ECE 523: Calculating Mutual Coupling

Double Circuit Line:

Two parallel lines, 100 feet apart (center to center). Each has flat spacing

A. Resistance Matrix

AC Resistance from table:

$Rac := 0.278 \frac{ohm}{mi}$ at 25 C and $freq := 60Hz$

$CarsonsResistConst := 9.869 \times 10^{-7} \frac{ohm}{m \cdot Hz}$

$Rd := CarsonsResistConst \cdot freq$ $Rd = 0.0953 \cdot \frac{ohm}{mi}$

$Rself := Rac + Rd$ $Rself = 0.3733 \cdot \frac{ohm}{mi}$

$$R' := \begin{pmatrix} Rself & Rd & Rd & Rd & Rd & Rd \\ Rd & Rself & Rd & Rd & Rd & Rd \\ Rd & Rd & Rself & Rd & Rd & Rd \\ Rd & Rd & Rd & Rself & Rd & Rd \\ Rd & Rd & Rd & Rd & Rself & Rd \\ Rd & Rd & Rd & Rd & Rd & Rself \end{pmatrix}$$

$$R' = \begin{pmatrix} 0.3733 & 0.0953 & 0.0953 & 0.0953 & 0.0953 & 0.0953 \\ 0.0953 & 0.3733 & 0.0953 & 0.0953 & 0.0953 & 0.0953 \\ 0.0953 & 0.0953 & 0.3733 & 0.0953 & 0.0953 & 0.0953 \\ 0.0953 & 0.0953 & 0.0953 & 0.3733 & 0.0953 & 0.0953 \\ 0.0953 & 0.0953 & 0.0953 & 0.0953 & 0.3733 & 0.0953 \\ 0.0953 & 0.0953 & 0.0953 & 0.0953 & 0.0953 & 0.3733 \end{pmatrix} \cdot \frac{\text{ohm}}{\text{mi}}$$

B. Inductance Matrix

$\mu o := 4 \cdot \pi \cdot 10^{-7} \frac{H}{m}$ $\rho := 100 \text{ohm} \cdot \text{m}$

Calculate GMR from conductor diameter

$\text{dia} := 0.528\text{in}$ $\text{GMR} := e^{\frac{-1}{4}} \cdot \frac{\text{dia}}{2}$ $\text{GMR} = 0.01713 \cdot \text{ft}$

Conductor GMR from table:

$\text{Ds} := 0.01668\text{ft}$

$\text{De} := 2160 \cdot \frac{\text{ft} \cdot \text{Hz}^{0.5}}{(\text{ohm} \cdot \text{m})^{0.5}}$

$\text{De} := \text{De} \cdot \sqrt{\frac{\rho}{\text{freq}}}$ $\text{De} = 2788.55 \cdot \text{ft}$

$Da1b1 := 10ft \quad Da1c1 := 20ft \quad Db1c1 := 10ft \quad Da1a2 := 100ft \quad Db1b2 := 100ft \quad Dc1c2 := 100ft$

$Da2b2 := 10ft \quad Da2c2 := 20ft \quad Db2c2 := 10ft \quad Da1b2 := 110ft \quad Da1c2 := 120ft \quad Db1a2 := 90ft$

$Db1c2 := 100ft \quad Dc1a2 := 80ft \quad Dc1b2 := 90ft$

$$L' := \frac{\mu o}{2\cdot\pi}\cdot\begin{pmatrix} \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Da1b1}\right) & \ln\left(\frac{De}{Da1c1}\right) & \ln\left(\frac{De}{Da1a2}\right) & \ln\left(\frac{De}{Da1b2}\right) & \ln\left(\frac{De}{Da1c2}\right) \\ \ln\left(\frac{De}{Da1b1}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Db1c1}\right) & \ln\left(\frac{De}{Db1a2}\right) & \ln\left(\frac{De}{Db1b2}\right) & \ln\left(\frac{De}{Db1c2}\right) \\ \ln\left(\frac{De}{Da1c1}\right) & \ln\left(\frac{De}{Db1c1}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Dc1a2}\right) & \ln\left(\frac{De}{Dc1b2}\right) & \ln\left(\frac{De}{Dc1c2}\right) \\ \ln\left(\frac{De}{Da1a2}\right) & \ln\left(\frac{De}{Db1a2}\right) & \ln\left(\frac{De}{Dc1a2}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Da2b2}\right) & \ln\left(\frac{De}{Da2c2}\right) \\ \ln\left(\frac{De}{Da1b2}\right) & \ln\left(\frac{De}{Db1b2}\right) & \ln\left(\frac{De}{Dc1b2}\right) & \ln\left(\frac{De}{Da2b2}\right) & \ln\left(\frac{De}{Ds}\right) & \ln\left(\frac{De}{Db2c2}\right) \\ \ln\left(\frac{De}{Da1c2}\right) & \ln\left(\frac{De}{Db1c2}\right) & \ln\left(\frac{De}{Dc1c2}\right) & \ln\left(\frac{De}{Da2c2}\right) & \ln\left(\frac{De}{Db2c2}\right) & \ln\left(\frac{De}{Ds}\right) \end{pmatrix}$$

$$L' = \begin{pmatrix} 3.87 & 1.81 & 1.59 & 1.07 & 1.04 & 1.01 \\ 1.81 & 3.87 & 1.81 & 1.11 & 1.07 & 1.07 \\ 1.59 & 1.81 & 3.87 & 1.14 & 1.11 & 1.07 \\ 1.07 & 1.11 & 1.14 & 3.87 & 1.81 & 1.59 \\ 1.04 & 1.07 & 1.11 & 1.81 & 3.87 & 1.81 \\ 1.01 & 1.07 & 1.07 & 1.59 & 1.81 & 3.87 \end{pmatrix}\cdot\frac{mH}{mi}$$

$$Z' := R' + j\cdot 2\cdot\pi\cdot freq\cdot L'$$

$$Z' = \begin{pmatrix} 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.599i & 0.095 + 0.404i & 0.095 + 0.392i & 0.095 + 0.382i \\ 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.417i & 0.095 + 0.404i & 0.095 + 0.404i \\ 0.095 + 0.599i & 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.431i & 0.095 + 0.417i & 0.095 + 0.404i \\ 0.095 + 0.404i & 0.095 + 0.417i & 0.095 + 0.431i & 0.373 + 1.459i & 0.095 + 0.683i & 0.095 + 0.599i \\ 0.095 + 0.392i & 0.095 + 0.404i & 0.095 + 0.417i & 0.095 + 0.683i & 0.373 + 1.459i & 0.095 + 0.683i \\ 0.095 + 0.382i & 0.095 + 0.404i & 0.095 + 0.404i & 0.095 + 0.599i & 0.095 + 0.683i & 0.373 + 1.459i \end{pmatrix} \cdot \frac{ohm}{mi}$$

If length = 40 miles:

$$Zline := Z'\cdot 40mi$$

$$Zline = \begin{pmatrix} 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 23.965i & 3.812 + 16.154i & 3.812 + 15.691i & 3.812 + 15.269i \\ 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 16.665i & 3.812 + 16.154i & 3.812 + 16.154i \\ 3.812 + 23.965i & 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 17.237i & 3.812 + 16.665i & 3.812 + 16.154i \\ 3.812 + 16.154i & 3.812 + 16.665i & 3.812 + 17.237i & 14.932 + 58.374i & 3.812 + 27.33i & 3.812 + 23.965i \\ 3.812 + 15.691i & 3.812 + 16.154i & 3.812 + 16.665i & 3.812 + 27.33i & 14.932 + 58.374i & 3.812 + 27.33i \\ 3.812 + 15.269i & 3.812 + 16.154i & 3.812 + 16.154i & 3.812 + 23.965i & 3.812 + 27.33i & 14.932 + 58.374i \end{pmatrix} \Omega$$

$$a := 1\cdot e^{j\cdot\frac{2\cdot\pi}{3}} \qquad A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$$

$Za := submatrix(Zline, 0, 2, 0, 2)$ $Zb := submatrix(Zline, 0, 2, 3, 5)$

$Zc := submatrix(Zline, 3, 5, 0, 2)$ $Zd := submatrix(Zline, 3, 5, 3, 5)$

$Za012 := A_{012}^{-1} \cdot Za \cdot A_{012}$ $Zb012 := A_{012}^{-1} \cdot Zb \cdot A_{012}$

$Zc012 := A_{012}^{-1} \cdot Zc \cdot A_{012}$ $Zd012 := A_{012}^{-1} \cdot Zd \cdot A_{012}$

Build the matrix by stacking and augmenting submatrices

$Z012left := stack(Za012, Zc012)$

$Z012right := stack(Zb012, Zd012)$

$Z_{012} := augment(Z012left, Z012right)$

Note the off-diagonal subblocks and their coupling.

$$Z_{012} = \begin{pmatrix} 22.555 + 110.79i & 0.971 - 0.561i & -0.971 - 0.561i & 11.435 + 48.713i & 0.27 + 0.671i & -0.27 + 0.671i \\ -0.971 - 0.561i & 11.12 + 32.166i & -1.942 + 1.121i & 0.313 - 0.8i & 0.139 - 0.126i & 0.043 + 0.129i \\ 0.971 - 0.561i & 1.942 + 1.121i & 11.12 + 32.166i & -0.313 - 0.8i & -0.043 + 0.129i & -0.139 - 0.126i \\ 11.435 + 48.713i & -0.313 - 0.8i & 0.313 - 0.8i & 22.555 + 110.79i & 0.971 - 0.561i & -0.971 - 0.561i \\ -0.27 + 0.671i & -0.139 - 0.126i & 0.043 + 0.129i & -0.971 - 0.561i & 11.12 + 32.166i & -1.942 + 1.121i \\ 0.27 + 0.671i & -0.043 + 0.129i & 0.139 - 0.126i & 0.971 - 0.561i & 1.942 + 1.121i & 11.12 + 32.166i \end{pmatrix} \Omega$$

Now try transposing one or both of the lines

$$Rp := \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

First, use the exact same scheme on both lines, so the Zb and Zc are kept simpler.

$$f1 := \frac{1}{3} \quad f2 := \frac{1}{3} \quad f3 := \frac{1}{3}$$

$$Zanet := f1 \cdot Za + f2 \cdot Rp^{-1} \cdot Za \cdot Rp + f3 \cdot Rp \cdot Za \cdot Rp^{-1}$$

$$Zbnet := f1 \cdot Zb + f2 \cdot Rp^{-1} \cdot Zb \cdot Rp + f3 \cdot Rp \cdot Zb \cdot Rp^{-1}$$

$$Zcnet := f1 \cdot Zc + f2 \cdot Rp^{-1} \cdot Zc \cdot Rp + f3 \cdot Rp \cdot Zc \cdot Rp^{-1}$$

$$Zdnet := f1 \cdot Zd + f2 \cdot Rp^{-1} \cdot Zd \cdot Rp + f3 \cdot Rp \cdot Zd \cdot Rp^{-1}$$

$$Zanet = \begin{pmatrix} 14.93 + 58.37i & 3.81 + 26.21i & 3.81 + 26.21i \\ 3.81 + 26.21i & 14.93 + 58.37i & 3.81 + 26.21i \\ 3.81 + 26.21i & 3.81 + 26.21i & 14.93 + 58.37i \end{pmatrix} \Omega$$

$$Zbnet = \begin{pmatrix} 3.81 + 16.15i & 3.81 + 16.36i & 3.81 + 16.2i \\ 3.81 + 16.2i & 3.81 + 16.15i & 3.81 + 16.36i \\ 3.81 + 16.36i & 3.81 + 16.2i & 3.81 + 16.15i \end{pmatrix} \Omega$$

$$Zcnet = \begin{pmatrix} 3.81 + 16.15i & 3.81 + 16.2i & 3.81 + 16.36i \\ 3.81 + 16.36i & 3.81 + 16.15i & 3.81 + 16.2i \\ 3.81 + 16.2i & 3.81 + 16.36i & 3.81 + 16.15i \end{pmatrix} \Omega$$

$$Zdnet = \begin{pmatrix} 14.93 + 58.37i & 3.81 + 26.21i & 3.81 + 26.21i \\ 3.81 + 26.21i & 14.93 + 58.37i & 3.81 + 26.21i \\ 3.81 + 26.21i & 3.81 + 26.21i & 14.93 + 58.37i \end{pmatrix} \Omega$$

$$Za0121 := A_{012}^{-1} \cdot Zanet \cdot A_{012}$$

$$Zb0121 := A_{012}^{-1} \cdot Zbnet \cdot A_{012}$$

$$Zc0121 := A_{012}^{-1} \cdot Zcnet \cdot A_{012}$$

$$Zd0121 := A_{012}^{-1} \cdot Zdnet \cdot A_{012}$$

Build the matrix by stacking and augmenting submatrices

$Z012left1 := stack(Za0121, Zc0121)$

$Z012right1 := stack(Zb0121, Zd0121)$

$Z_{0121} := augment(Z012left1, Z012right1)$

$$Z_{0121} = \begin{pmatrix} 22.555 + 110.79i & 0 & 0 & 11.435 + 48.713i & 0 & 0 \\ 0 & 11.12 + 32.166i & 0 & 0 & 0.139 - 0.126i & 0 \\ 0 & 0 & 11.12 + 32.166i & 0 & 0 & -0.139 - 0.126i \\ 11.435 + 48.713i & 0 & 0 & 22.555 + 110.79i & 0 & 0 \\ 0 & -0.139 - 0.126i & 0 & 0 & 11.12 + 32.166i & 0 \\ 0 & 0 & 0.139 - 0.126i & 0 & 0 & 11.12 + 32.166i \end{pmatrix} \Omega$$

- Notice what this has done to the cross-coupling terms. The positive and negative sequence coupling between the two lines is very small relative to the zero sequence cross-coupling.

Consider transposing only one line

$Za2 := Za$

$Zb2 := f1 \cdot Zb + f2 \cdot Zb \cdot Rp + f3 \cdot Zb \cdot Rp^{-1}$

$Zd2 := f1 \cdot Zd + f2 \cdot Rp^{-1} \cdot Zd \cdot Rp + f3 \cdot Rp \cdot Zd \cdot Rp^{-1}$

$Zc2 := f1 \cdot Zc + f2 \cdot Rp^{-1} \cdot Zc + f3 \cdot Rp \cdot Zc$

$$Za2 = \begin{pmatrix} 14.93 + 58.37i & 3.81 + 27.33i & 3.81 + 23.97i \\ 3.81 + 27.33i & 14.93 + 58.37i & 3.81 + 27.33i \\ 3.81 + 23.97i & 3.81 + 27.33i & 14.93 + 58.37i \end{pmatrix} \Omega$$

$$Zb2 = \begin{pmatrix} 3.81 + 15.7i & 3.81 + 15.7i & 3.81 + 15.7i \\ 3.81 + 16.32i & 3.81 + 16.32i & 3.81 + 16.32i \\ 3.81 + 16.69i & 3.81 + 16.69i & 3.81 + 16.69i \end{pmatrix} \Omega$$

$$Zc2 = \begin{pmatrix} 3.81 + 15.7i & 3.81 + 16.32i & 3.81 + 16.69i \\ 3.81 + 15.7i & 3.81 + 16.32i & 3.81 + 16.69i \\ 3.81 + 15.7i & 3.81 + 16.32i & 3.81 + 16.69i \end{pmatrix} \Omega$$

$$Zd2 = \begin{pmatrix} 14.93 + 58.37i & 3.81 + 26.21i & 3.81 + 26.21i \\ 3.81 + 26.21i & 14.93 + 58.37i & 3.81 + 26.21i \\ 3.81 + 26.21i & 3.81 + 26.21i & 14.93 + 58.37i \end{pmatrix} \Omega$$

$Za0122 := A_{012}^{-1} \cdot Za2 \cdot A_{012}$

$Zb0122 := A_{012}^{-1} \cdot Zb2 \cdot A_{012}$

$Zc0122 := A_{012}^{-1} \cdot Zc2 \cdot A_{012}$

$Zd0122 := A_{012}^{-1} \cdot Zd2 \cdot A_{012}$

Build the matrix by stacking and augmenting submatrices

$Z012left2 := stack(Za0122, Zc0122)$

$Z012right2 := stack(Zb0122, Zd0122)$

$Z0122 := augment(Z012left2, Z012right2)$

$$Z0122 = \begin{pmatrix} 22.555 + 110.79i & 0.971 - 0.561i & -0.971 - 0.561i & 11.435 + 48.713i & 0 & 0 \\ -0.971 - 0.561i & 11.12 + 32.166i & -1.942 + 1.121i & 0.313 - 0.8i & 0 & 0 \\ 0.971 - 0.561i & 1.942 + 1.121i & 11.12 + 32.166i & -0.313 - 0.8i & 0 & 0 \\ 11.435 + 48.713i & -0.313 - 0.8i & 0.313 - 0.8i & 22.555 + 110.79i & 0 & 0 \\ 0 & 0 & 0 & 0 & 11.12 + 32.166i & 0 \\ 0 & 0 & 0 & 0 & 0 & 11.12 + 32.166i \end{pmatrix} \Omega$$