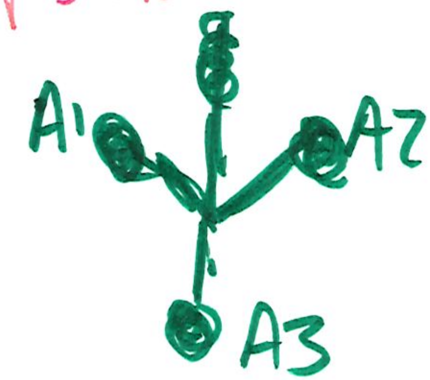
ECE 523
Symmetrical Components

Session 26

$$\underset{3\times3}{Zequiv := Za} - \underset{3\times6}{Zbnew}\cdot \underset{6\times6}{Zdnew}^{-1}\cdot \underset{6\times3}{Zcnew}$$

$$Zequiv = \begin{pmatrix} 0.1343 + 1.0626i & 0.0953 + 0.577i & 0.0953 + 0.493i \\ 0.0953 + 0.577i & 0.1343 + 1.0625i & 0.0953 + 0.577i \\ 0.0953 + 0.493i & 0.0953 + 0.577i & 0.1343 + 1.0626i \end{pmatrix} \cdot \frac{ohm}{mi}$$

$$GMR_{Bundle} = \sqrt[n]{D_s \cdot d_{12} \cdot d_{13} \cdots d_{1n}}$$

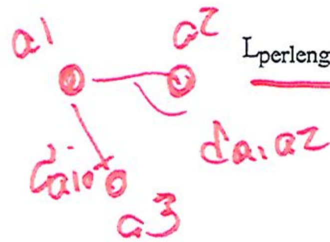


Now if we had instead used the approximations of using the GMR of the bundle

conductor GMR

$$Rsbundle := (Ds\cdot Da1a2\cdot Da1a3)^{\frac{1}{3}}$$

Geometric mean radius of Bundle



$$L_{perlengthnew} := \frac{\mu_0}{2\cdot\pi}\cdot\begin{pmatrix} \ln\left(\frac{D_e}{Rsbundle}\right) & \ln\left(\frac{D_e}{Da1b1}\right) & \ln\left(\frac{D_e}{Da1c1}\right) \\ \ln\left(\frac{D_e}{Da1b1}\right) & \ln\left(\frac{D_e}{Rsbundle}\right) & \ln\left(\frac{D_e}{Db1c1}\right) \\ \ln\left(\frac{D_e}{Da1c1}\right) & \ln\left(\frac{D_e}{Db1c1}\right) & \ln\left(\frac{D_e}{Rsbundle}\right) \end{pmatrix}$$

The resistance matrix must also be modified since there are now parallel conductors

# of conductors in bundle

$$R_{perlengthnew} := \begin{pmatrix} \frac{Rac_{bund3}}{3} + R_d & R_d & R_d \\ R_d & \frac{Rac_{bund3}}{3} + R_d & R_d \\ R_d & R_d & \frac{Rac_{bund3}}{3} + R_d \end{pmatrix}$$

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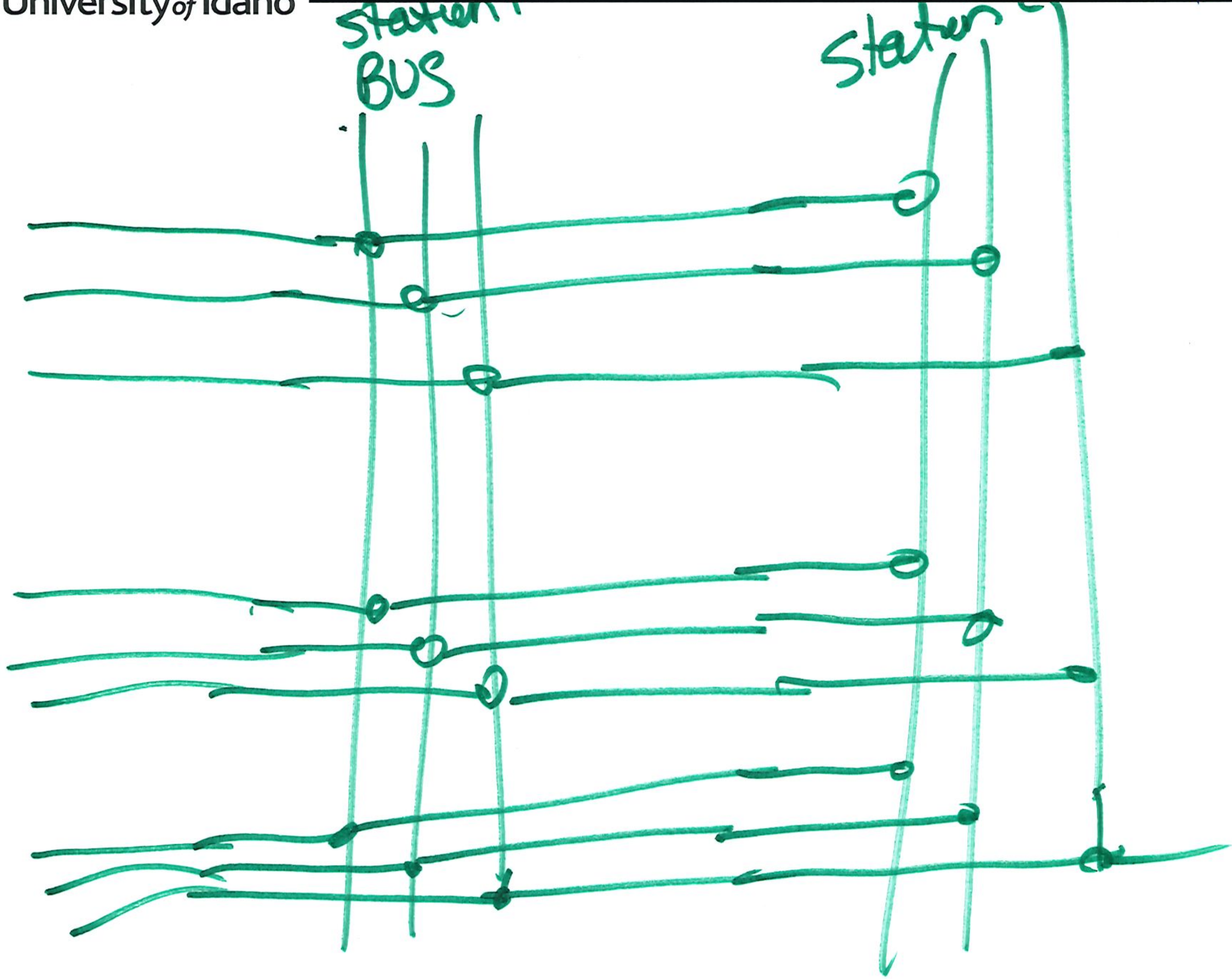

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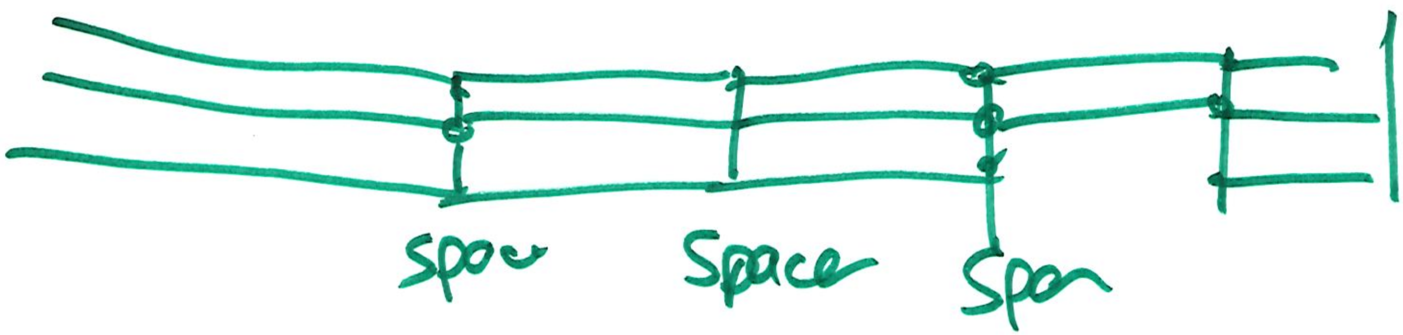

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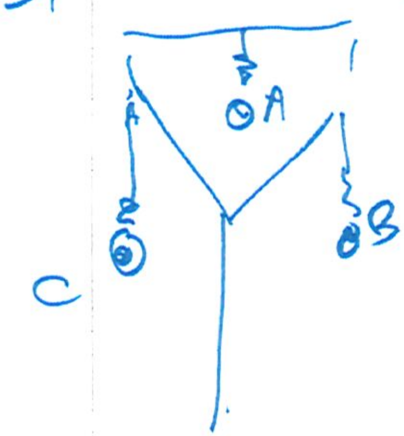
method vector potentials

A B C

$$P = \frac{1}{2\pi\varepsilon_0} \begin{bmatrix} \ln\left(\frac{H_{aa'}}{r}\right) & \ln\left(\frac{H_{ab'}}{D_{ab}}\right) & \ln\left(\frac{H_{ac'}}{D_{ac}}\right) \\ & \ln\left(\frac{H_{bb'}}{r}\right) & \\ & & \ln\left(\frac{H_{cc'}}{r}\right) \end{bmatrix}$$

$2\pi\varepsilon_r\varepsilon_0$

$= 1$



A' B' C'

P has units $\frac{m}{F}$

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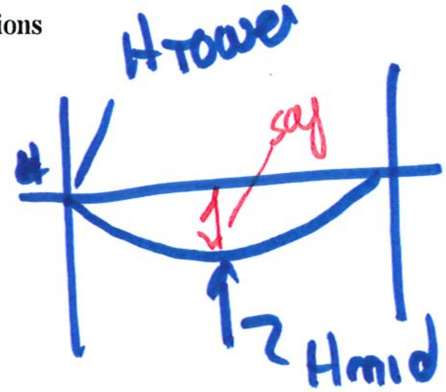
Overhead Line Capacitance Calculations

A. Start with the same line used for the series impedance calculations



- Conductor radius:

$$\text{dia} := 0.528\text{in} \qquad r := \frac{\text{dia}}{2}$$

- Space between phase conductors

$$\text{Dab} := 10\text{ft} \qquad \text{Dac} := 20\text{ft} \qquad \text{Dbc} := 10\text{ft}$$

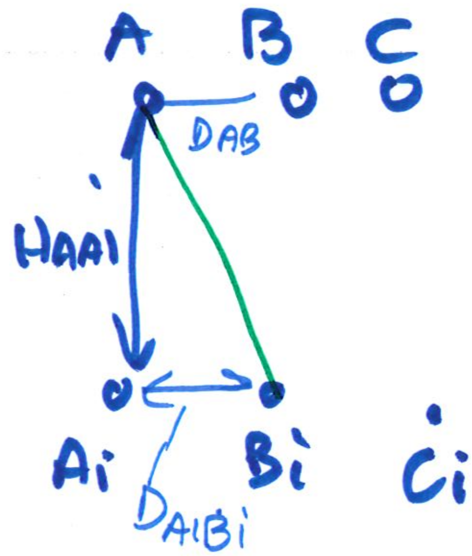
- Height calculations and distance to image conductors

$$H_{tower} := 45\text{ft} \qquad \text{Sag} := 15\text{ft}$$

$$\text{Ha} := H_{tower} - \frac{2}{3}\cdot\text{Sag} \qquad \text{Ha} = 35\cdot\text{ft} \qquad \text{Hb} := \text{Ha} \qquad \text{Hc} := \text{Ha}$$

$$\text{Haai} := 2\cdot\text{Ha} \qquad \text{Haai} = 70\cdot\text{ft}$$

$$\text{Hbbi} := 2\cdot\text{Hb} \qquad \text{Hcci} := 2\cdot\text{Hc}$$



$$\text{Habi} := \sqrt{(2\cdot\text{Ha})^2 + \text{Dab}^2} \qquad \text{Habi} = 70.71\cdot\text{ft}$$

$$\text{Haci} := \sqrt{(2\cdot\text{Ha})^2 + \text{Dac}^2} \qquad \text{Haci} = 72.8\cdot\text{ft}$$

$$\text{Hbci} := \sqrt{(2\cdot\text{Hc})^2 + \text{Dbc}^2} \qquad \text{Hbci} = 70.71\cdot\text{ft}$$

$$P := \frac{1}{(2\cdot\pi\cdot\varepsilon_0)}\begin{pmatrix} \ln\left(\frac{\text{Haai}}{r}\right) & \ln\left(\frac{\text{Habi}}{\text{Dab}}\right) & \ln\left(\frac{\text{Haci}}{\text{Dac}}\right) \\ \ln\left(\frac{\text{Habi}}{\text{Dab}}\right) & \ln\left(\frac{\text{Hbbi}}{r}\right) & \ln\left(\frac{\text{Hbci}}{\text{Dbc}}\right) \\ \ln\left(\frac{\text{Haci}}{\text{Dac}}\right) & \ln\left(\frac{\text{Hbci}}{\text{Dbc}}\right) & \ln\left(\frac{\text{Hcci}}{r}\right) \end{pmatrix} \qquad P = \begin{pmatrix} 0.14 & 0.04 & 0.02 \\ 0.04 & 0.14 & 0.04 \\ 0.02 & 0.04 & 0.14 \end{pmatrix}\cdot\frac{\text{m}}{\text{pF}}$$

P is matrix of "Potential Coefficients"

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$C' := P^{-1}$

$$C' = \begin{pmatrix} 11.93 & -2.58 & -1.29 \\ -2.58 & 12.35 & -2.58 \\ -1.29 & -2.58 & 11.93 \end{pmatrix} \cdot \frac{nF}{mi}$$

Length := 40mi

$C_{untran} := C' \cdot Length$

$$C_{untran} = \begin{pmatrix} 0.48 & -0.1 & -0.05 \\ -0.1 & 0.49 & -0.1 \\ -0.05 & -0.1 & 0.48 \end{pmatrix} \cdot \mu F$$

Transformation Matrix:

$a := 1 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}}$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$C_{012} := A_{012}^{-1} \cdot C_{untran} \cdot A_{012}$

C0 C1

$$C_{012} = \begin{pmatrix} 0.31 & 0.01 + 0.01i & 0.01 - 0.01i \\ 0.01 - 0.01i & 0.57 & -0.02 - 0.03i \\ 0.01 + 0.01i & -0.02 + 0.03i & 0.57 \end{pmatrix} \cdot \mu F$$

C2

- Note the complex capacitance values in the off diagonal terms.

$C_0 := C_{012_{0,0}}$ $\quad C_0 = 310.93 \cdot nF$

$C_1 := C_{012_{1,1}}$ $\quad C_1 = 568.93 \cdot nF$ $\quad C_{012_{1,1}} - C_{012_{2,2}} = 0 \cdot nF$ $\quad$ So C2=C1

- If we use the per phase capacitance formulas:

$Dm := (Dab \cdot Dac \cdot Dbc)^{\frac{1}{3}}$ $\quad Dm = 12.6 \cdot ft$

$c_{phase} := \dfrac{2 \cdot \pi \cdot \varepsilon_0}{\ln\left(\frac{Dm}{r}\right)}$ $\quad c_{phase} = 8.761 \cdot \frac{pF}{m}$ $\quad c_{phase} \cdot Length = 563.95 \cdot nF$

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$C0\_W := 40mi \cdot C\_zero\_west \qquad C0\_W = 311.552 \cdot nF \qquad C_0 = 310.93 \cdot nF$

Transposed Line Examples:

$$Rp := \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

original Rp

$$Rp_{26W} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Transpose Case 1:

$f1 := 0.2 \qquad f2 := 0.3 \qquad f3 := 0.5$

$$C_{net} := f1 \cdot C_{untran} + f2 \cdot Rp^{-1} \cdot C_{untran} \cdot Rp + f3 \cdot Rp \cdot C_{untran} \cdot Rp^{-1}$$

$$C_{net} = \begin{pmatrix} 0.48 & -0.08 & -0.09 \\ -0.08 & 0.48 & -0.09 \\ -0.09 & -0.09 & 0.49 \end{pmatrix} \cdot \mu F$$

$$C_{0121} := A_{012}^{-1} \cdot C_{net} \cdot A_{012}$$

$$C_{0121} = \begin{pmatrix} 0.31 & 5.84 \times 10^{-4} - 0i & 5.84 \times 10^{-4} + 0i \\ 5.84 \times 10^{-4} + 0i & 0.57 & -0 + 0.01i \\ 5.84 \times 10^{-4} - 0i & -0 - 0.01i & 0.57 \end{pmatrix} \cdot \mu F$$

Transposition Case 2:

$f13 := 0.4 \qquad f23 := 0.6 \qquad f33 := 0.0$

$$C_{net3} := f13 \cdot C_{untran} + f23 \cdot Rp^{-1} \cdot C_{untran} \cdot Rp + f33 \cdot Rp \cdot C_{untran} \cdot Rp^{-1}$$

$$C_{net3} = \begin{pmatrix} 0.49 & -0.1 & -0.08 \\ -0.1 & 0.48 & -0.07 \\ -0.08 & -0.07 & 0.48 \end{pmatrix} \cdot \mu F$$

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$C_{0123} := A_{012}^{-1} \cdot C_{net3} \cdot A_{012}$

$$C_{0123} = \begin{pmatrix} 0.31 & -0+0i & -0-0i \\ -0-0i & 0.57 & 0.02-0.01i \\ -0+0i & 0.02+0.01i & 0.57 \end{pmatrix} \cdot \mu F$$

Transposition Case 3:

$f16 := \frac{1}{3}$ $\quad f26 := \frac{1}{3}$ $\quad f36 := \frac{1}{3}$

$$C_{net6} := f16 \cdot C_{untran} + f26 \cdot Rp^{-1} \cdot C_{untran} \cdot Rp + f36 \cdot Rp \cdot C_{untran} \cdot Rp^{-1}$$

$$C_{net6} = \begin{pmatrix} 0.48 & -0.09 & -0.09 \\ -0.09 & 0.48 & -0.09 \\ -0.09 & -0.09 & 0.48 \end{pmatrix} \cdot \mu F$$

$C_{0126} := A_{012}^{-1} \cdot C_{net6} \cdot A_{012}$

$$C_{0126} = \begin{pmatrix} 0.3109 & 0 & 0 \\ 0 & 0.5689 & 0 \\ 0 & 0 & 0.5689 \end{pmatrix} \cdot \mu F$$

Line with two conductor bundles

Conductor data from table:

$dia2 := 1.108in$ $\quad rad2 := \frac{dia2}{2}$ $\quad rad2 = 0.05 \cdot ft$

Spacing:

Within the bundle:

$Da1a2 := 1.5ft$ $\quad Db1b2 := 1.5ft$ $\quad Dc1c2 := 1.5ft$

Between Phases

$Da1b1 := 24ft$ $\quad Da1b2 := Da1b1 + Db1b2$ $\quad Da1b2 = 25.5 \cdot ft$

$Da2b1 := Da1b1 - Da1a2$ $\quad Da2b1 = 22.5 \cdot ft$ $\quad Da2b2 := 24ft$

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If length = 40 miles:

$C_{line\_bu} := C_{bund} \cdot 40mi$

A1 A2
0 0
V'A1 VA2
VA2-VA1

PA PB

$$C_{line\_bu} = \begin{pmatrix} 0.68 & -0.03 & -0.01 & -0.35 & -0.03 & -0.01 \\ -0.03 & 0.69 & -0.03 & -0.04 & -0.34 & -0.03 \\ -0.01 & -0.03 & 0.68 & -0.01 & -0.04 & -0.35 \\ -0.35 & -0.04 & -0.01 & 0.68 & -0.03 & -0.01 \\ -0.03 & -0.34 & -0.04 & -0.03 & 0.69 & -0.03 \\ -0.01 & -0.03 & -0.35 & -0.01 & -0.03 & 0.68 \end{pmatrix} \cdot \mu F$$

Pc Pd

Matrix Reduction (note this is done to the P matrix:

$Pa := submatrix(P_u, 0, 2, 0, 2)$

$$Pa = \begin{pmatrix} 0.13 & 0.02 & 0.01 \\ 0.02 & 0.13 & 0.02 \\ 0.01 & 0.02 & 0.13 \end{pmatrix} \cdot \frac{m}{pF}$$

$Pb := submatrix(P_u, 0, 2, 3, 5)$

$$Pb = \begin{pmatrix} 0.07 & 0.02 & 0.01 \\ 0.02 & 0.07 & 0.02 \\ 0.01 & 0.02 & 0.07 \end{pmatrix} \cdot \frac{m}{pF}$$

$Pc := submatrix(P_u, 3, 5, 0, 2)$

$$Pc = \begin{pmatrix} 0.07 & 0.02 & 0.01 \\ 0.02 & 0.07 & 0.02 \\ 0.01 & 0.02 & 0.07 \end{pmatrix} \cdot \frac{m}{pF}$$

$Pd := submatrix(P_u, 3, 5, 3, 5)$

$$Pd = \begin{pmatrix} 0.13 & 0.02 & 0.01 \\ 0.02 & 0.13 & 0.02 \\ 0.01 & 0.02 & 0.13 \end{pmatrix} \cdot \frac{m}{pF}$$

Modify matrix by performing: Qa'b'c'-Qabc and Vabc + Va'b'c' similar to what was done for series Zabc

$Pbnew := Pb - Pa$ $\quad$ $Pcnew := Pc - Pa$ $\quad$ $Pdnew := Pa - Pb - Pc + Pd$

Reduce to equivalent 3x3 matrix

$Pequiv := Pa - Pbnew \cdot Pdnew^{-1} \cdot Pcnew$

Reduced P

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$$\mathrm{Pequiv} = \begin{pmatrix} 0.1 & 0.02 & 0.01 \\ 0.02 & 0.1 & 0.02 \\ 0.01 & 0.02 & 0.1 \end{pmatrix} \cdot \frac{\mathrm{m}}{\mathrm{pF}} \qquad \mathrm{Cequiv} := \mathrm{Pequiv}^{-1}$$

$$\mathrm{Cequiv} = \begin{pmatrix} 10.43 & -1.97 & -0.67 \\ -1.97 & 10.76 & -1.97 \\ -0.67 & -1.97 & 10.43 \end{pmatrix} \cdot \frac{\mathrm{pF}}{\mathrm{m}} \qquad \mathrm{Cequiv\_len} := \mathrm{Cequiv} \cdot \mathrm{Length}$$

effective radius

$$C_{0128} := A_{012}^{-1} \cdot \mathrm{Cequiv\_len} \cdot A_{012}$$

$$C_{0128} = \begin{pmatrix} 0.48 & 0.01+0.02i & 0.01-0.02i \\ 0.01-0.02i & 0.78 & -0.03-0.05i \\ 0.01+0.02i & -0.03+0.05i & 0.78 \end{pmatrix} \cdot \mu\mathrm{F}$$

Note that the original C012 was:

$$C_{012} = \begin{pmatrix} 0.31 & 0.01+0.01i & 0.01-0.01i \\ 0.01-0.01i & 0.57 & -0.02-0.03i \\ 0.01+0.01i & -0.02+0.03i & 0.57 \end{pmatrix} \cdot \mu\mathrm{F}$$

Compare what the bundling has done to the sequence capacitances

Now if we had instead used the approximations of using the GMR of the bundle

$$\mathrm{Rsbundle} := \sqrt{\mathrm{rad2} \cdot \mathrm{Da1a2}}$$

$$\mathrm{Pequivbund} := \frac{1}{2 \cdot \pi \cdot \varepsilon_0} \cdot \begin{pmatrix} \ln\left(\frac{\mathrm{Haai}}{\mathrm{Rsbundle}}\right) & \ln\left(\frac{\mathrm{Habi}}{\mathrm{Da1b1}}\right) & \ln\left(\frac{\mathrm{Haci}}{\mathrm{Da1c1}}\right) \\ \ln\left(\frac{\mathrm{Habi}}{\mathrm{Da1b1}}\right) & \ln\left(\frac{\mathrm{Hbbi}}{\mathrm{Rsbundle}}\right) & \ln\left(\frac{\mathrm{Hbci}}{\mathrm{Db1c1}}\right) \\ \ln\left(\frac{\mathrm{Haci}}{\mathrm{Da1c1}}\right) & \ln\left(\frac{\mathrm{Hbci}}{\mathrm{Db1c1}}\right) & \ln\left(\frac{\mathrm{Hcci}}{\mathrm{Rsbundle}}\right) \end{pmatrix}$$

$$\mathrm{Cequivbund} := \mathrm{Pequivbund}^{-1} \cdot \mathrm{Length}$$

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$$\mathrm{Cequivbund} = \begin{pmatrix} 667.35 & -124.16 & -25.76 \\ -124.16 & 689.46 & -124.16 \\ -25.76 & -124.16 & 667.35 \end{pmatrix} \cdot \mathrm{nF}$$

Error between these methods:

$$\mathrm{Error} := \mathrm{Cequiv\_len} - \mathrm{Cequivbund} \qquad \mathrm{Error} = \begin{pmatrix} 4.098 & -2.685 & -17.278 \\ -2.685 & 3.243 & -2.685 \\ -17.278 & -2.685 & 4.098 \end{pmatrix} \cdot \mathrm{nF}$$

Or calculating the sequence capacitance

$$C_{0128e} := A_{012}^{-1} \cdot \mathrm{Cequivbund} \cdot A_{012}$$

$$C_{0128e} = \begin{pmatrix} 492 & 12.72 + 22.03i & 12.72 - 22.03i \\ 12.72 - 22.03i & 766.08 & -36.49 - 63.2i \\ 12.72 + 22.03i & -36.49 + 63.2i & 766.08 \end{pmatrix} \cdot \mathrm{nF}$$

$$\mathrm{Err} := C_{0128} - C_{0128e}$$

$$\mathrm{Err} = \begin{pmatrix} -11.285 & -2.29 - 3.966i & -2.29 + 3.966i \\ -2.29 + 3.966i & 11.362 & 5.007 + 8.672i \\ -2.29 - 3.966i & 5.007 - 8.672i & 11.362 \end{pmatrix} \cdot \mathrm{nF}$$

This is comparable to what we saw for the impedance

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Now consider a static wire case:

- Consider two GW options

$\mathrm{dia}_{gw\_375in} := 0.385\mathrm{in}$ $\quad\quad$ $\mathrm{dia}_{gw1F} := 0.346\mathrm{in}$

$\mathrm{Dagw} := \sqrt{(10\mathrm{ft})^2 + (15\mathrm{ft})^2}$ $\quad\quad$ $\mathrm{Dcgw} := \mathrm{Dagw}$ $\quad\quad$ $\mathrm{Dbgw} := 15\mathrm{ft}$

$\mathrm{Hgw}_{tower} := 60\mathrm{ft}$ $\quad\quad$ $\mathrm{Sag}_{gw} := 15\mathrm{ft}$

- Note that sag for ground wire is often different than for phase conductors. This will impact distance between phase conductors and ground wire

$\mathrm{Hgw} := \mathrm{Hgw}_{tower} - \frac{2}{3}\cdot \mathrm{Sag}_{gw}$ $\quad\quad$ $\mathrm{Hgw} = 50\cdot\mathrm{ft}$ $\quad\quad$ $\mathrm{Hgwgwi} := 2\cdot\mathrm{Hgw}$

$\mathrm{Hagwi} := \sqrt{(2\cdot\mathrm{Ha} + \mathrm{Dbgw})^2 + \mathrm{Dab}^2}$ $\quad\quad$ $\mathrm{Hagwi} = 85.59\cdot\mathrm{ft}$

$\mathrm{Hcgwi} := \sqrt{(2\cdot\mathrm{Hc} + \mathrm{Dbgw})^2 + \mathrm{Dbc}^2}$ $\quad\quad$ $\mathrm{Hcgwi} = 85.59\cdot\mathrm{ft}$

$\mathrm{Hbgwi} := 2\cdot\mathrm{Hb} + \mathrm{Dbgw}$ $\quad\quad$ $\mathrm{Hbgwi} = 85\cdot\mathrm{ft}$

First Ground Wire Case: 3/8" copperweld

PA $\quad\quad$ PB

$$\mathrm{Pgw1} := \frac{1}{2\pi\cdot\varepsilon_0}\cdot\left(\begin{array}{ccc|c}
\ln\left(\frac{\mathrm{Haai}}{r}\right) & \ln\left(\frac{\mathrm{Habi}}{\mathrm{Dab}}\right) & \ln\left(\frac{\mathrm{Haci}}{\mathrm{Dac}}\right) & \ln\left(\frac{\mathrm{Hagwi}}{\mathrm{Dagw}}\right) \\
\ln\left(\frac{\mathrm{Habi}}{\mathrm{Dab}}\right) & \ln\left(\frac{\mathrm{Hbbi}}{r}\right) & \ln\left(\frac{\mathrm{Hbci}}{\mathrm{Dbc}}\right) & \ln\left(\frac{\mathrm{Hbgwi}}{\mathrm{Dbgw}}\right) \\
\ln\left(\frac{\mathrm{Haci}}{\mathrm{Dac}}\right) & \ln\left(\frac{\mathrm{Hbci}}{\mathrm{Dbc}}\right) & \ln\left(\frac{\mathrm{Hcci}}{r}\right) & \ln\left(\frac{\mathrm{Hcgwi}}{\mathrm{Dcgw}}\right) \\
\hline
\ln\left(\frac{\mathrm{Hagwi}}{\mathrm{Dagw}}\right) & \ln\left(\frac{\mathrm{Hbgwi}}{\mathrm{Dbgw}}\right) & \ln\left(\frac{\mathrm{Hcgwi}}{\mathrm{Dcgw}}\right) & \ln\left(\frac{\mathrm{Hgwgwi}}{\frac{\mathrm{dia}_{gw_375in}}{2}}\right)
\end{array}\right)$$

PC $\quad\quad$ PD

$$\mathrm{Pgw1} = \begin{pmatrix} 0.14 & 0.04 & 0.02 & 0.03 \\ 0.04 & 0.14 & 0.04 & 0.03 \\ 0.02 & 0.04 & 0.14 & 0.03 \\ 0.03 & 0.03 & 0.03 & 0.16 \end{pmatrix}\cdot\frac{\mathrm{m}}{\mathrm{pF}}$$

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Matrix Reduction (note this is done to the P matrix:

$$Pagw1 := \mathrm{submatrix}(Pgw1,0,2,0,2) \qquad Pagw1 = \begin{pmatrix} 0.14 & 0.04 & 0.02 \\ 0.04 & 0.14 & 0.04 \\ 0.02 & 0.04 & 0.14 \end{pmatrix} \cdot \frac{m}{pF}$$

$$Pbgw1 := \mathrm{submatrix}(Pgw1,0,2,3,3) \qquad Pbgw1 = \begin{pmatrix} 0.03 \\ 0.03 \\ 0.03 \end{pmatrix} \cdot \frac{m}{pF}$$

$$Pcgw1 := \mathrm{submatrix}(Pgw1,3,3,0,2)$$

$$Pcgw1 = \begin{pmatrix} 0.03 & 0.03 & 0.03 \end{pmatrix} \cdot \frac{m}{pF}$$

$$Pdgw1 := \mathrm{submatrix}(Pgw1,3,3,3,3)$$

$$Pdgw1 = (0.16) \cdot \frac{m}{pF}$$

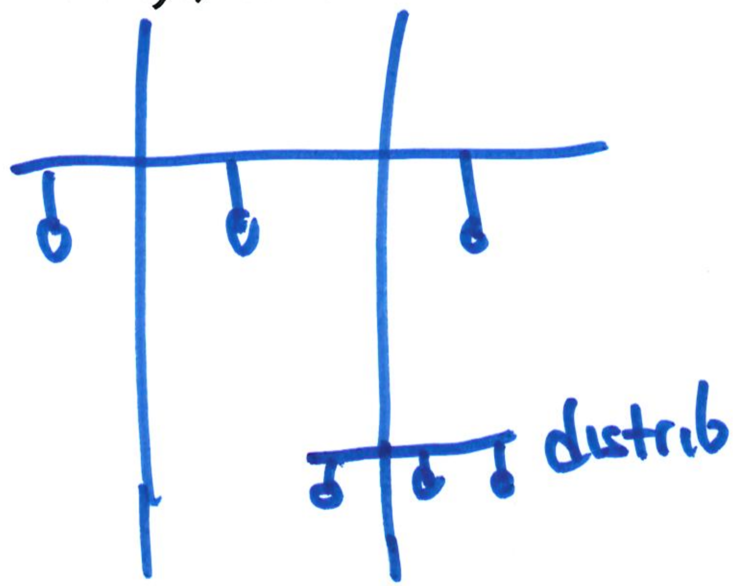
Reduce to equivalent 3x3 matrix

$$Pequivgw1 := Pagw1 - Pbgw1 \cdot Pdgw1^{-1} \cdot Pcgw1 \qquad Pequivgw1 = \begin{pmatrix} 0.14 & 0.03 & 0.02 \\ 0.03 & 0.14 & 0.03 \\ 0.02 & 0.03 & 0.14 \end{pmatrix} \cdot \frac{m}{pF}$$

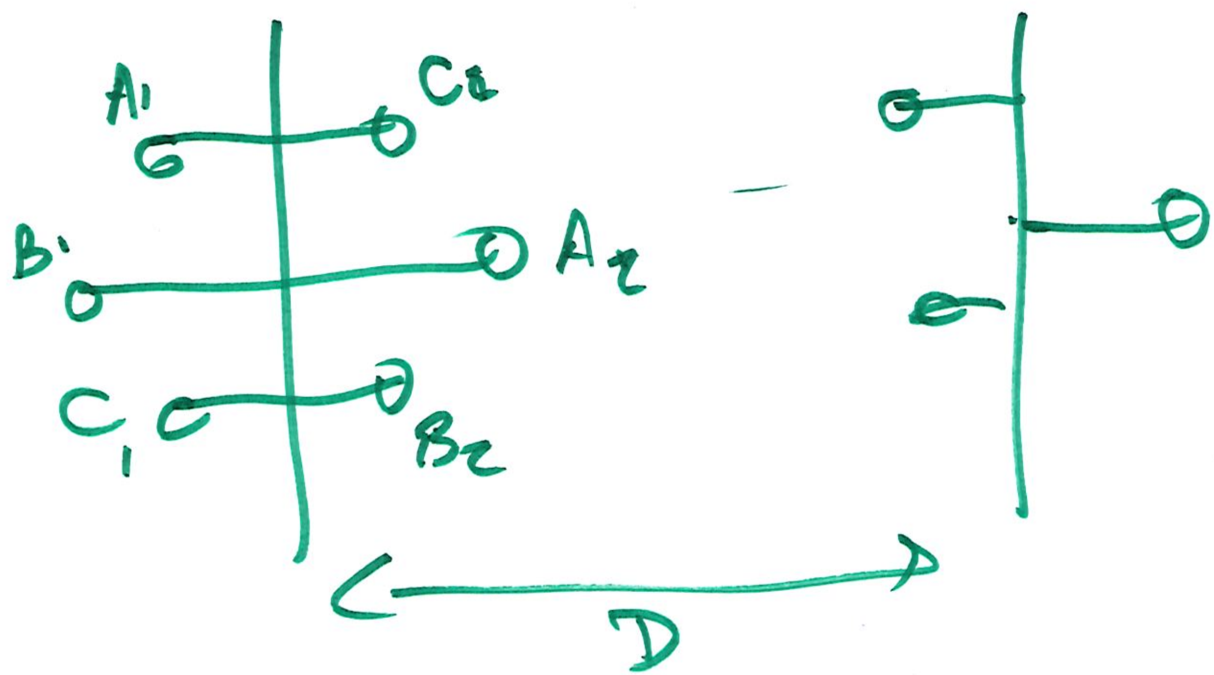
$$Cequivgw1 := Pequivgw1^{-1} \cdot Length \qquad Cequivgw1 = \begin{pmatrix} 0.49 & -0.09 & -0.04 \\ -0.09 & 0.5 & -0.09 \\ -0.04 & -0.09 & 0.49 \end{pmatrix} \cdot \mu F$$

$$C_{012gw1} := A_{012}^{-1} \cdot Cequivgw1 \cdot A_{012}$$

$$C_{012gw1} = \begin{pmatrix} 337.06 & 5.4 + 9.35i & 5.4 - 9.35i \\ 5.4 - 9.35i & 568.96 & -20.11 - 34.83i \\ 5.4 + 9.35i & -20.11 + 34.83i & 568.96 \end{pmatrix} \cdot nF$$

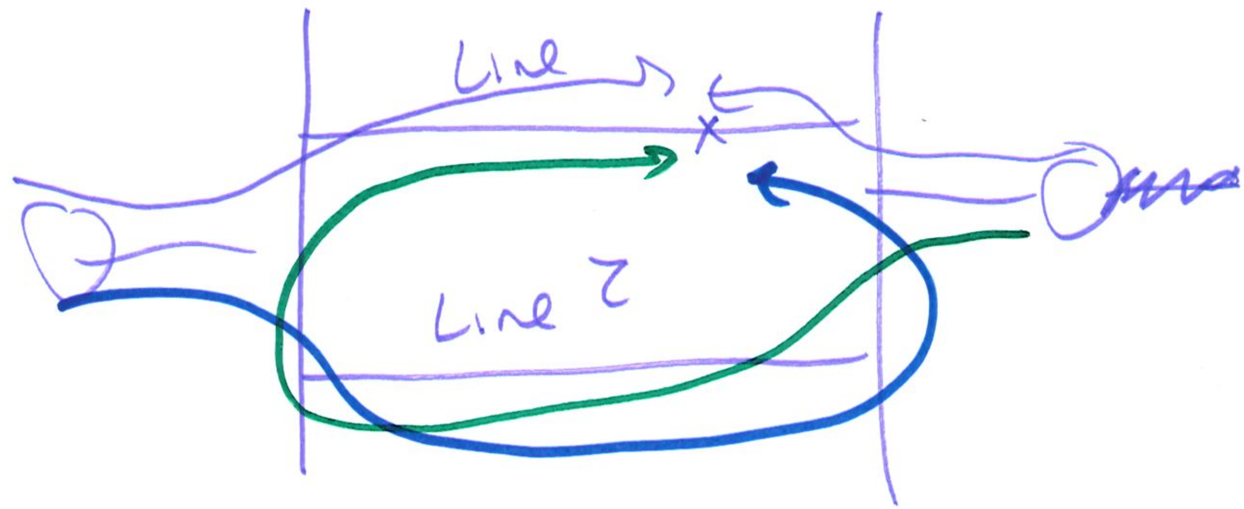


- magnetic connect
- little or ~~not~~ no electrical



Parallel Lines

1. Line model & parameters
2. system connection



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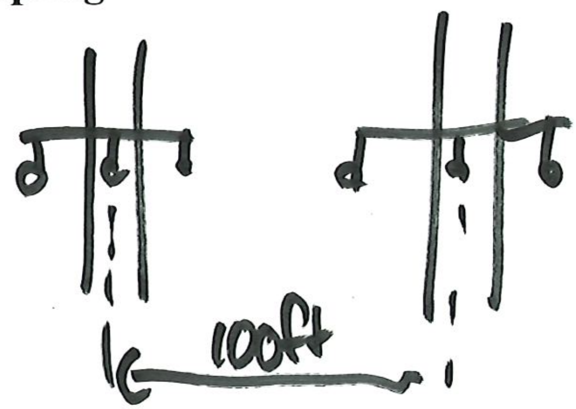
ECE 523: Calculating Mutual Coupling

Double Circuit Line:



Two parallel lines, 100 feet apart (center to center). Each has flat spacing

A. Resistance Matrix

AC Resistance from table:

$$Rac := 0.278\frac{ohm}{mi} \qquad \text{at 25 C and} \qquad freq := 60Hz$$

$$CarsonsResistConst := 9.869 \times 10^{-7}\frac{ohm}{m\cdot Hz}$$

$$Rd := CarsonsResistConst\cdot freq \qquad Rd = 0.0953\cdot\frac{ohm}{mi}$$

$$Rself := Rac + Rd \qquad Rself = 0.3733\cdot\frac{ohm}{mi}$$

L_1 $\qquad$ $L_{1\to 2}$

$$R' := \begin{pmatrix} Rself & Rd & Rd & Rd & Rd & Rd \\ Rd & Rself & Rd & Rd & Rd & Rd \\ Rd & Rd & Rself & Rd & Rd & Rd \\ Rd & Rd & Rd & Rself & Rd & Rd \\ Rd & Rd & Rd & Rd & Rself & Rd \\ Rd & Rd & Rd & Rd & Rd & Rself \end{pmatrix}$$

$L_{2\to 1}$ $\qquad$ L_2

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$$Z' := R' + j \cdot 2 \cdot \pi \cdot freq \cdot L'$$

$$Z' = \begin{pmatrix} 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.599i & 0.095 + 0.404i & 0.095 + 0.392i & 0.095 + 0.382i \\ 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.417i & 0.095 + 0.404i & 0.095 + 0.404i \\ 0.095 + 0.599i & 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.431i & 0.095 + 0.417i & 0.095 + 0.404i \\ 0.095 + 0.404i & 0.095 + 0.417i & 0.095 + 0.431i & 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.599i \\ 0.095 + 0.392i & 0.095 + 0.404i & 0.095 + 0.417i & 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.683i \\ 0.095 + 0.382i & 0.095 + 0.404i & 0.095 + 0.404i & 0.095 + 0.599i & 0.095 + 0.683i & 0.373 + 1.459i \end{pmatrix} \cdot \frac{ohm}{mi}$$

If length = 40 miles:

$$Zline := Z' \cdot 40mi$$

Line 1 Line1 → Line2

$$Zline = \begin{pmatrix} 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 23.965i & 3.812 + 16.154i & 3.812 + 15.691i & 3.812 + 1 \\ 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 16.665i & 3.812 + 16.154i & 3.812 + 1 \\ 3.812 + 23.965i & 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 17.237i & 3.812 + 16.665i & 3.812 + 1 \\ 3.812 + 16.154i & 3.812 + 16.665i & 3.812 + 17.237i & 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 2 \\ 3.812 + 15.691i & 3.812 + 16.154i & 3.812 + 16.665i & 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 2 \\ 3.812 + 15.269i & 3.812 + 16.154i & 3.812 + 16.154i & 3.812 + 23.965i & 3.812 + 27.33i & 14.932 + 5 \end{pmatrix}$$

Line2 → Line Line 2

$$a := 1 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}}$$

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

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3x3

Za := submatrix(Zline, 0, 2, 0, 2)   Zb := submatrix(Zline, 0, 2, 3, 5)

Zc := submatrix(Zline, 3, 5, 0, 2)   Zd := submatrix(Zline, 3, 5, 3, 5)

$Za012 := A_{012}^{-1} \cdot Za \cdot A_{012}$ $Zb012 := A_{012}^{-1} \cdot Zb \cdot A_{012}$

$Zc012 := A_{012}^{-1} \cdot Zc \cdot A_{012}$ $Zd012 := A_{012}^{-1} \cdot Zd \cdot A_{012}$

Build the matrix by stacking and augmenting submatrices

Z012left := stack(Za012, Zc012)

Z012right := stack(Zb012, Zd012)

Z_{012} := augment(Z012left, Z012right)

stack

Augment

Note the off-diagonal subblocks and their coupling.

zero to zero 2

- zero to 1 for pos or 2

$$
\begin{array}{cccccc}
Z0L1 & Z1L1 & Z2L1 & & & \\
\end{array}
$$

$$
Z_{012} = \begin{pmatrix}
22.555 + 110.79i & 0.971 - 0.561i & -0.971 - 0.561i & 11.435 + 48.713i & 0.27 + 0.671i & -0.27 + 0.671i \\
-0.971 - 0.561i & 11.12 + 32.166i & -1.942 + 1.121i & 0.313 - 0.8i & 0.139 - 0.126i & 0.043 + 0.129i \\
0.971 - 0.561i & 1.942 + 1.121i & 11.12 + 32.166i & -0.313 - 0.8i & -0.043 + 0.129i & -0.139 - 0.126 \\
11.435 + 48.713i & -0.313 - 0.8i & 0.313 - 0.8i & 22.555 + 110.79i & 0.971 - 0.561i & -0.971 - 0.561 \\
-0.27 + 0.671i & -0.139 - 0.126i & 0.043 + 0.129i & -0.971 - 0.561i & 11.12 + 32.166i & -1.942 + 1.121 \\
0.27 + 0.671i & -0.043 + 0.129i & 0.139 - 0.126i & 0.971 - 0.561i & 1.942 + 1.121i & 11.12 + 32.166i
\end{pmatrix}
$$

unchanged from doing line 1 alone

Zero sequence mutual couplin

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Build the matrix by stacking and augmenting submatrices

$$Z012left1 := stack(Za0121, Zc0121)$$

$$Z012right1 := stack(Zb0121, Zd0121)$$

$$Z_{0121} := augment(Z012left1, Z012right1)$$

$$Z_{0121} = \left(\begin{array}{ccc|ccc} 22.555+110.79i & 0 & 0 & 11.435+48.713i & 0 & 0 \\ 0 & 11.12+32.166i & 0 & 0 & 0.139-0.126i & 0 \\ 0 & 0 & 11.12+32.166 & 0 & 0 & -0.139-0.126 \\ \hline 11.435+48.713i & 0 & 0 & 22.555+110.79i & 0 & 0 \\ 0 & -0.139-0.126i & 0 & 0 & 11.12+32.166i & 0 \\ 0 & 0 & 0.139-0.126i & 0 & 0 & 11.12+32.166 \end{array}\right.$$

unchanged

- Notice what this has done to the cross-coupling terms. The positive and negative sequence coupling betw is very small relative to the zero sequence cross-coupling.

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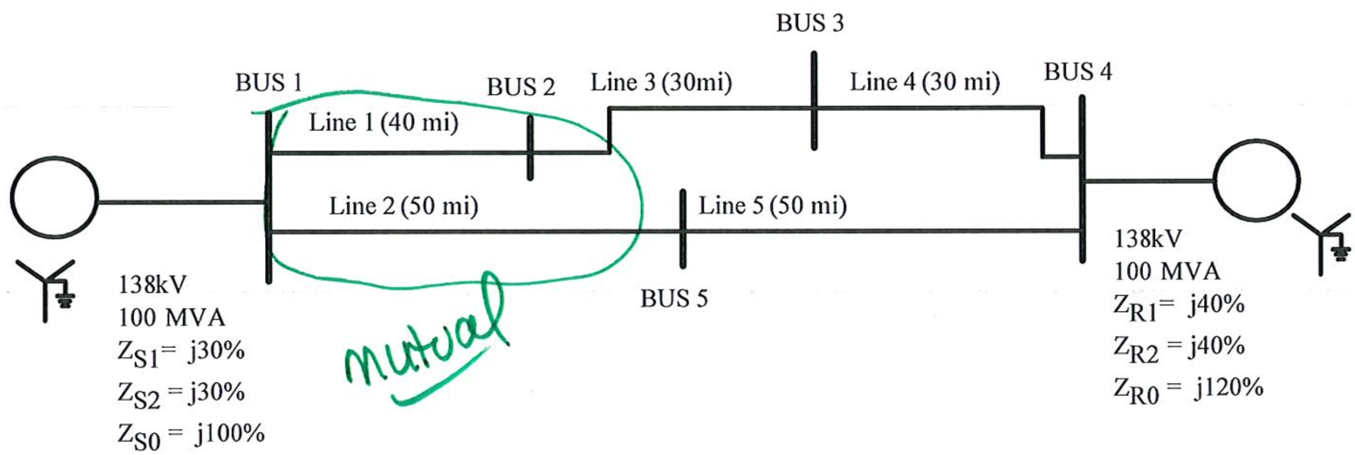
Mutual Coupling in Fault Analysis

$MVA := 1000kW \qquad a := 1 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}} \qquad A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$

$pu := 1$

$S_{base} := 100MVA \qquad V_{base} := 138kV$

$Z_{base} := \frac{V_{base}^2}{S_{base}} \qquad Z_{base} = 190.44\,\Omega \qquad I_{base} := \frac{S_{base}}{\sqrt{3} \cdot V_{base}} \qquad I_{base} = 418.37\,A$



- Source impedances:

  $Z_{S1} := j \cdot 0.3pu \qquad Z_{R1} := j \cdot 0.4pu$

  $Z_{S0} := j \cdot 1.0pu \qquad Z_{R0} := j \cdot 1.2pu$

- Line impedances from lecture 32 handout:

- Line 1, 40 mile length

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$Z_{0Line1} := (22.55548 + j\cdot 110.79032)ohm$

$Z_{1Line1} := (11.12 + j\cdot 32.16612)ohm$

} from other handout

$Z_{1L1} := \frac{Z_{1Line1}}{Z_{base}}$ $\qquad Z_{1L1} = (0.058 + 0.169i)\cdot pu$

$Z_{0L1} := \frac{Z_{0Line1}}{Z_{base}}$ $\qquad Z_{0L1} = (0.118 + 0.582i)\cdot pu$

- Line 2, 50 mile length

$Z_{0Line2} := 1.2\cdot(22.55548 + j\cdot 110.79032)ohm$

$Z_{1Line2} := 1.2\cdot(11.12 + j\cdot 32.16612)ohm$

$Z_{1L2} := \frac{Z_{1Line2}}{Z_{base}}$ $\qquad Z_{1L2} = (0.07 + 0.203i)\cdot pu$

$Z_{0L2} := \frac{Z_{0Line2}}{Z_{base}}$ $\qquad Z_{0L2} = (0.142 + 0.698i)\cdot pu$

- Line 1 and line 2 have 40 miles that are mutually coupled (just use zero sequence coupling)

$Z_{0M\_12} := (11.43548 + j\cdot 48.71333)ohm$ - other handout

$Z_{0M12} := \frac{Z_{0M\_12}}{Z_{base}}$ $\qquad Z_{0M12} = (0.06 + 0.256i)\cdot pu$

- Line 3 is 30 miles with the same tower configuration as above, 20 miles of which is not close enough for mutual coupli

$Z_{1L3} := 0.75\cdot Z_{1L1}$

Note that 30 miles is 3/4 of the length of line 1

$Z_{0L3} := 0.75\cdot Z_{0L1}$

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- Line 2 and line 3 have 10 miles that are mutually coupled (just use zero sequence coupling)

$$Z_{0M23} := 0.25 \cdot \left(Z_{0M12}\right)$$

- Line 4 is 30 miles with same tower configuration that is not close enough for mutual coupling

$$Z_{1L4} := 0.75 \cdot Z_{1L1}$$

$$Z_{0L4} := 0.75 \cdot Z_{0L1}$$

- Line 5 is 50 miles with same tower configuration that is not close enough for mutual coupling

$$Z_{1L5} := 1.2 \cdot Z_{1L1}$$

$$Z_{0L5} := 1.2 Z_{0L1}$$

- First, create the positive sequence Y_{bus} for fault study using the common formulation approach. Note that there is no m the positive sequence.

POS SEQ

$$Y_{bus1} := \begin{pmatrix} \frac{1}{Z_{S1}} + \frac{1}{Z_{1L1}} + \frac{1}{Z_{1L2}} & \frac{-1}{Z_{1L1}} & 0 & 0 & \frac{-1}{Z_{1L2}} \\ \frac{-1}{Z_{1L1}} & \frac{1}{Z_{1L1}} + \frac{1}{Z_{1L3}} & \frac{-1}{Z_{1L3}} & 0 & 0 \\ 0 & \frac{-1}{Z_{1L3}} & \frac{1}{Z_{1L3}} + \frac{1}{Z_{1L4}} & \frac{-1}{Z_{1L4}} & 0 \\ 0 & 0 & \frac{-1}{Z_{1L4}} & \frac{1}{Z_{R1}} + \frac{1}{Z_{1L4}} + \frac{1}{Z_{1L5}} & \frac{-1}{Z_{1L5}} \\ \frac{-1}{Z_{1L2}} & 0 & 0 & \frac{-1}{Z_{1L5}} & \frac{1}{Z_{1L2}} + \frac{1}{Z_{1L5}} \end{pmatrix}$$

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$$Y_{bus1} = \begin{pmatrix} 3.352-13.029i & -1.828+5.288i & 0 & 0 & -1.524+4.407i \\ -1.828+5.288i & 4.266-12.34i & -2.438+7.051i & 0 & 0 \\ 0 & -2.438+7.051i & 4.875-14.103i & -2.438+7.051i & 0 \\ 0 & 0 & -2.438+7.051i & 3.961-13.958i & -1.524+4.407i \\ -1.524+4.407i & 0 & 0 & -1.524+4.407i & 3.047-8.814i \end{pmatrix} \cdot pu$$

- Now build the Ybus, using the incidence matrix approach (again, no mutual coupling for positive sequence)

$$Y_{pr1} := \begin{array}{cc} & \text{Mapping to digraph} \\ \begin{pmatrix} \frac{1}{Z_{S1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{Z_{R1}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{Z_{1L1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{Z_{1L2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{Z_{1L3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{Z_{1L4}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{Z_{1L5}} \end{pmatrix} & \begin{array}{c} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \\ \mathbf{d} \\ \mathbf{e} \\ \mathbf{f} \\ \mathbf{g} \end{array} \end{array}$$

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Incidence matrix

- Each row corresponds to a branch
- Each column corresponds to a node (note that matrix is not generally square)
    a. Enter a 0 if no connection between the node and branch for that cell
    b. Enter a (+1) if the current is leaving a node
    c. Enter a (-1) if the current is entering a node
    d. No row and column associated with ground node

$$
\begin{array}{r c l}
 & \text{Node number: } \; 1 \quad 2 \quad 3 \quad 4 \quad 5 & \text{Branch} \\
 & & \mathbf{a} \\
 & & \mathbf{b} \\
 & & \mathbf{c} \\
A_{incid} := & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix} & \mathbf{d} \\
 & & \mathbf{e} \\
 & & \mathbf{f} \\
 & & \mathbf{g}
\end{array}
$$

$$Y_{bus1\_alt} := A_{incid}{}^{T} \cdot Y_{pr1} \cdot A_{incid}$$

$$Y_{bus1\_alt} = \begin{pmatrix} 3.352 - 13.029i & -1.828 + 5.288i & 0 & 0 & -1.524 + 4.407i \\ -1.828 + 5.288i & 4.266 - 12.34i & -2.438 + 7.051i & 0 & 0 \\ 0 & -2.438 + 7.051i & 4.875 - 14.103i & -2.438 + 7.051i & 0 \\ 0 & 0 & -2.438 + 7.051i & 3.961 - 13.958i & -1.524 + 4.407i \\ -1.524 + 4.407i & 0 & 0 & -1.524 + 4.407i & 3.047 - 8.814i \end{pmatrix}$$

pos seq

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$$Y_{bus1\_alt} - Y_{bus1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Y_{bus2} := Y_{bus1}$$

- Now for zero sequence matrix. Now we need to include the zero sequence mutual coupling
  - It will be put into the Zpr0 matrix as off-diagonal terms
  - The sign of the offdiagonal terms depends on the sign of the associated links in the digraph (positive signs if the ar way and negative otherwise.

Mapping to digraph

$$Z_{pr0M} := \begin{array}{c} \begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\ \begin{pmatrix} Z_{S0} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{R0} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{0L1} & Z_{0M12} & 0 & 0 & 0 \\ 0 & 0 & Z_{0M12} & Z_{0L2} & -Z_{0M23} & 0 & 0 \\ 0 & 0 & 0 & -Z_{0M23} & Z_{0L3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_{0L4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_{0L5} \end{pmatrix} \end{array} \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \end{array}$$

C & D have same directn

negative bec- C & E opposit direct

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and for later comparisons, the same matrix, with no mutual terms.

$$Z_{pr0\_noM} := \begin{pmatrix} Z_{S0} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{R0} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{0L1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_{0L2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_{0L3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_{0L4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_{0L5} \end{pmatrix}$$

$$Y_{bus0\_M} := A_{incid}^{T} \cdot Z_{pr0M}^{-1} \cdot A_{incid}$$

$$Y_{bus0\_NoM} := A_{incid}^{T} \cdot Z_{pr0\_noM}^{-1} \cdot A_{incid}$$

$$Y_{bus0\_M} = \begin{pmatrix} 0.46-3.17i & -0.29+1.37i & 0.03-0.14i & 0 & -0.2+0.93i \\ -0.29+1.37i & 0.8-4i & -0.44+2.13i & 0 & -0.08+0.5i \\ 0.03-0.14i & -0.44+2.13i & 0.9-4.44i & -0.45+2.2i & -0.04+0.25i \\ 0 & 0 & -0.45+2.2i & 0.73-4.41i & -0.28+1.38i \\ -0.2+0.93i & -0.08+0.5i & -0.04+0.25i & -0.28+1.38i & 0.6-3.05i \end{pmatrix}$$

Matches Powerworld Matrix:

Name	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5
BUS1	0.46 - j3.17	-0.29 + j1.37	0.03 - j0.14		-0.20 + j0.
BUS2	-0.29 + j1.37	0.80 - j4.00	-0.44 + j2.13		-0.08 + j0.
BUS3	0.03 - j0.14	-0.44 + j2.13	0.90 - j4.44	-0.45 + j2.20	-0.04 + j0.
BUS4			-0.45 + j2.20	0.73 - j4.41	-0.28 + j1.
BUS5	-0.20 + j0.93	-0.08 + j0.50	-0.04 + j0.25	-0.28 + j1.38	0.60 - j3.(