ECE 523
Symmetrical Components

Session 27

$$Y_{bus1\_alt} - Y_{bus1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Y_{bus2} := Y_{bus1}$$

- Now for zero sequence matrix. Now we need to include the zero sequence mutual coupling
  - It will be put into the Zpr0 matrix as off-diagonal terms
  - The sign of the offdiagonal terms depends on the sign of the associated links in the digraph (positive signs if the arrows point the same way and negative otherwise.

Mapping to digraph

$$Z_{pr0M} := \begin{pmatrix} Z_{S0} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{R0} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{0L1} & Z_{0M12} & 0 & 0 & 0 \\ 0 & 0 & Z_{0M12} & Z_{0L2} & -Z_{0M23} & 0 & 0 \\ 0 & 0 & 0 & -Z_{0M23} & Z_{0L3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_{0L4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_{0L5} \end{pmatrix} \begin{matrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \\ \mathbf{d} \\ \mathbf{e} \\ \mathbf{f} \\ \mathbf{g} \end{matrix}$$

The resulting screen with the data entered looks like this:

Mutual Impedances

Records Set Columns Options

Filter Advanced Mutual Impedance Find... Remove Quick Filter

	L1 From Bus	L1 To Bus	L1 Ckt ID	L2 From Bus	L2 To Bus	L2 Ckt ID	Mutual R	Mutual X	L1 Mut. Start	L1 Mut. End	L2 Mut. Start	L2 Mut. End
1	1	2	1	1	5	1	0.06000	0.25580	0.000	1.000	0.000	0.800
2	2	3	1	1	5	1	0.01500	0.06390	0.000	0.333	0.800	1.000

per unit

- L1 From Bus, L1 To Bus, and L1 Ckt ID are the "From Bus" number, "To Bus" number, and circuit identifier for the first mutually coupled line
- L2 From Bus, L2 To Bus, and L2 Ckt ID are the "From Bus" number, "To Bus" number, and circuit identifier for the second mutually coupled line
- Mutual R and Mutual X are the per unit values from Z0M for the two lines in question, they were calculated in the other handout from last time.
  - Polarity dot is assumed to be at the From Bus for each line, so designate the lines based on this
- L1 Mutual Start and L1 Mutual End are the starting and end points for line 1, where 1.000 represents 100% of the line length
  - Program uses these for analyzing faults within the line section
- L2 Mutual Start and L2 Mutual End are the starting and end points for line 2, where 1.000 represents 100% of the line length
  - Program uses these for analyzing faults within the line section

and for later comparisons, the same matrix, with no mutual terms.

$$Z_{pr0\_noM} := \begin{pmatrix} Z_{S0} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{R0} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{0L1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_{0L2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_{0L3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_{0L4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_{0L5} \end{pmatrix}$$

y

$$Y_{bus0\_M} := A_{incid}^{T} \cdot Z_{pr0M}^{-1} \cdot A_{incid}$$

$$Y_{bus0\_NoM} := A_{incid}^{T} \cdot Z_{pr0\_noM}^{-1} \cdot A_{incid}$$

$$Y_{bus0\_M} = \begin{pmatrix} 0.46 - 3.17i & -0.29 + 1.37i & 0.03 - 0.14i & 0 & -0.2 + 0.93i \\ -0.29 + 1.37i & 0.8 - 4i & -0.44 + 2.13i & 0 & -0.08 + 0.5i \\ 0.03 - 0.14i & -0.44 + 2.13i & 0.9 - 4.44i & -0.45 + 2.2i & -0.04 + 0.25i \\ 0 & 0 & -0.45 + 2.2i & 0.73 - 4.41i & -0.28 + 1.38i \\ -0.2 + 0.93i & -0.08 + 0.5i & -0.04 + 0.25i & -0.28 + 1.38i & 0.6 - 3.05i \end{pmatrix}$$

Matches Powerworld Matrix:

Name	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5
BUS1	0.46 - j3.17	-0.29 + j1.37	0.03 - j0.14		-0.20 + j0.93
BUS2	-0.29 + j1.37	0.80 - j4.00	-0.44 + j2.13		-0.08 + j0.50
BUS3	0.03 - j0.14	-0.44 + j2.13	0.90 - j4.44	-0.45 + j2.20	-0.04 + j0.25
BUS4			-0.45 + j2.20	0.73 - j4.41	-0.28 + j1.38
BUS5	-0.20 + j0.93	-0.08 + j0.50	-0.04 + j0.25	-0.28 + j1.38	0.60 - j3.05

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Case 2: With mutual coupling:

$$I_{0\_M} := \frac{V_f}{Z_{Bus1_{4,4}} + Z_{Bus2_{4,4}} + Z_{0\_M_{4,4}}} \qquad \left|I_{0\_M}\right| = 0.694 \cdot pu \qquad \arg\left(I_{0\_M}\right) = -84.48 \cdot deg$$

$$I_{1\_M} := I_{0\_M} \qquad I_{2\_M} := I_{1\_M}$$

$$I_{ABC\_SLG\_M} := A_{012} \cdot \begin{pmatrix} I_{0\_M} \\ I_{1\_M} \\ I_{2\_M} \end{pmatrix} \qquad \overrightarrow{\left|I_{ABC\_SLG\_M}\right|} = \begin{pmatrix} 2.081 \\ 0 \\ 0 \end{pmatrix} \qquad \overrightarrow{\arg\left(I_{ABC\_SLG\_M}\right)} = \begin{pmatrix} -84.48 \\ 13.173 \\ 13.173 \end{pmatrix} \cdot deg$$

- Very close match to Powerworld results

$$\Delta V1_M := Z_{Bus1} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_{1\_M} \end{pmatrix} \qquad V1_M := \begin{pmatrix} 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \end{pmatrix} + \Delta V1_M$$

$$\overrightarrow{\left|V1_M\right|} = \begin{pmatrix} 0.885 \\ 0.882 \\ 0.879 \\ 0.877 \\ 0.809 \end{pmatrix} \qquad \overrightarrow{\arg\left(V1_M\right)} = \begin{pmatrix} -0.777 \\ -0.746 \\ -0.722 \\ -0.699 \\ 0.431 \end{pmatrix} \cdot deg$$

same as usual

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$$\Delta V2_M := Z_{Bus2} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_{2\_M} \end{pmatrix} \qquad V2_M := \Delta V2_M$$

$$\Delta V0_M := Z_{0\_M} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -I_{0\_M} \end{pmatrix} \qquad V0_M := \Delta V0_M$$

$$V_{ABC\_B1\_M} := A_{012} \cdot \begin{pmatrix} V0_{M_0} \\ V1_{M_0} \\ V2_{M_0} \end{pmatrix} \qquad \overrightarrow{\left|V_{ABC\_B1\_M}\right|} = \begin{pmatrix} 0.4253 \\ 1.1558 \\ 1.1159 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg\left(V_{ABC\_B1\_M}\right)} = \begin{pmatrix} -8.412 \\ -129.471 \\ 131.182 \end{pmatrix} \cdot deg$$

also same approach

$$V_{ABC\_B2\_M} := A_{012} \cdot \begin{pmatrix} V0_{M_1} \\ V1_{M_1} \\ V2_{M_1} \end{pmatrix} \qquad \overrightarrow{\left|V_{ABC\_B2\_M}\right|} = \begin{pmatrix} 0.3423 \\ 1.1973 \\ 1.1708 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg\left(V_{ABC\_B2\_M}\right)} = \begin{pmatrix} -8.838 \\ -132.399 \\ 133.597 \end{pmatrix} \cdot deg$$

Compare to Powerworld

voltages match

Name	Phase Volt A	Phase Volt B	Phase Volt C	Phase Ang A	Phase Ang B	Phase Ang C
BUS1	0.4253	1.15581	1.11585	-8.41	-129.47	131.18
BUS2	0.34229	1.19733	1.17075	-8.84	-132.4	133.6
BUS3	0.33257	1.19963	1.17171	-9.06	-132.45	133.71
BUS4	0.3491	1.18713	1.15225	-9.17	-131.47	133.02
BUS5	0	1.28049	1.25553	0	-136.32	137.53

- First get the positive and negative sequence currents in each of the three branches involved in the mutual coupled lines

$I_{1\_L1\_M} := \frac{V1_{M_0} - V1_{M_1}}{Z_{1L1}}$ $\quad |I_{1\_L1\_M}| = 0.019$ $\quad \arg(I_{1\_L1\_M}) = -79.972 \cdot deg$

$I_{2\_L1\_M} := \frac{V2_{M_0} - V2_{M_1}}{Z_{1L1}}$ $\quad |I_{2\_L1\_M}| = 0.019$ $\quad \arg(I_{2\_L1\_M}) = -79.972 \cdot deg$

$I_{1\_L2\_M} := \frac{V1_{M_0} - V1_{M_4}}{Z_{1L2}}$ $\quad |I_{1\_L2\_M}| = 0.366$ $\quad \arg(I_{1\_L2\_M}) = -84.241 \cdot deg$

$I_{2\_L2\_M} := \frac{V2_{M_0} - V2_{M_4}}{Z_{1L2}}$ $\quad |I_{2\_L2\_M}| = 0.366$ $\quad \arg(I_{2\_L2\_M}) = -84.241 \cdot deg$

pos & neg sequence – same approach as usual.

$$I_{1\_L3\_M} := \frac{V1_{M_1} - V1_{M_2}}{Z_{1L3}} \qquad \left|I_{1\_L3\_M}\right| = 0.019 \cdot pu \qquad \arg\left(I_{1\_L3\_M}\right) = -79.972 \cdot deg$$

$$I_{2\_L3\_M} := \frac{V2_{M_1} - V2_{M_2}}{Z_{1L3}} \qquad \left|I_{2\_L3\_M}\right| = 0.019 \cdot pu \qquad \arg\left(I_{2\_L3\_M}\right) = -79.972 \cdot deg$$

- The zero sequence current calculation is little more tricky. This is based on equation 12.19 in the text book
- We need terms from the matrix of impedance primitives, recall:

$$Z_{pr0M} := \begin{pmatrix} Z_{S0} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{R0} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{0L1} & Z_{0M12} & 0 & 0 & 0 \\ 0 & 0 & Z_{0M12} & Z_{0L2} & -Z_{0M23} & 0 & 0 \\ 0 & 0 & 0 & -Z_{0M23} & Z_{0L3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_{0L4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_{0L5} \end{pmatrix}$$

We need the 3x3 block in the middle of the matrix.

$$Z_{mutual0} := \text{submatrix}\left(Z_{pr0M}, 2, 4, 2, 4\right) \qquad Z_{mutual0} = \begin{pmatrix} 0.118 + 0.582i & 0.06 + 0.256i & 0 \\ 0.06 + 0.256i & 0.142 + 0.698i & -0.015 - 0.064i \\ 0 & -0.015 - 0.064i & 0.089 + 0.436i \end{pmatrix}$$

Mutual Coupling in Fault Analysis

$MVA := 1000kW$

$pu := 1$

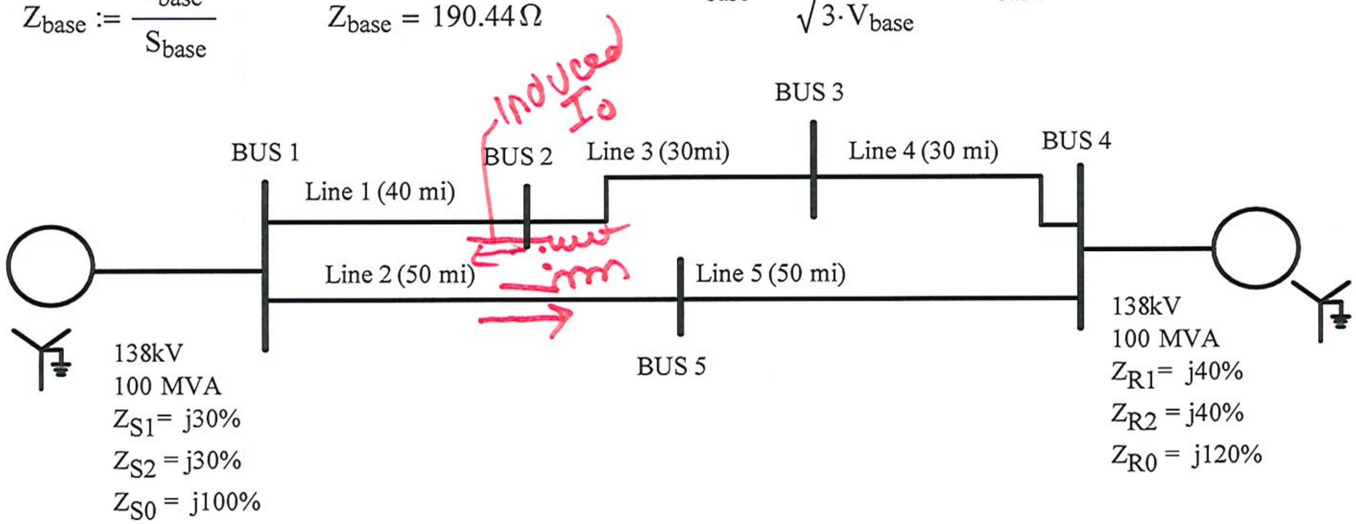
$a := 1 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}}$

$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix}$

$S_{base} := 100MVA$ $\quad V_{base} := 138kV$

$Z_{base} := \frac{V_{base}^2}{S_{base}}$ $\quad Z_{base} = 190.44\Omega$

$I_{base} := \frac{S_{base}}{\sqrt{3} \cdot V_{base}}$ $\quad I_{base} = 418.37A$



- Source impedances:

  $Z_{S1} := j \cdot 0.3pu$ $\quad Z_{R1} := j \cdot 0.4pu$

  $Z_{S0} := j \cdot 1.0pu$ $\quad Z_{R0} := j \cdot 1.2pu$

- Line impedances from lecture 32 handout:

- Line 1, 40 mile length

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- zero sequence current for the three branches with mutual coupling) don't need for other branches

$$\begin{pmatrix} I_{0\_L1\_M} \\ I_{0\_L2\_M} \\ I_{0\_L3\_M} \end{pmatrix} := Z_{mutual0}^{-1} \cdot \begin{pmatrix} V0_{M_0} - V0_{M_1} \\ V0_{M_0} - V0_{M_4} \\ V0_{M_2} - V0_{M_1} \end{pmatrix}$$

Back to the digraph...

- Note sign on I_{0_L3}, dot polarity

$|I_{0\_L1\_M}| = 0.048 \qquad \arg(I_{0\_L1\_M}) = 90.677 \cdot deg$

$|I_{0\_L2\_M}| = 0.4$ faulted line $\qquad \arg(I_{0\_L2\_M}) = -84.435 \cdot deg$

$|I_{0\_L3\_M}| = 0.048 \qquad \arg(-I_{0\_L3\_M}) = 90.677 \cdot deg$

— Unfaulted

$$I_{ABC\_L1\_M} := A_{012} \cdot \begin{pmatrix} I_{0\_L1\_M} \\ I_{1\_L1\_M} \\ I_{2\_L1\_M} \end{pmatrix}$$

$$\overrightarrow{|I_{ABC\_L1\_M}|} = \begin{pmatrix} 0.013 \\ 0.0669 \\ 0.0669 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg(I_{ABC\_L1\_M})} = \begin{pmatrix} 62.941 \\ 93.277 \\ 93.277 \end{pmatrix} \cdot deg$$

$$\overrightarrow{|I_{ABC\_L1\_NoM}|} = \begin{pmatrix} 0.0487 \\ 0.007 \\ 0.007 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg(I_{ABC\_L1\_NoM})} = \begin{pmatrix} -80.275 \\ 102.864 \\ 102.864 \end{pmatrix} \cdot deg$$

$$I_{ABC\_L2\_M} := A_{012} \cdot \begin{pmatrix} I_{0\_L2\_M} \\ I_{1\_L2\_M} \\ I_{2\_L2\_M} \end{pmatrix}$$

$$\overrightarrow{|I_{ABC\_L2\_M}|} = \begin{pmatrix} 1.1325 \\ 0.0336 \\ 0.0336 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg(I_{ABC\_L2\_M})} = \begin{pmatrix} -84.309 \\ -86.547 \\ -86.547 \end{pmatrix} \cdot deg$$

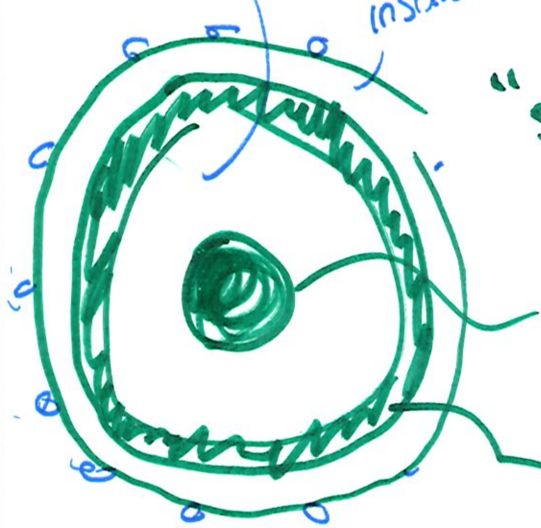
$$\overrightarrow{|I_{ABC\_L2\_NoM}|} = \begin{pmatrix} 1.0853 \\ 0.0073 \\ 0.0073 \end{pmatrix} \cdot pu \qquad \overrightarrow{\arg(I_{ABC\_L3\_NoM})} = \begin{pmatrix} -80.275 \\ 102.864 \\ 102.864 \end{pmatrix} \cdot deg$$

- Underground/undersea cables

Built from single core cables

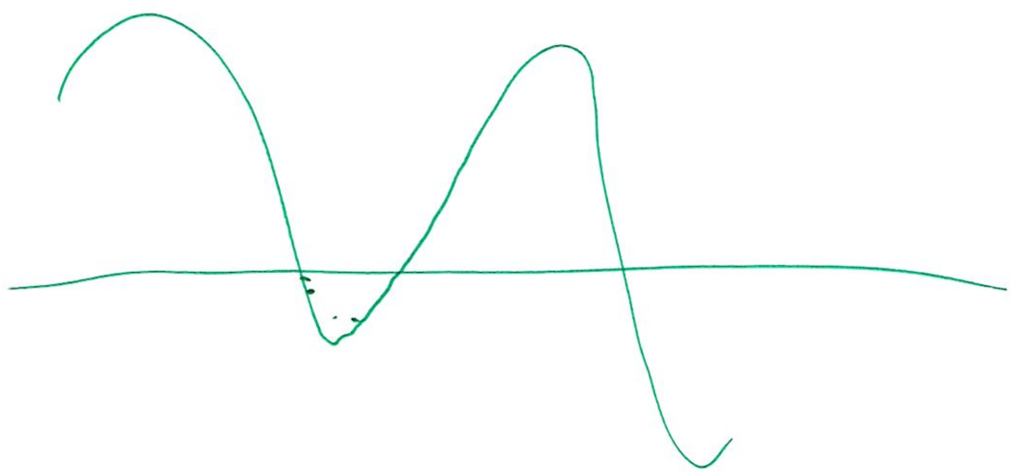
- circuit model for fault analysis -
  - → use standard 60Hz/50Hz models

- for power flow, use for lines over 15km

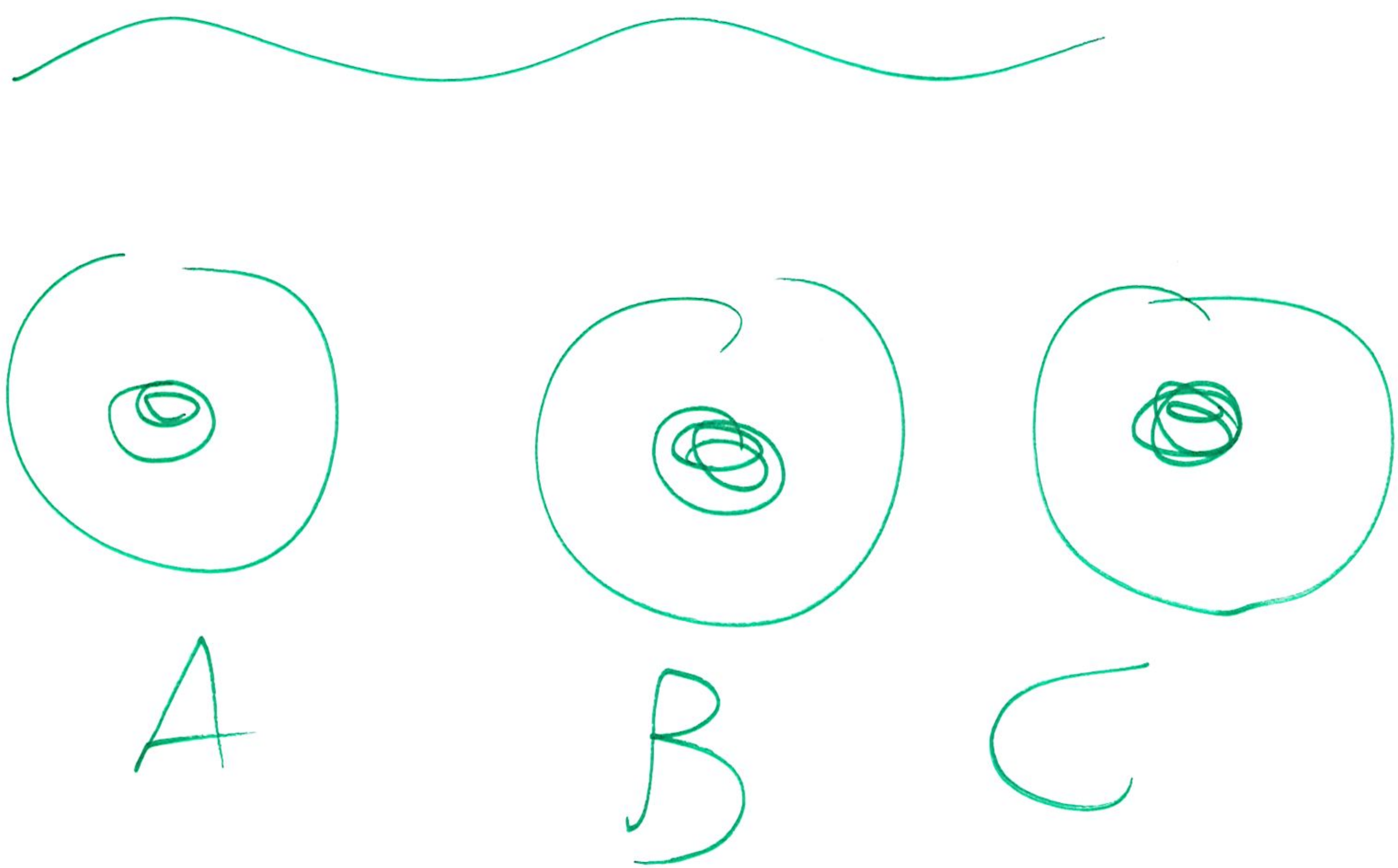


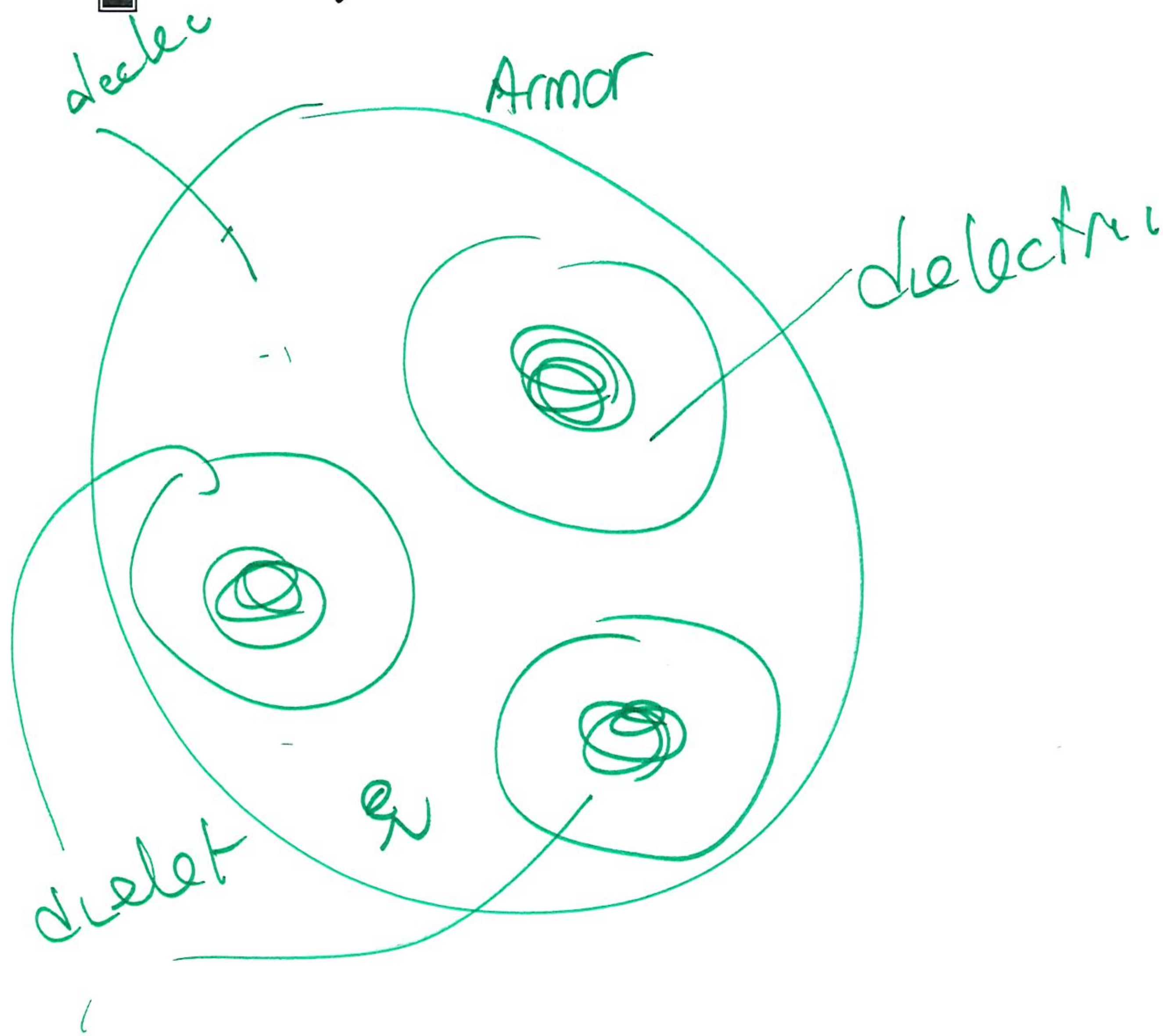
cross linked polyethelene

XLPE



3Ø designs

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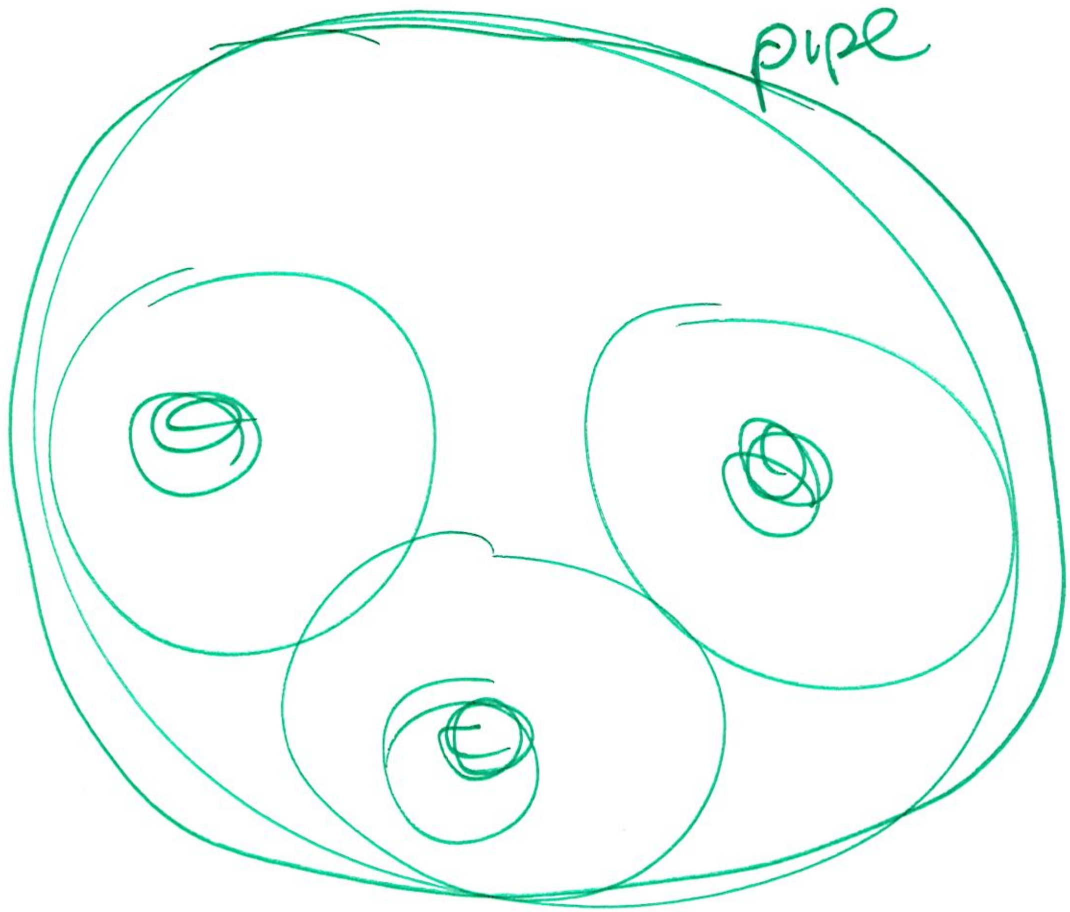
pipe

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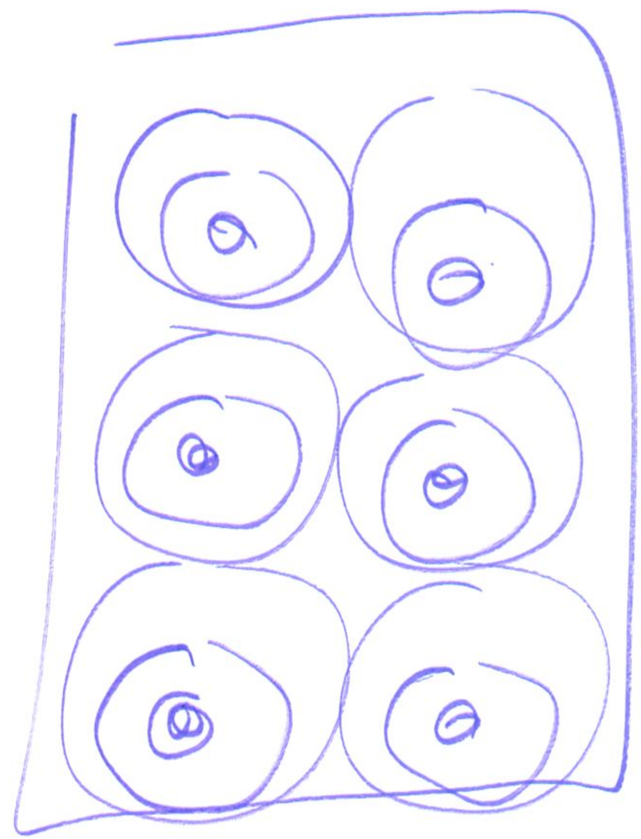
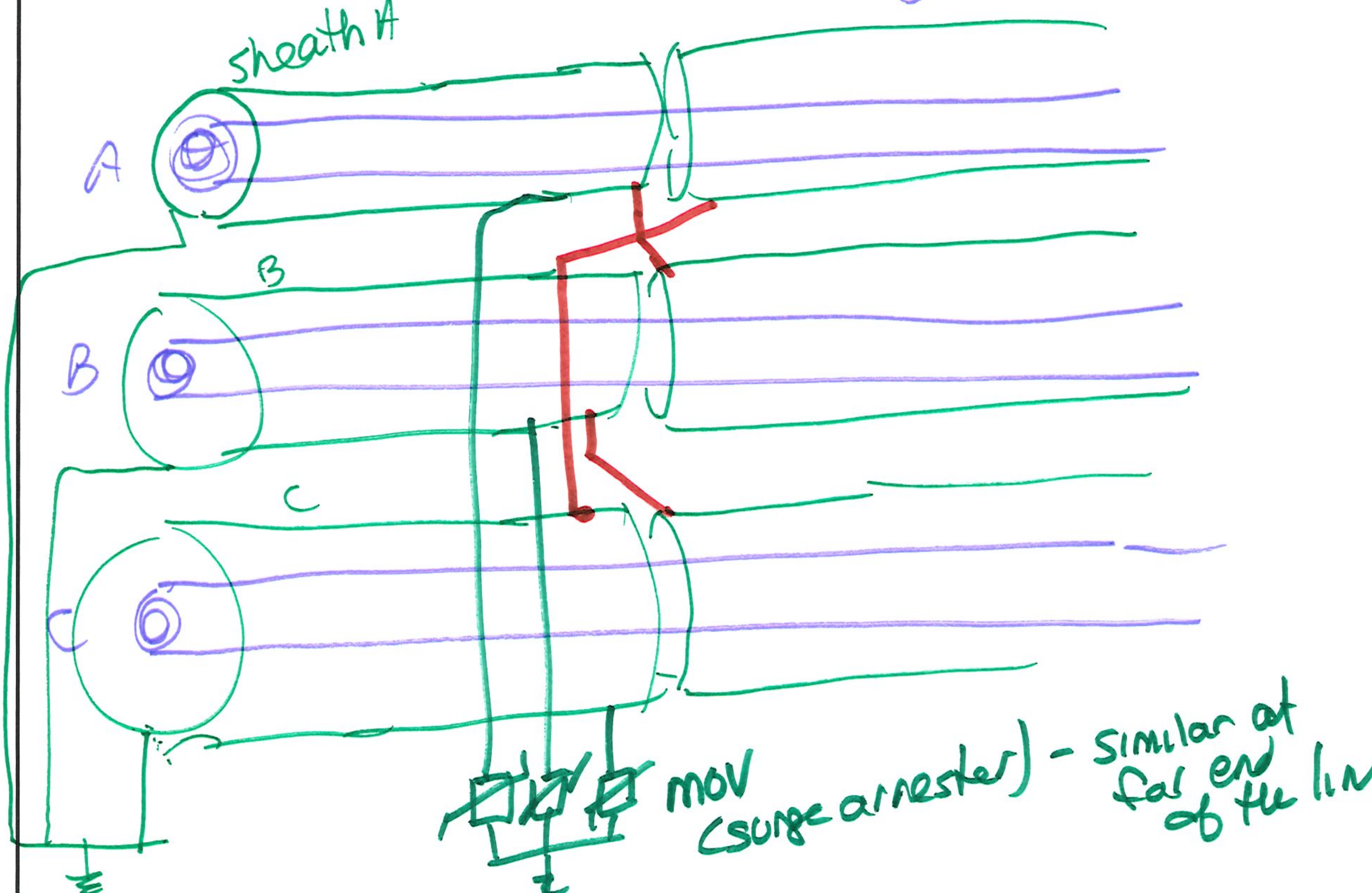


Sheath - cross bonding



Cable sequence impedances

- Best option - if available
  - → get data from manufacture
  - → ABC
  - → 012

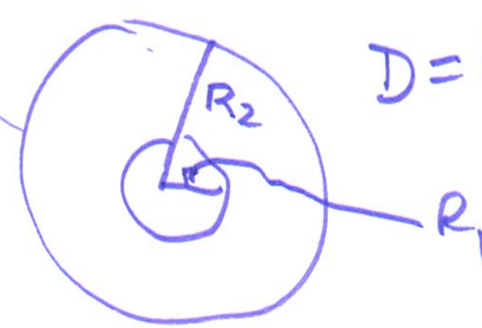
Calculating L, C, R per unit length

R → from data sheets

R_{AC}

$$L_{core} = \frac{\mu_0}{2\pi} \ln\left(\frac{D}{GMR}\right)$$

GMR: Geometric mean radius



$D = R_2 - R_1$

R_1: outer radius of core

R_2: inner radius of sheath