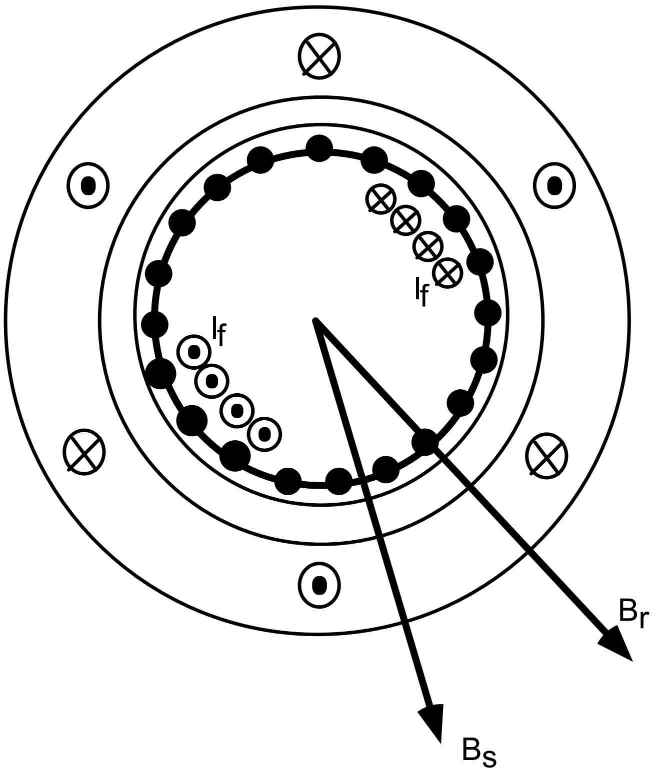
ECE 523: Synchronous Machine Steady-state Equivalent Circuit



Rotor flux pulls the stator flux
in a generator, vice versa for a motor

$$\tau = k\cdot\left(B_s \times B_r\right)$$

Electric Circuit Equivalent:

$$v_a(t) = r_a \cdot i_a(t) + \frac{d}{dt}\lambda_a$$

r_a is the copper loss in the stator winding.

- λ is called a flux linkage.
- If we ignore saturation then

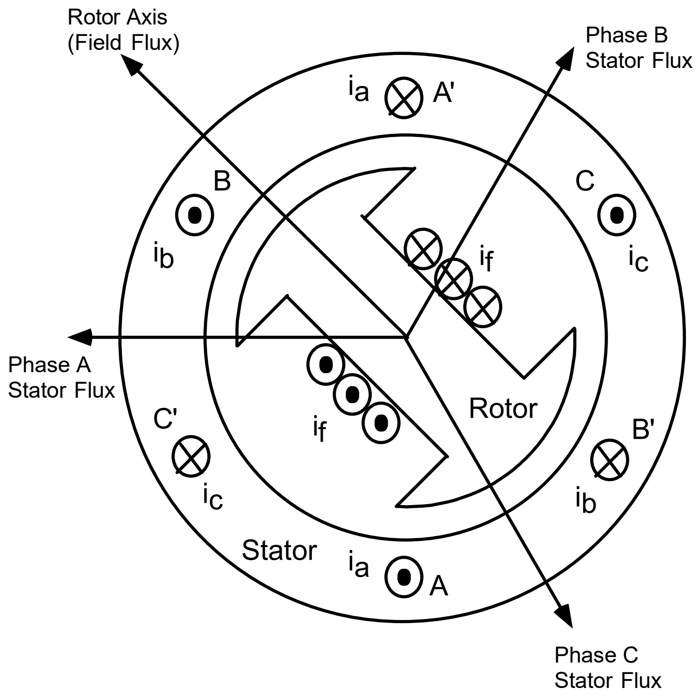
$$\lambda = N\cdot\phi$$

- Faraday's law, for a transformer:

$$e = \frac{d}{dt}\lambda = N\cdot\frac{d}{dt}\phi$$

- However, for a rotating machine, the number of turns coupled by the flux will also vary since the windings are distributed.
- So the Faraday's Law equation becomes:

$$e = \frac{d}{dt}\lambda = N\cdot\frac{d}{dt}\phi(t) + \phi\cdot\frac{d}{dt}N(t)$$



- The machine case is further complicated by the coupling between phases, so λ_a will be impacted by the currents in the other phases and in the field circuit
- Recall that when we discussed magnetic circuits, we defined a term called "Reluctance"

$$L = \frac{N^2}{Rel}$$

- We can use L to related the flux linkages to the currents in each coupled circuit

$$\lambda_a = L_{aa}\cdot i_a(t) + L_{ab}\cdot i_b(t) + L_{ac}\cdot i_c(t) + L_{aF}\cdot i_f$$

  - L_{aa} is the self inductance of phase A
  - L_{ab} is mutual inductance between phases A and B
  - L_{ac} is mutual inductance between phases A and C
  - L_{aF} is mutual inductance between phases A the field winding F

  We will find later that each of these inductances each have a constant part and a part that varies with time as the rotor turns

- As a first approximation, we can break the L_{aa} into:

$$L_{aa} = L_{aa0} + L_{al}$$ where: $$L_{aa0} = \frac{N_s^2}{2\cdot Rel_{ag}}$$

N_s = Stator_turns

Reluctance across the air gap

L_{al} = Leakage

$$L_{ab} = L_{aa0} \cdot \cos(120deg)$$

$$L_{ac} = L_{aa0} \cdot \cos(-120deg)$$

Note that only the self term has leakage (leakage is not part of the mutual inductance.

$$L_{aF} = L_f \cdot \cos(\theta_0 + \omega \cdot t)$$

note that this varies with time.

$$L_f = \frac{N_f \cdot N_s}{2Rel_{ag}}$$

Then we can rewrite the flux linkage equation as:

$$\lambda_a = L_{aa0} \cdot \left[i_a(t) - \left(\frac{i_b(t)}{2} \right) - \left(\frac{i_c(t)}{2} \right) \right] + L_{al} \cdot i_a(t) + L_F \cdot i_f \cdot \cos(\theta_0 + \omega \cdot t)$$

- Note impact of cos(+120) and cos(-120)

Assume balanced three phase circuit:

$$i_a + i_b + i_c = 0 \qquad \text{or:} \qquad i_a = -(i_b + i_c)$$

So we can rewrite the expression as:

$$\lambda_a = \frac{3}{2} \cdot L_{aa0} \cdot i_a(t) + L_{al} \cdot i_a(t) + L_F \cdot i_f \cdot \cos(\theta_0 + \omega \cdot t)$$

Then the voltage equation becomes:

$$v_a(t) = r_a \cdot i_a(t) + \frac{d}{dt}\lambda_a = r_a \cdot i_a(t) + \frac{3}{2} \cdot L_{aa0} \cdot \frac{d}{dt} i_a(t) + L_{al} \cdot \frac{d}{dt} i_a(t) - \omega \cdot L_F \cdot i_f \cdot \sin(\theta_0 + \omega \cdot t)$$

Note derivative of cosine term

Define:

- Voltage due to B_r

$$e_a(t) = -\omega \cdot L_F \cdot i_f \cdot \sin(\theta_0 + \omega \cdot t) = \omega \cdot L_F \cdot i_f \cdot \cos\left(\frac{\pi}{2} + \theta_0 + \omega \cdot t \right)$$

Define: $$\delta = \frac{\pi}{2} + \theta_0$$

Phasor form: $$\overline{E_a} = |E_a| \cdot e^{j \cdot \delta}$$

- Voltage due to B_s (armature reaction)

$$\frac{3}{2}\cdot L_{aa0}\cdot\frac{d}{dt}i_a(t)$$

Now define the **direct axis synchronous reactance:**

$$X_d = 2\cdot\pi\cdot 60\,\text{Hz}\left(\frac{3}{2}\cdot L_{aa0} + L_{al}\right)$$

Dominated by L_{aa0} since leakage is small

So the voltage equation becomes:

$$v_a(t) = r_a\cdot i_a(t) + L_s\cdot\frac{d}{dt}i_a(t) + e_a(t)$$

Or in phasor form:

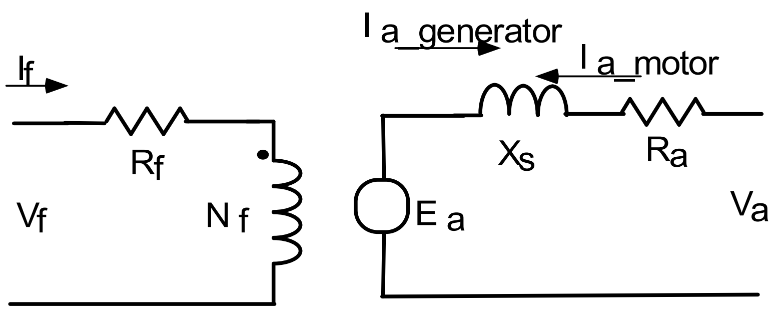
$$\overline{V_a} = r_a\cdot\overline{I_a} + j\cdot X_s\cdot\overline{I_a} + \overline{E_a}$$

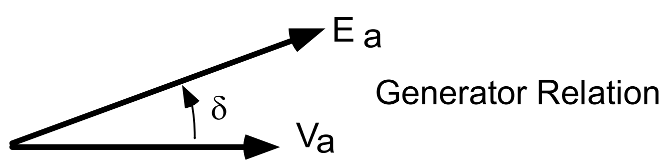
Think back to dc machine, this implies current entering machine (motor operation)

Generator equation:

$$\overline{V_a} = \overline{E_a}\cdot -r\cdot\overline{I_a} - j\cdot X_s\cdot\overline{I_a}$$

Per Phase Equivalent Circuit (assumes Y connected):

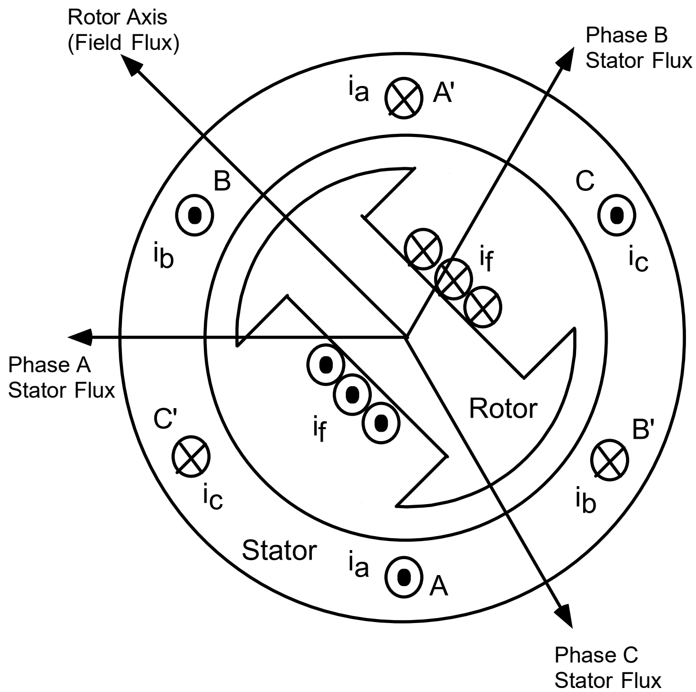




- For a large machine it is generally possible to neglect R_a
- Normally the X/R ratio is over 20

Salient Pole Machine Equations

Inductance Equations:



Direct axis inductance of phase "s" (round rotor term)

$$L_{ss} = h \cdot k \cdot {N_s}^2 \qquad k = \frac{\mu_0 \cdot r \cdot l_{eff} \cdot \pi}{4}$$

or we could say: $$L_{ss} = \frac{{N_s}^2}{2 \cdot Rel_{ag}}$$

Saliency adjustment term:

$$L_\Delta = \frac{\Delta h}{2} \cdot k \cdot {N_s}^2$$

Coupling to rotor:

$$L_{sf} = \left(h + \frac{\Delta h}{2}\right) \cdot k \cdot N_s \cdot N_f \qquad L_m = \frac{3}{2} \cdot L_{ss}$$

Self inductances:

$$L_{aa}(\theta_r) = (L_{ss} + L_{ls}) - L_\Delta \cdot \cos(2 \cdot \theta_r)$$

$$L_{bb}(\theta_r) = (L_{ss} + L_{ls}) - L_\Delta \cdot \cos\left[2 \cdot \left(\theta_r - \frac{2\pi}{3}\right)\right] = (L_{ss} + L_{ls}) - L_\Delta \cdot \cos\left(2 \cdot \theta_r + \frac{2 \cdot \pi}{3}\right)$$

$$L_{cc}(\theta_r) = (L_{ss} + L_{ls}) - L_\Delta \cdot \cos\left[2 \cdot \left(\theta_r + \frac{2\pi}{3}\right)\right] = (L_{ss} + L_{ls}) - L_\Delta \cdot \cos\left(2 \cdot \theta_r - \frac{2 \cdot \pi}{3}\right)$$

Stator to stator mutual inductances

$$L_{ab}(\theta_r) = \frac{-L_{ss}}{2} - L_\Delta \cdot \cos\left(2 \cdot \theta_r - \frac{2 \cdot \pi}{3}\right) \qquad L_{ac}(\theta_r) = \frac{-L_{ss}}{2} - L_\Delta \cdot \cos\left(2 \cdot \theta_r + \frac{2 \cdot \pi}{3}\right)$$

$$L_{bc}(\theta_r) = \frac{-L_{ss}}{2} - L_\Delta \cdot \cos(2 \cdot \theta_r)$$

all symmetric: $Lab = Lba$ and so on.

Stator to rotor mutual inductances

$$L_{af}(\theta_r) = -L_{sf}\cdot \sin(\theta_r) \qquad L_{bf}(\theta_r) = -L_{sf}\cdot \sin\left(\theta_r - \frac{2\cdot\pi}{3}\right)$$

$$L_{cf}(\theta_r) = -L_{sf}\cdot \sin\left(\theta_r + \frac{2\cdot\pi}{3}\right)$$

Rotor inductance:

$$L_{ff}(\theta_r) = \left(h + \frac{\Delta h}{2}\right)\cdot k\cdot N_f^{\,2} + L_{lf}$$

Electric Circuit Equivalent (generator convention):

$$v_a(t) = -r_a\cdot i_a(t) - \frac{d}{dt}\lambda_a$$

$$\lambda_a(t) = \lambda_{aarm} + \lambda_{af} = L_{aa}\cdot i_a(t) + L_{ab}\cdot i_b(t) + L_{ac}\cdot i_c(t) + L_{aF}\cdot i_f$$

$$\lambda_a(t) = \left(\frac{3}{2}\cdot L_{ss} + L_{ls}\right)\cdot\sqrt{2}\cdot \left|I_s\right| \cdot\cos\left(\omega_e\cdot t - \theta_i\right) - \frac{3}{2}\cdot L_\Delta\cdot\sqrt{2}\,\left|I_s\right|\cdot\cos\left(\omega_e\cdot t + \theta_i\right) + L_{sf}\cdot I_f\cdot\cos\left(\omega_e\cdot t + \frac{\pi}{2}\right)$$

Phasor form:

$$\overline{\Lambda_a} = L_{sf}\cdot\frac{I_f}{\sqrt{2}}\,e^{j\cdot\frac{\pi}{2}} + \left(\frac{3}{2}\cdot L_{ss} + L_{ls}\right)\cdot\left|I_s\right|\,e^{j\cdot\theta_i} - \frac{3}{2}\cdot L_\Delta\cdot\left|I_s\right|\cdot e^{-j\cdot\theta_i}$$

Field term   Round rotor term   Salient pole adjustment term

$$\overline{I_f} = \frac{I_f}{\sqrt{2}}\cdot e^{j\cdot\frac{\pi}{2}}$$

"Phasor" for field current representing angle of rotor axis

$$\overline{\Lambda_{af}} = L_{sf}\cdot\overline{I_f}$$

$$\overline{\Lambda_{aarm}} = \left(\frac{3}{2}\cdot L_{ss} + L_{ls}\right)\cdot\left|I_s\right|\,e^{-j\cdot\theta_i} - \frac{3}{2}\cdot L_\Delta\cdot\left|I_s\right|\cdot e^{-j\cdot\theta_i}$$

$$\overline{\Lambda_{aarm}} = \left(\frac{3}{2}\cdot L_{ss} + L_{ls}\right)\cdot\left(\cos(\theta_i) - j\cdot\sin(\theta_i)\right) - \frac{3}{2}\cdot L_{\Delta}\cdot\left|I_s\right|\cdot\left(\cos(\theta_i) + j\cdot\sin(\theta_i)\right)$$

$$\overline{\Lambda_{aarm}} = \left(\frac{3}{2}\cdot L_{ss} + L_{ls} - \frac{3}{2}\cdot L_{\Delta}\right)\cdot\left|I_s\right|\cdot\left(\cos(\theta_i)\right) - j\left(\frac{3}{2}\cdot L_{ss} + L_{ls} + \frac{3}{2}\cdot L_{\Delta}\right)\cdot\left|I_s\right|\cdot\sin(\theta_i)$$

Define quadrature and direct axis inductances:

$$L_q = \frac{3}{2}\cdot L_{ss} + L_{ls} - \frac{3}{2}\cdot L_{\Delta}$$

$$L_d = \frac{3}{2}\cdot L_{ss} + L_{ls} + \frac{3}{2}\cdot L_{\Delta}$$

Note the cause of the difference between L_d and L_q

Define quadrature and direct axis currents:

$$\overline{I_{aq}} = \left|I_s\right|\cdot\cos(\theta_i)\, e^{j\cdot 0}$$

$$\overline{I_{ad}} = -j\left|I_s\right|\cdot\sin(\theta_i)\, e^{j\cdot 0} = \left|I_s\right|\cdot\sin(\theta_i)\cdot e^{-j\cdot\frac{\pi}{2}}$$

Therefore:

$$\overline{\Lambda_{aarm}} = L_q\cdot\overline{I_{aq}} + L_d\cdot\overline{I_{ad}}$$

$$\overline{\Lambda_a} = \overline{\Lambda_{af}} + \overline{\Lambda_{aq}} + \overline{\Lambda_{ad}} = L_{sf}\cdot\overline{I_f} + L_q\cdot\overline{I_{aq}} + L_d\cdot\overline{I_{ad}}$$

recall the angle of the field current

$$\overline{\Lambda_{aq}} = L_q\cdot\overline{I_{aq}}$$

$$\overline{\Lambda_{ad}} = L_{sf}\cdot\overline{I_f} + L_d\cdot\overline{I_{ad}}$$

Back to the voltage equation:

$$v_a(t) = -r_a\cdot i_a(t) - \frac{d}{dt}\lambda_a$$

as a phasor equation:

$$\overline{V_a} = -r_a \cdot \overline{I_a} - j \cdot \omega_e \cdot \overline{\Lambda_a} = -r_a \cdot \overline{I_a} - j \cdot \omega_e \cdot \overline{\Lambda_{aq}} - j \cdot \omega_e \cdot \overline{\Lambda_{ad}}$$

$$\overline{V_a} = -r_a \cdot \overline{I_a} - j \cdot \omega_e \cdot L_q \cdot \overline{I_{aq}} - j \cdot \omega_e \cdot L_d \cdot \overline{I_{ad}} - j \cdot \omega_e \cdot L_{sf} \cdot \overline{I_f}$$

$$X_q = \omega_e \cdot L_q$$

$$X_d = \omega_e \cdot L_d$$

$$X_{sf} = \omega_e \cdot L_{sf}$$

$$\overline{E_a} = \omega_e \cdot L_{sf} \cdot I_f \cdot e^{j \cdot \left(\frac{\pi}{2} - \frac{\pi}{2}\right)} = X_{sf} \cdot \left|I_f\right| \cdot e^{j \cdot 0}$$

Note that this is reference angle in this derivation

$$V_a = \overline{E_a} - r_a \cdot \overline{I_a} - j \cdot X_q \cdot \overline{I_{aq}} - j \cdot X_d \cdot \overline{I_{ad}}$$