hgw_{tower} - \frac{2}{3} \cdot Sag_{gw}$ $\qquad$ $Hgw = 50 \cdot ft$ $\qquad$ $Hgwgwi := 2 \cdot Hgw$

$Hagwi := \sqrt{(2 \cdot Ha + Dbgw)^2 + Dab^2}$ $\qquad$ $Hagwi = 85.59 \cdot ft$

$Hcgwi := \sqrt{(2 \cdot Hc + Dbgw)^2 + Dbc^2}$ $\qquad$ $Hcgwi = 85.59 \cdot ft$

$Hbgwi := 2 \cdot Hb + Dbgw$ $\qquad$ $Hbgwi = 85 \cdot ft$

First Ground Wire Case: 3/8" copperweld

$$Pgw1 := \frac{1}{2\pi \cdot \varepsilon_0} \cdot \begin{pmatrix} \ln\left(\frac{Haai}{r}\right) & \ln\left(\frac{Habi}{Dab}\right) & \ln\left(\frac{Haci}{Dac}\right) & \ln\left(\frac{Hagwi}{Dagw}\right) \\ \ln\left(\frac{Habi}{Dab}\right) & \ln\left(\frac{Hbbi}{r}\right) & \ln\left(\frac{Hbci}{Dbc}\right) & \ln\left(\frac{Hbgwi}{Dbgw}\right) \\ \ln\left(\frac{Haci}{Dac}\right) & \ln\left(\frac{Hbci}{Dbc}\right) & \ln\left(\frac{Hcci}{r}\right) & \ln\left(\frac{Hcgwi}{Dcgw}\right) \\ \ln\left(\frac{Hagwi}{Dagw}\right) & \ln\left(\frac{Hbgwi}{Dbgw}\right) & \ln\left(\frac{Hcgwi}{Dcgw}\right) & \ln\left(\frac{Hgwgwi}{\frac{dia_{gw\_375in}}{2}}\right) \end{pmatrix}$$

$$Pgw1 = \begin{pmatrix} 0.14 & 0.04 & 0.02 & 0.03 \\ 0.04 & 0.14 & 0.04 & 0.03 \\ 0.02 & 0.04 & 0.14 & 0.03 \\ 0.03 & 0.03 & 0.03 & 0.16 \end{pmatrix} \cdot \frac{m}{pF}$$

Matrix Reduction (note this is done to the P matrix:

$Pagw1 := \mathrm{submatrix}(Pgw1, 0, 2, 0, 2)$

$$Pagw1 = \begin{pmatrix} 0.14 & 0.04 & 0.02 \\ 0.04 & 0.14 & 0.04 \\ 0.02 & 0.04 & 0.14 \end{pmatrix} \cdot \frac{m}{pF}$$

$Pbgw1 := \mathrm{submatrix}(Pgw1, 0, 2, 3, 3)$

$$Pbgw1 = \begin{pmatrix} 0.03 \\ 0.03 \\ 0.03 \end{pmatrix} \cdot \frac{m}{pF}$$

$Pcgw1 := \mathrm{submatrix}(Pgw1, 3, 3, 0, 2)$

$$Pcgw1 = \begin{pmatrix} 0.03 & 0.03 & 0.03 \end{pmatrix} \cdot \frac{m}{pF}$$

$Pdgw1 := \mathrm{submatrix}(Pgw1, 3, 3, 3, 3)$

$$Pdgw1 = (0.16) \cdot \frac{m}{pF}$$

Reduce to equivalent 3x3 matrix

$Pequivgw1 := Pagw1 - Pbgw1 \cdot Pdgw1^{-1} \cdot Pcgw1$

$$Pequivgw1 = \begin{pmatrix} 0.14 & 0.03 & 0.02 \\ 0.03 & 0.14 & 0.03 \\ 0.02 & 0.03 & 0.14 \end{pmatrix} \cdot \frac{m}{pF}$$

$Cequivgw1 := Pequivgw1^{-1} \cdot Length$

$$Cequivgw1 = \begin{pmatrix} 0.49 & -0.09 & -0.04 \\ -0.09 & 0.5 & -0.09 \\ -0.04 & -0.09 & 0.49 \end{pmatrix} \cdot \mu F$$

$C_{012gw1} := A_{012}^{-1} \cdot Cequivgw1 \cdot A_{012}$

$$C_{012gw1} = \begin{pmatrix} 337.06 & 5.4 + 9.35i & 5.4 - 9.35i \\ 5.4 - 9.35i & 568.96 & -20.11 - 34.83i \\ 5.4 + 9.35i & -20.11 + 34.83i & 568.96 \end{pmatrix} \cdot nF$$

Note that the original C012 was:

$$C_{012} = \begin{pmatrix} 310.93 & 5.84+10.12i & 5.84-10.12i \\ 5.84-10.12i & 568.93 & -20.1-34.81i \\ 5.84+10.12i & -20.1+34.81i & 568.93 \end{pmatrix} \cdot nF$$

All of the change is basically in the zero sequence component.

Second Ground Wire Case: 1F copperweld

$$Pgw2 := \frac{1}{2\pi\cdot\varepsilon_0}\cdot\begin{pmatrix} \ln\left(\frac{Haai}{r}\right) & \ln\left(\frac{Habi}{Dab}\right) & \ln\left(\frac{Haci}{Dac}\right) & \ln\left(\frac{Hagwi}{Dagw}\right) \\ \ln\left(\frac{Habi}{Dab}\right) & \ln\left(\frac{Hbbi}{r}\right) & \ln\left(\frac{Hbci}{Dbc}\right) & \ln\left(\frac{Hbgwi}{Dbgw}\right) \\ \ln\left(\frac{Haci}{Dac}\right) & \ln\left(\frac{Hbci}{Dbc}\right) & \ln\left(\frac{Hcci}{r}\right) & \ln\left(\frac{Hcgwi}{Dcgw}\right) \\ \ln\left(\frac{Hagwi}{Dagw}\right) & \ln\left(\frac{Hbgwi}{Dbgw}\right) & \ln\left(\frac{Hcgwi}{Dcgw}\right) & \ln\left(\frac{Hgwgwi}{\frac{dia_{gw1F}}{2}}\right) \end{pmatrix}$$

$$Pgw2 = \begin{pmatrix} 0.14 & 0.04 & 0.02 & 0.03 \\ 0.04 & 0.14 & 0.04 & 0.03 \\ 0.02 & 0.04 & 0.14 & 0.03 \\ 0.03 & 0.03 & 0.03 & 0.16 \end{pmatrix} \cdot \frac{m}{pF}$$

Matrix Reduction (note this is done to the P matrix:

$$Pagw2 := \text{submatrix}(Pgw2, 0, 2, 0, 2) \qquad Pagw2 = \begin{pmatrix} 0.14 & 0.04 & 0.02 \\ 0.04 & 0.14 & 0.04 \\ 0.02 & 0.04 & 0.14 \end{pmatrix} \cdot \frac{m}{pF}$$

$Pbgw2 := submatrix(Pgw2, 0, 2, 3, 3)$

$$Pbgw2 = \begin{pmatrix} 0.03 \\ 0.03 \\ 0.03 \end{pmatrix} \cdot \frac{m}{pF}$$

$Pcgw2 := submatrix(Pgw2, 3, 3, 0, 2)$

$$Pcgw2 = \begin{pmatrix} 0.03 & 0.03 & 0.03 \end{pmatrix} \cdot \frac{m}{pF}$$

$Pdgw2 := submatrix(Pgw2, 3, 3, 3, 3)$

$$Pdgw2 = (0.16) \cdot \frac{m}{pF}$$

Reduce to equivalent 3x3 matrix

$Pequivgw2 := Pagw2 - Pbgw2 \cdot Pdgw2^{-1} \cdot Pcgw2$

$$Pequivgw2 = \begin{pmatrix} 0.14 & 0.03 & 0.02 \\ 0.03 & 0.14 & 0.03 \\ 0.02 & 0.03 & 0.14 \end{pmatrix} \cdot \frac{m}{pF}$$

$Cequivgw2 := Pequivgw2^{-1} \cdot Length$

$$Cequivgw2 = \begin{pmatrix} 0.49 & -0.09 & -0.04 \\ -0.09 & 0.5 & -0.09 \\ -0.04 & -0.09 & 0.49 \end{pmatrix} \cdot \mu F$$

$$C012_{gw2} := A_{012}^{-1} \cdot Cequivgw2 \cdot A_{012}$$

$$C012_{gw2} = \begin{pmatrix} 336.71 & 5.41 + 9.36i & 5.41 - 9.36i \\ 5.41 - 9.36i & 568.96 & -20.11 - 34.83i \\ 5.41 + 9.36i & -20.11 + 34.83i & 568.96 \end{pmatrix} \cdot nF$$

Compare to first GW

$$C_{012gw1} = \begin{pmatrix} 337.06 & 5.4 + 9.35i & 5.4 - 9.35i \\ 5.4 - 9.35i & 568.96 & -20.11 - 34.83i \\ 5.4 + 9.35i & -20.11 + 34.83i & 568.96 \end{pmatrix} \cdot nF$$

$$diff\_gw := submatrix\left(C_{012gw1}, 0, 0, 0, 0\right) - submatrix\left(C012_{gw2}, 0, 0, 0, 0\right)$$

$diff\_gw = (0.34) \cdot nF$ Due to different conductor size