Mutual Coupling in Fault Analysis

$MVA := 1000kW$

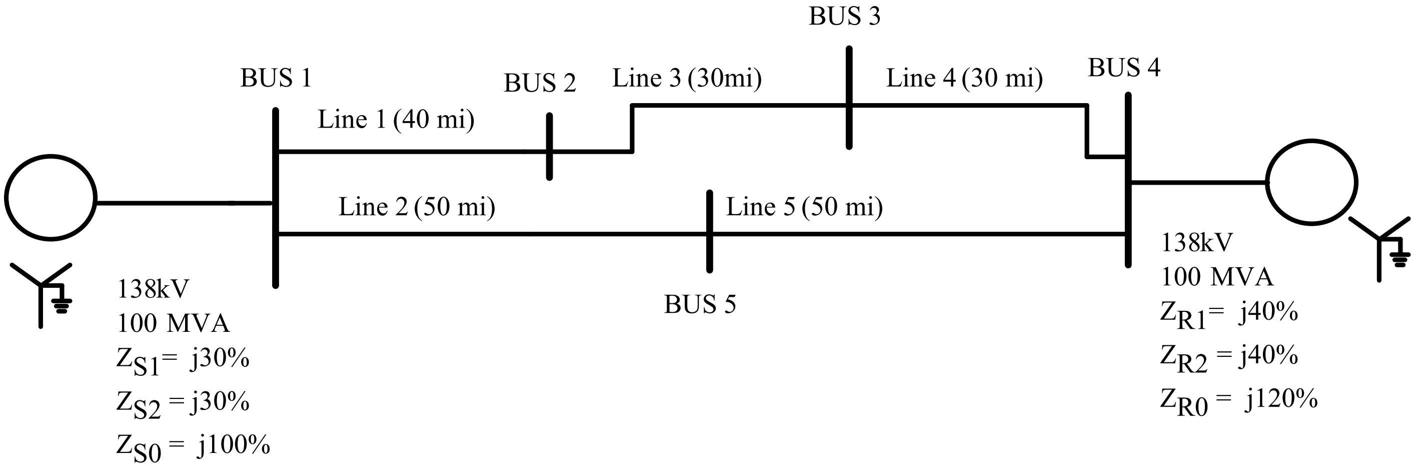
$pu := 1$

$a := 1 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}}$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$S_{base} := 100MVA$ $\qquad V_{base} := 138kV$

$$Z_{base} := \frac{V_{base}^{\ 2}}{S_{base}} \qquad Z_{base} = 190.44\,\Omega \qquad I_{base} := \frac{S_{base}}{\sqrt{3} \cdot V_{base}} \qquad I_{base} = 418.37\,A$$



- Source impedances:

$Z_{S1} := j \cdot 0.3pu$ $\qquad Z_{R1} := j \cdot 0.4pu$

$Z_{S0} := j \cdot 1.0pu$ $\qquad Z_{R0} := j \cdot 1.2pu$

- Line impedances from lecture 32 handout:

\- Line 1, 40 mile length

$Z_{0Line1} := (22.55548 + j\cdot 110.79032)ohm$

$Z_{1Line1} := (11.12 + j\cdot 32.16612)ohm$

$Z_{1L1} := \frac{Z_{1Line1}}{Z_{base}}$ $Z_{1L1} = (0.058 + 0.169i)\cdot pu$

$Z_{0L1} := \frac{Z_{0Line1}}{Z_{base}}$ $Z_{0L1} = (0.118 + 0.582i)\cdot pu$

- Line 2, 50 mile length

$Z_{0Line2} := 1.2\cdot(22.55548 + j\cdot 110.79032)ohm$

$Z_{1Line2} := 1.2\cdot(11.12 + j\cdot 32.16612)ohm$

$Z_{1L2} := \frac{Z_{1Line2}}{Z_{base}}$ $Z_{1L2} = (0.07 + 0.203i)\cdot pu$

$Z_{0L2} := \frac{Z_{0Line2}}{Z_{base}}$ $Z_{0L2} = (0.142 + 0.698i)\cdot pu$

- Line 1 and line 2 have 40 miles that are mutually coupled (just use zero sequence coupling)

$Z_{0M\_12} := (11.43548 + j\cdot 48.71333)ohm$

$Z_{0M12} := \frac{Z_{0M\_12}}{Z_{base}}$ $Z_{0M12} = (0.06 + 0.256i)\cdot pu$

- Line 3 is 30 miles with the same tower configuration as above, 20 miles of which is not close enough for mutual coupling.

$Z_{1L3} := 0.75\cdot Z_{1L1}$

Note that 30 miles is 3/4 of the length of line 1

$Z_{0L3} := 0.75\cdot Z_{0L1}$

  - Line 2 and line 3 have 10 miles that are mutually coupled (just use zero sequence coupling)

    $Z_{0M23} := 0.25\cdot\left(Z_{0M12}\right)$

- Line 4 is 30 miles with same tower configuration that is not close enough for mutual coupling

  $Z_{1L4} := 0.75\cdot Z_{1L1}$

  $Z_{0L4} := 0.75\cdot Z_{0L1}$

- Line 5 is 50 miles with same tower configuration that is not close enough for mutual coupling

  $Z_{1L5} := 1.2\cdot Z_{1L1}$

  $Z_{0L5} := 1.2Z_{0L1}$

- First, create the positive sequence Y_{bus} for fault study using the common formulation approach. Note that there is no mutual coupling for the positive sequence.

$$Y_{bus1} := \begin{pmatrix} \frac{1}{Z_{S1}} + \frac{1}{Z_{1L1}} + \frac{1}{Z_{1L2}} & \frac{-1}{Z_{1L1}} & 0 & 0 & \frac{-1}{Z_{1L2}} \\ \frac{-1}{Z_{1L1}} & \frac{1}{Z_{1L1}} + \frac{1}{Z_{1L3}} & \frac{-1}{Z_{1L3}} & 0 & 0 \\ 0 & \frac{-1}{Z_{1L3}} & \frac{1}{Z_{1L3}} + \frac{1}{Z_{1L4}} & \frac{-1}{Z_{1L4}} & 0 \\ 0 & 0 & \frac{-1}{Z_{1L4}} & \frac{1}{Z_{R1}} + \frac{1}{Z_{1L4}} + \frac{1}{Z_{1L5}} & \frac{-1}{Z_{1L5}} \\ \frac{-1}{Z_{1L2}} & 0 & 0 & \frac{-1}{Z_{1L5}} & \frac{1}{Z_{1L2}} + \frac{1}{Z_{1L5}} \end{pmatrix}$$

$$Y_{bus1} = \begin{pmatrix} 3.352-13.029i & -1.828+5.288i & 0 & 0 & -1.524+4.407i \\ -1.828+5.288i & 4.266-12.34i & -2.438+7.051i & 0 & 0 \\ 0 & -2.438+7.051i & 4.875-14.103i & -2.438+7.051i & 0 \\ 0 & 0 & -2.438+7.051i & 3.961-13.958i & -1.524+4.407i \\ -1.524+4.407i & 0 & 0 & -1.524+4.407i & 3.047-8.814i \end{pmatrix} \cdot pu$$

- Now build the Ybus, using the incidence matrix approach (again, no mutual coupling for positive sequence)

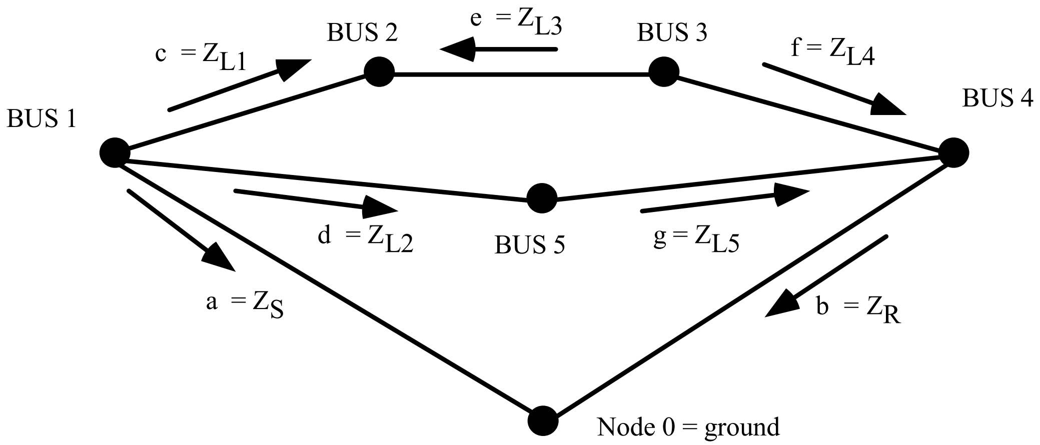
$$Y_{pr1} := \begin{array}{cc} & \text{Mapping to digraph} \\ \begin{pmatrix} \frac{1}{Z_{S1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{Z_{R1}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{Z_{1L1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{Z_{1L2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{Z_{1L3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{Z_{1L4}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{Z_{1L5}} \end{pmatrix} & \begin{array}{c} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \\ \mathbf{d} \\ \mathbf{e} \\ \mathbf{f} \\ \mathbf{g} \end{array} \end{array}$$

- Note that the matrix Ypr, actually comes from the primitive impedance matrix. We will modify this matrix later to add zero sequence mutual coupling.

$$Z_{pr1} := \begin{pmatrix} Z_{S1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{R1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{1L1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_{1L2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_{1L3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_{1L4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_{1L5} \end{pmatrix}$$

$$Y_{pr1}^{-1} - Z_{pr1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- Directed graph representating the system (we will use the polarities of the arrows for the incidence matrix)



Incidence matrix

- Each row corresponds to a branch
- Each column corresponds to a node (note that matrix is not generally square)
  - a. Enter a 0 if no connection between the node and branch for that cell
  - b. Enter a (+1) if the current is leaving a node
  - c. Enter a (-1) if the current is entering a node
  - d. No row and column associated with ground node

$$\begin{array}{r|rrrrr|l}
\text{Node number:} & 1 & 2 & 3 & 4 & 5 & \text{Branch} \\
 & 1 & 0 & 0 & 0 & 0 & \mathbf{a} \\
 & 0 & 0 & 0 & 1 & 0 & \mathbf{b} \\
 & 1 & -1 & 0 & 0 & 0 & \mathbf{c} \\
A_{incid} := & 1 & 0 & 0 & 0 & -1 & \mathbf{d} \\
 & 0 & -1 & 1 & 0 & 0 & \mathbf{e} \\
 & 0 & 0 & 1 & -1 & 0 & \mathbf{f} \\
 & 0 & 0 & 0 & -1 & 1 & \mathbf{g}
\end{array}$$

$$Y_{bus1\_alt} := A_{incid}{}^{T} \cdot Y_{pr1} \cdot A_{incid}$$

$$Y_{bus1\_alt} = \begin{pmatrix}
3.352 - 13.029i & -1.828 + 5.288i & 0 & 0 & -1.524 + 4.407i \\
-1.828 + 5.288i & 4.266 - 12.34i & -2.438 + 7.051i & 0 & 0 \\
0 & -2.438 + 7.051i & 4.875 - 14.103i & -2.438 + 7.051i & 0 \\
0 & 0 & -2.438 + 7.051i & 3.961 - 13.958i & -1.524 + 4.407i \\
-1.524 + 4.407i & 0 & 0 & -1.524 + 4.407i & 3.047 - 8.814i
\end{pmatrix}$$

$$Y_{bus1\_alt} - Y_{bus1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Y_{bus2} := Y_{bus1}$$

- Now for zero sequence matrix. Now we need to include the zero sequence mutual coupling
  - It will be put into the Zpr0 matrix as off-diagonal terms
  - The sign of the offdiagonal terms depends on the sign of the associated links in the digraph (positive signs if the arrows point the same way and negative otherwise.

Mapping to digraph

$$Z_{pr0M} := \begin{array}{c} \begin{pmatrix} Z_{S0} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{R0} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{0L1} & Z_{0M12} & 0 & 0 & 0 \\ 0 & 0 & Z_{0M12} & Z_{0L2} & -Z_{0M23} & 0 & 0 \\ 0 & 0 & 0 & -Z_{0M23} & Z_{0L3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_{0L4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_{0L5} \end{pmatrix} \end{array} \begin{array}{c} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \\ \mathbf{d} \\ \mathbf{e} \\ \mathbf{f} \\ \mathbf{g} \end{array}$$

and for later comparisons, the same matrix, with no mutual terms.

$$Z_{pr0\_noM} := \begin{pmatrix} Z_{S0} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{R0} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{0L1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_{0L2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_{0L3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_{0L4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_{0L5} \end{pmatrix}$$

$$Y_{bus0\_M} := A_{incid}^{T} \cdot Z_{pr0M}^{-1} \cdot A_{incid}$$

$$Y_{bus0\_NoM} := A_{incid}^{T} \cdot Z_{pr0\_noM}^{-1} \cdot A_{incid}$$

$$Y_{bus0\_M} = \begin{pmatrix} 0.46 - 3.17i & -0.29 + 1.37i & 0.03 - 0.14i & 0 & -0.2 + 0.93i \\ -0.29 + 1.37i & 0.8 - 4i & -0.44 + 2.13i & 0 & -0.08 + 0.5i \\ 0.03 - 0.14i & -0.44 + 2.13i & 0.9 - 4.44i & -0.45 + 2.2i & -0.04 + 0.25i \\ 0 & 0 & -0.45 + 2.2i & 0.73 - 4.41i & -0.28 + 1.38i \\ -0.2 + 0.93i & -0.08 + 0.5i & -0.04 + 0.25i & -0.28 + 1.38i & 0.6 - 3.05i \end{pmatrix}$$

Matches Powerworld Matrix:

Name	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5
BUS1	0.46 - j3.17	-0.29 + j1.37	0.03 - j0.14		-0.20 + j0.93
BUS2	-0.29 + j1.37	0.80 - j4.00	-0.44 + j2.13		-0.08 + j0.50
BUS3	0.03 - j0.14	-0.44 + j2.13	0.90 - j4.44	-0.45 + j2.20	-0.04 + j0.25
BUS4			-0.45 + j2.20	0.73 - j4.41	-0.28 + j1.38
BUS5	-0.20 + j0.93	-0.08 + j0.50	-0.04 + j0.25	-0.28 + j1.38	0.60 - j3.05

$$Y_{bus0\_NoM} = \begin{pmatrix} 0.616-4.026i & -0.336+1.651i & 0 & 0 & -0.28+1.375i \\ -0.336+1.651i & 0.784-3.851i & -0.448+2.201i & 0 & 0 \\ 0 & -0.448+2.201i & 0.896-4.401i & -0.448+2.201i & 0 \\ 0 & 0 & -0.448+2.201i & 0.728-4.409i & -0.28+1.375i \\ -0.28+1.375i & 0 & 0 & -0.28+1.375i & 0.56-2.751i \end{pmatrix}$$

$$Z_{0\_M} := Y_{bus0\_M}{}^{-1}$$

$$Z_{0\_NoM} := Y_{bus0\_NoM}{}^{-1}$$

$$Z_{0\_M} = \begin{pmatrix} 0.019+0.676i & -1.425\times10^{-3}+0.539i & -0.013+0.459i & -0.023+0.389i & -6.347\times10^{-3}+0.507i \\ -1.425\times10^{-3}+0.539i & 0.068+0.882i & 0.034+0.715i & 1.71\times10^{-3}+0.553i & 0.016+0.615i \\ -0.013+0.459i & 0.034+0.715i & 0.069+0.901i & 0.016+0.649i & 0.016+0.622i \\ -0.023+0.389i & 1.71\times10^{-3}+0.553i & 0.016+0.649i & 0.028+0.733i & 7.617\times10^{-3}+0.592i \\ -6.347\times10^{-3}+0.507i & 0.016+0.615i & 0.016+0.622i & 7.617\times10^{-3}+0.592i & 0.068+0.887i \end{pmatrix}$$

$$Z_{0\_NoM} = \begin{pmatrix} 0.017+0.657i & 2.047\times10^{-3}+0.559i & -9.21\times10^{-3}+0.485i & -0.02+0.411i & -1.706\times10^{-3}+0.534i \\ 2.047\times10^{-3}+0.559i & 0.071+0.896i & 0.034+0.713i & -2.456\times10^{-3}+0.529i & -2.047\times10^{-4}+0.544i \\ -9.21\times10^{-3}+0.485i & 0.034+0.713i & 0.067+0.884i & 0.011+0.618i & 9.21\times10^{-4}+0.552i \\ -0.02+0.411i & -2.456\times10^{-3}+0.529i & 0.011+0.618i & 0.025+0.707i & 2.047\times10^{-3}+0.559i \\ -1.706\times10^{-3}+0.534i & -2.047\times10^{-4}+0.544i & 9.21\times10^{-4}+0.552i & 2.047\times10^{-3}+0.559i & 0.071+0.896i \end{pmatrix}$$

$Z_{Bus1} := Y_{bus1}^{-1}$ $\qquad$ $Z_{Bus2} := Z_{Bus1}$

Now lets look at a SLG fault at Bus 5:

Assume no prefault powerflow, and 1.0pu voltage everywhere prefault.... $\qquad$ $V_f := 1.0\text{pu}$

Case 1: No mutual coupling:

$$I_{0\_NoM} := \frac{V_f}{Z_{Bus1_{4,4}} + Z_{Bus2_{4,4}} + Z_{0\_NoM_{4,4}}} \qquad \left|I_{0\_NoM}\right| = 0.69\cdot\text{pu} \qquad \arg\left(I_{0\_NoM}\right) = -84.39\cdot\text{deg}$$

$I_{1\_NoM} := I_{0\_NoM}$ $\qquad$ $I_{2\_NoM} := I_{1\_NoM}$

$$I_{ABC\_SLG\_NoM} := A_{012}\cdot\begin{pmatrix} I_{0\_NoM} \\ I_{1\_NoM} \\ I_{2\_NoM} \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_SLG\_NoM}\right|} = \begin{pmatrix} 2.069 \\ 0 \\ 0 \end{pmatrix} \qquad \overrightarrow{\arg\left(I_{ABC\_SLG\_NoM}\right)} = \begin{pmatrix} -84.39 \\ 19.537 \\ 19.537 \end{pmatrix}\cdot\text{deg}$$

$$\Delta V1_{NoM} := Z_{Bus1}\cdot\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_{1\_NoM} \end{pmatrix} \qquad V1_{NoM} := \begin{pmatrix} 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \end{pmatrix} + \Delta V1_{NoM}$$

$$\overrightarrow{\left|V1_{NoM}\right|} = \begin{pmatrix} 0.886 \\ 0.883 \\ 0.88 \\ 0.878 \\ 0.81 \end{pmatrix} \qquad \overrightarrow{\arg\left(V1_{NoM}\right)} = \begin{pmatrix} -0.783 \\ -0.753 \\ -0.73 \\ -0.706 \\ 0.407 \end{pmatrix} \cdot deg$$

$$\Delta V2_{NoM} := Z_{Bus2} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_{2\_NoM} \end{pmatrix} \qquad V2_{NoM} := \Delta V2_{NoM}$$

$$\Delta V0_{NoM} := Z_{0\_NoM} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_{0\_NoM} \end{pmatrix} \qquad V0_{NoM} := \Delta V0_{NoM}$$

$$V_{ABC\_B1\_NoM} := A_{012} \cdot \begin{pmatrix} {V0_{NoM}}_{0} \\ {V1_{NoM}}_{0} \\ {V2_{NoM}}_{0} \end{pmatrix} \qquad \overrightarrow{\left|V_{ABC\_B1\_NoM}\right|} = \begin{pmatrix} 0.4097 \\ 1.1663 \\ 1.1284 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg\left(V_{ABC\_B1\_NoM}\right)} = \begin{pmatrix} -8.623 \\ -130.178 \\ 131.822 \end{pmatrix} \cdot deg$$

$$V_{ABC\_B2\_NoM} := A_{012} \cdot \begin{pmatrix} V0_{NoM_1} \\ V1_{NoM_1} \\ V2_{NoM_1} \end{pmatrix} \qquad \overrightarrow{\left|V_{ABC\_B2\_NoM}\right|} = \begin{pmatrix} 0.3962 \\ 1.1687 \\ 1.1307 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg\left(V_{ABC\_B2\_NoM}\right)} = \begin{pmatrix} -8.714 \\ -130.308 \\ 131.962 \end{pmatrix} \cdot deg$$

$$I_{1\_L1\_NoM} := \frac{V1_{NoM_0} - V1_{NoM_1}}{Z_{1L1}}$$

$$I_{2\_L1\_NoM} := \frac{V2_{NoM_0} - V2_{NoM_1}}{Z_{1L1}}$$

$$I_{0\_L1\_NoM} := \frac{V0_{NoM_0} - V0_{NoM_1}}{Z_{0L1}}$$

$$I_{ABC\_L1\_NoM} := A_{012} \cdot \begin{pmatrix} I_{0\_L1\_NoM} \\ I_{1\_L1\_NoM} \\ I_{2\_L1\_NoM} \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_L1\_NoM}\right|} = \begin{pmatrix} 0.0487 \\ 0.007 \\ 0.007 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_L1\_NoM}\right)} = \begin{pmatrix} -80.275 \\ 102.864 \\ 102.864 \end{pmatrix} \cdot deg$$

$$I_{1\_L2\_NoM} := \frac{V1_{NoM_0} - V1_{NoM_4}}{Z_{1L2}}$$

$$I_{2\_L2\_NoM} := \frac{V2_{NoM_0} - V2_{NoM_4}}{Z_{1L2}}$$

$$I_{0\_L2\_NoM} := \frac{V0_{NoM_0} - V0_{NoM_4}}{Z_{0L2}}$$

$$I_{ABC\_L2\_NoM} := A_{012}\cdot\begin{pmatrix} I_{0\_L2\_NoM} \\ I_{1\_L2\_NoM} \\ I_{2\_L2\_NoM} \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_L2\_NoM}\right|} = \begin{pmatrix} 1.0853 \\ 0.0073 \\ 0.0073 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_L2\_NoM}\right)} = \begin{pmatrix} -84.197 \\ 102.864 \\ 102.864 \end{pmatrix}\cdot deg$$

$$I_{1\_L3\_NoM} := \frac{V1_{NoM_1} - V1_{NoM_2}}{Z_{1L3}}$$

$$I_{2\_L3\_NoM} := \frac{V2_{NoM_1} - V2_{NoM_2}}{Z_{1L3}}$$

$$I_{0\_L3\_NoM} := \frac{V0_{NoM_1} - V0_{NoM_2}}{Z_{0L3}}$$

$$I_{ABC\_L3\_NoM} := A_{012}\cdot\begin{pmatrix} I_{0\_L3\_NoM} \\ I_{1\_L3\_NoM} \\ I_{2\_L3\_NoM} \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_L3\_NoM}\right|} = \begin{pmatrix} 0.0487 \\ 0.007 \\ 0.007 \end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_L3\_NoM}\right)} = \begin{pmatrix} -80.275 \\ 102.864 \\ 102.864 \end{pmatrix}\cdot deg$$

$$I_{ABC\_L3\_NoM} + I_{ABC\_L1\_NoM} = \begin{pmatrix} 0.016 - 0.096i \\ -3.116\times 10^{-3} + 0.014i \\ -3.116\times 10^{-3} + 0.014i \end{pmatrix}$$

Case 2: With mutual coupling:

$$I_{0\_M} := \frac{V_f}{Z_{Bus1_{4,4}} + Z_{Bus2_{4,4}} + Z_{0\_M_{4,4}}} \qquad \left|I_{0\_M}\right| = 0.694 \cdot pu \qquad \arg\left(I_{0\_M}\right) = -84.48 \cdot deg$$

$$I_{1\_M} := I_{0\_M} \qquad I_{2\_M} := I_{1\_M}$$

$$I_{ABC\_SLG\_M} := A_{012} \cdot \begin{pmatrix} I_{0\_M} \\ I_{1\_M} \\ I_{2\_M} \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_SLG\_M}\right|} = \begin{pmatrix} 2.081 \\ 0 \\ 0 \end{pmatrix} \qquad \overrightarrow{\arg\left(I_{ABC\_SLG\_M}\right)} = \begin{pmatrix} -84.48 \\ 13.173 \\ 13.173 \end{pmatrix} \cdot deg$$

- Very close match to Powerworld results

$$\Delta V1_M := Z_{Bus1} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_{1\_M} \end{pmatrix} \qquad V1_M := \begin{pmatrix} 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \end{pmatrix} + \Delta V1_M$$

$$\overrightarrow{\left|V1_M\right|} = \begin{pmatrix} 0.885 \\ 0.882 \\ 0.879 \\ 0.877 \\ 0.809 \end{pmatrix} \qquad \overrightarrow{\arg\left(V1_M\right)} = \begin{pmatrix} -0.777 \\ -0.746 \\ -0.722 \\ -0.699 \\ 0.431 \end{pmatrix} \cdot deg$$

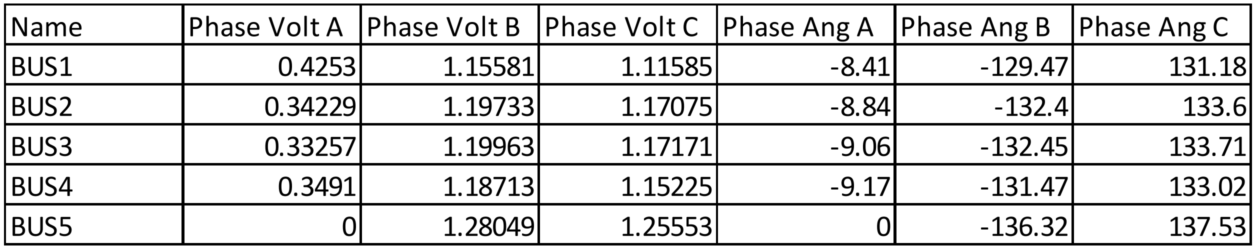
$$\Delta V2_M := Z_{Bus2}\cdot\begin{pmatrix}0\\0\\0\\0\\-I_{2\_M}\end{pmatrix} \qquad V2_M := \Delta V2_M$$

$$\Delta V0_M := Z_{0\_M}\cdot\begin{pmatrix}0\\0\\0\\0\\-I_{0\_M}\end{pmatrix} \qquad V0_M := \Delta V0_M$$

$$V_{ABC\_B1\_M} := A_{012}\cdot\begin{pmatrix}V0_{M_0}\\V1_{M_0}\\V2_{M_0}\end{pmatrix} \qquad \overrightarrow{\left|V_{ABC\_B1\_M}\right|} = \begin{pmatrix}0.4253\\1.1558\\1.1159\end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(V_{ABC\_B1\_M}\right)} = \begin{pmatrix}-8.412\\-129.471\\131.182\end{pmatrix}\cdot deg$$

$$V_{ABC\_B2\_M} := A_{012}\cdot\begin{pmatrix}V0_{M_1}\\V1_{M_1}\\V2_{M_1}\end{pmatrix} \qquad \overrightarrow{\left|V_{ABC\_B2\_M}\right|} = \begin{pmatrix}0.3423\\1.1973\\1.1708\end{pmatrix}\cdot pu \qquad \overrightarrow{\arg\left(V_{ABC\_B2\_M}\right)} = \begin{pmatrix}-8.838\\-132.399\\133.597\end{pmatrix}\cdot deg$$

Compare to Powerworld



Name	Phase Volt A	Phase Volt B	Phase Volt C	Phase Ang A	Phase Ang B	Phase Ang C
BUS1	0.4253	1.15581	1.11585	-8.41	-129.47	131.18
BUS2	0.34229	1.19733	1.17075	-8.84	-132.4	133.6
BUS3	0.33257	1.19963	1.17171	-9.06	-132.45	133.71
BUS4	0.3491	1.18713	1.15225	-9.17	-131.47	133.02
BUS5	0	1.28049	1.25553	0	-136.32	137.53

- First get the positive and negative sequence currents in each of the three branches involved in the mutual coupled lines

$$I_{1\_L1\_M} := \frac{V1_{M_0} - V1_{M_1}}{Z_{1L1}} \qquad \left|I_{1\_L1\_M}\right| = 0.019 \qquad \arg\left(I_{1\_L1\_M}\right) = -79.972 \cdot \text{deg}$$

$$I_{2\_L1\_M} := \frac{V2_{M_0} - V2_{M_1}}{Z_{1L1}} \qquad \left|I_{2\_L1\_M}\right| = 0.019 \qquad \arg\left(I_{2\_L1\_M}\right) = -79.972 \cdot \text{deg}$$

$$I_{1\_L2\_M} := \frac{V1_{M_0} - V1_{M_4}}{Z_{1L2}} \qquad \left|I_{1\_L2\_M}\right| = 0.366 \qquad \arg\left(I_{1\_L2\_M}\right) = -84.241 \cdot \text{deg}$$

$$I_{2\_L2\_M} := \frac{V2_{M_0} - V2_{M_4}}{Z_{1L2}} \qquad \left|I_{2\_L2\_M}\right| = 0.366 \qquad \arg\left(I_{2\_L2\_M}\right) = -84.241 \cdot \text{deg}$$

$$I_{1\_L3\_M} := \frac{V1_{M_1} - V1_{M_2}}{Z_{1L3}} \qquad \left|I_{1\_L3\_M}\right| = 0.019 \cdot pu \qquad \arg\left(I_{1\_L3\_M}\right) = -79.972 \cdot deg$$

$$I_{2\_L3\_M} := \frac{V2_{M_1} - V2_{M_2}}{Z_{1L3}} \qquad \left|I_{2\_L3\_M}\right| = 0.019 \cdot pu \qquad \arg\left(I_{2\_L3\_M}\right) = -79.972 \cdot deg$$

- The zero sequence calculation is little more tricky. This is based on equation 12.19 in the text book
- We need terms from the matrix of impedance primitives, recall:

$$Z_{pr0M} := \begin{pmatrix} Z_{S0} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{R0} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{0L1} & Z_{0M12} & 0 & 0 & 0 \\ 0 & 0 & Z_{0M12} & Z_{0L2} & -Z_{0M23} & 0 & 0 \\ 0 & 0 & 0 & -Z_{0M23} & Z_{0L3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_{0L4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_{0L5} \end{pmatrix}$$

We need the 3x3 block in the middle of the matrix.

$$Z_{mutual0} := \mathrm{submatrix}\left(Z_{pr0M}, 2, 4, 2, 4\right) \qquad Z_{mutual0} = \begin{pmatrix} 0.118 + 0.582i & 0.06 + 0.256i & 0 \\ 0.06 + 0.256i & 0.142 + 0.698i & -0.015 - 0.064i \\ 0 & -0.015 - 0.064i & 0.089 + 0.436i \end{pmatrix}$$

$$\begin{pmatrix} I_{0\_L1\_M} \\ I_{0\_L2\_M} \\ I_{0\_L3\_M} \end{pmatrix} := Z_{mutual0}^{-1} \cdot \begin{pmatrix} V0_{M_0} - V0_{M_1} \\ V0_{M_0} - V0_{M_4} \\ V0_{M_2} - V0_{M_1} \end{pmatrix}$$

$$\left|I_{0\_L1\_M}\right| = 0.048 \qquad \arg\left(I_{0\_L1\_M}\right) = 90.677 \cdot deg$$

$$\left|I_{0\_L2\_M}\right| = 0.4 \qquad \arg\left(I_{0\_L2\_M}\right) = -84.435 \cdot deg$$

- Note sign on I_{0_L3}, dot polarity

$$\left|I_{0\_L3\_M}\right| = 0.048 \qquad \arg\left(-I_{0\_L3\_M}\right) = 90.677 \cdot deg$$

$$I_{ABC\_L1\_M} := A_{012} \cdot \begin{pmatrix} I_{0\_L1\_M} \\ I_{1\_L1\_M} \\ I_{2\_L1\_M} \end{pmatrix}$$

$$\overrightarrow{\left|I_{ABC\_L1\_M}\right|} = \begin{pmatrix} 0.013 \\ 0.0669 \\ 0.0669 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_L1\_M}\right)} = \begin{pmatrix} 62.941 \\ 93.277 \\ 93.277 \end{pmatrix} \cdot deg$$

$$\overrightarrow{\left|I_{ABC\_L1\_NoM}\right|} = \begin{pmatrix} 0.0487 \\ 0.007 \\ 0.007 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_L1\_NoM}\right)} = \begin{pmatrix} -80.275 \\ 102.864 \\ 102.864 \end{pmatrix} \cdot deg$$

$$I_{ABC\_L2\_M} := A_{012} \cdot \begin{pmatrix} I_{0\_L2\_M} \\ I_{1\_L2\_M} \\ I_{2\_L2\_M} \end{pmatrix}$$

$$\overrightarrow{\left|I_{ABC\_L2\_M}\right|} = \begin{pmatrix} 1.1325 \\ 0.0336 \\ 0.0336 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_L2\_M}\right)} = \begin{pmatrix} -84.309 \\ -86.547 \\ -86.547 \end{pmatrix} \cdot deg$$

$$\overrightarrow{\left|I_{ABC\_L2\_NoM}\right|} = \begin{pmatrix} 1.0853 \\ 0.0073 \\ 0.0073 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_L3\_NoM}\right)} = \begin{pmatrix} -80.275 \\ 102.864 \\ 102.864 \end{pmatrix} \cdot deg$$

$$I_{ABC\_L3\_M} := A_{012} \cdot \begin{pmatrix} -I_{0\_L3\_M} \\ I_{1\_L3\_M} \\ I_{2\_L3\_M} \end{pmatrix}$$

$$\overrightarrow{\left|I_{ABC\_L3\_M}\right|} = \begin{pmatrix} 0.013 \\ 0.0669 \\ 0.0669 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_L3\_M}\right)} = \begin{pmatrix} 62.941 \\ 93.277 \\ 93.277 \end{pmatrix} \cdot deg$$

$$\overrightarrow{\left|I_{ABC\_L3\_NoM}\right|} = \begin{pmatrix} 0.0487 \\ 0.007 \\ 0.007 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_L3\_NoM}\right)} = \begin{pmatrix} -80.275 \\ 102.864 \\ 102.864 \end{pmatrix} \cdot deg$$

$$I_{ABC\_L3\_M} - I_{ABC\_L1\_M} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\overrightarrow{\left|I_{ABC\_L1\_M}\right|} = \begin{pmatrix} 0.013 \\ 0.0669 \\ 0.0669 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg\left(I_{ABC\_L1\_M}\right)} = \begin{pmatrix} 62.941 \\ 93.277 \\ 93.277 \end{pmatrix} \cdot deg$$

Powerworld Results

Phase Cur A From	Phase Ang A Fron	Phase Cur B From	Phase Ang B Fron	Phase Cur C From	Phase Ang C From
0.01303	62.95	0.06689	93.28	0.06689	93.28
1.13243	-84.31	0.03359	-86.54	0.03359	-86.54
0.01303	62.95	0.06689	93.28	0.06689	93.28
0.01303	62.95	0.06689	93.28	0.06689	93.28
0.94891	95.32	0.03359	-86.54	0.03359	-86.54