

## Synchronous Machine Three Phase Fault Example

$$\text{pu} := 1 \quad \text{MVA} := \text{MW} \quad V_{LL} := 13.8\text{kV} \quad S_{\text{rated}} := 20\text{MVA}$$

$$X''_d := 0.145\text{pu} \quad X_2 := X''_d \quad T''_d := 0.035\text{sec} \quad \omega := 2 \cdot \pi \cdot 60\text{Hz}$$

$$X'_d := 0.240\text{pu} \quad X_{\text{tran}} := 0.0\text{pu} \quad T'_d := 1\text{sec}$$

$$X_d := 1.10\text{pu} \quad V_{\text{term}} := 1.0\text{pu} \quad T_a := 0.2\text{sec}$$

$$L_2 := \frac{X_2}{\omega} \cdot \left( \frac{V_{LL}^2}{S_{\text{rated}}} \right) \quad L_2 = 3.66 \cdot \text{mH}$$

$$R_a := \frac{L_2}{T_a} \quad R_a = 0.02 \Omega$$

Since unloaded:  $E''_a := V_{\text{term}} \quad E'_a := V_{\text{term}} \quad E_a := V_{\text{term}}$

$$I_{\text{Base}} := \frac{S_{\text{rated}}}{\sqrt{3} \cdot V_{LL}} \quad I_{\text{Base}} = 836.74 \text{ A} \quad \bullet \text{ Also } I_{\text{rated}}$$

$$\frac{R_a}{\frac{V_{LL}^2}{S_{\text{rated}}}} = 0.00192$$

$$I''_a := \frac{E''_a}{j \cdot X''_d + j \cdot X_{\text{tran}}} \quad I''_a = -6.9i \cdot \text{pu} \quad |I''_a| \cdot I_{\text{Base}} = 5770.62 \text{ A}$$

$$I'_a := \frac{E'_a}{j \cdot X'_d + j \cdot X_{\text{tran}}} \quad I'_a = -4.17i \cdot \text{pu} \quad |I'_a| \cdot I_{\text{Base}} = 3486.41 \text{ A}$$

$$I_{\text{ss}} := \frac{E_a}{j \cdot X_d + j \cdot X_{\text{tran}}} \quad I_{\text{ss}} = -0.91i \cdot \text{pu} \quad |I_{\text{ss}}| \cdot I_{\text{Base}} = 760.67 \text{ A}$$

- Less than rated load current

$$I_{\text{doffsetmax}} := \sqrt{2} \cdot \frac{E''_a}{X''_d + X_{\text{tran}}} \quad I_{\text{doffsetmax}} = 9.75 \cdot \text{pu} \quad I_{\text{doffsetmax}} \cdot I_{\text{Base}} = 8160.89 \text{ A}$$

$$t := 0\text{sec}, 1 \cdot 10^{-4}\text{sec}.. 2\text{sec}$$

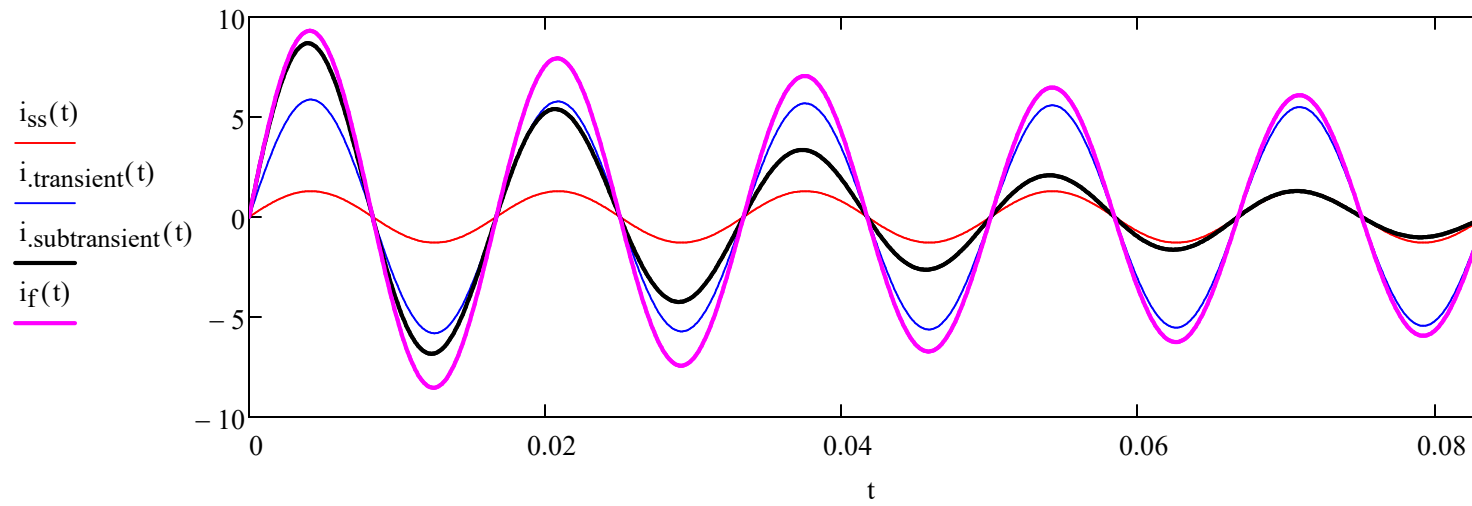
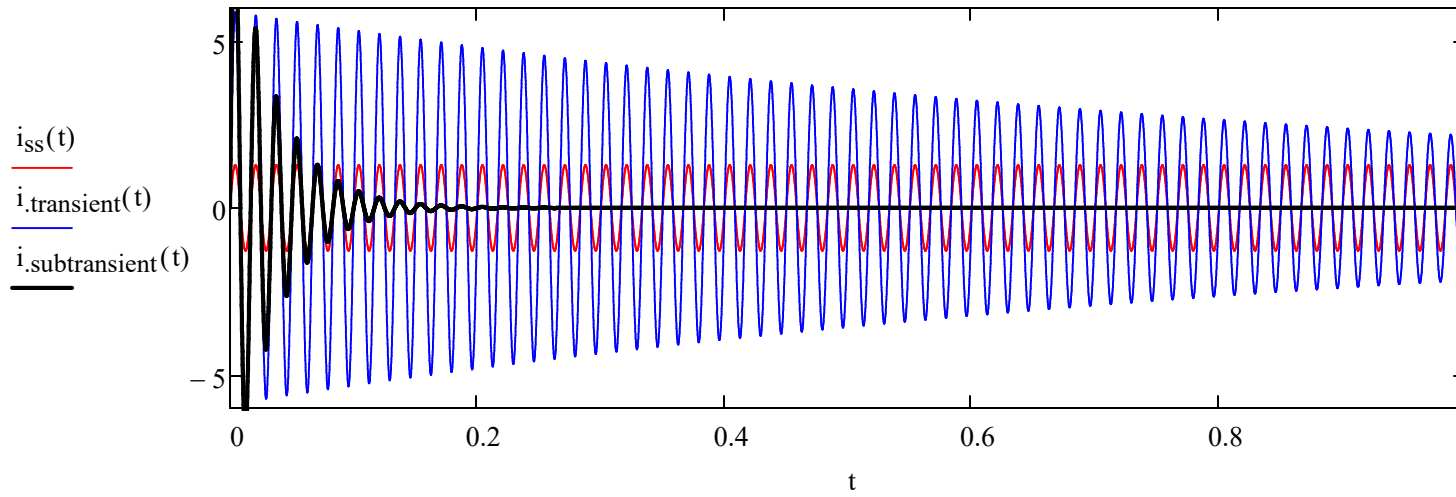
$$i_{ss}(t) := \sqrt{2} \cdot |I_{ss}| \cdot \cos(\omega \cdot t - 90\text{deg})$$

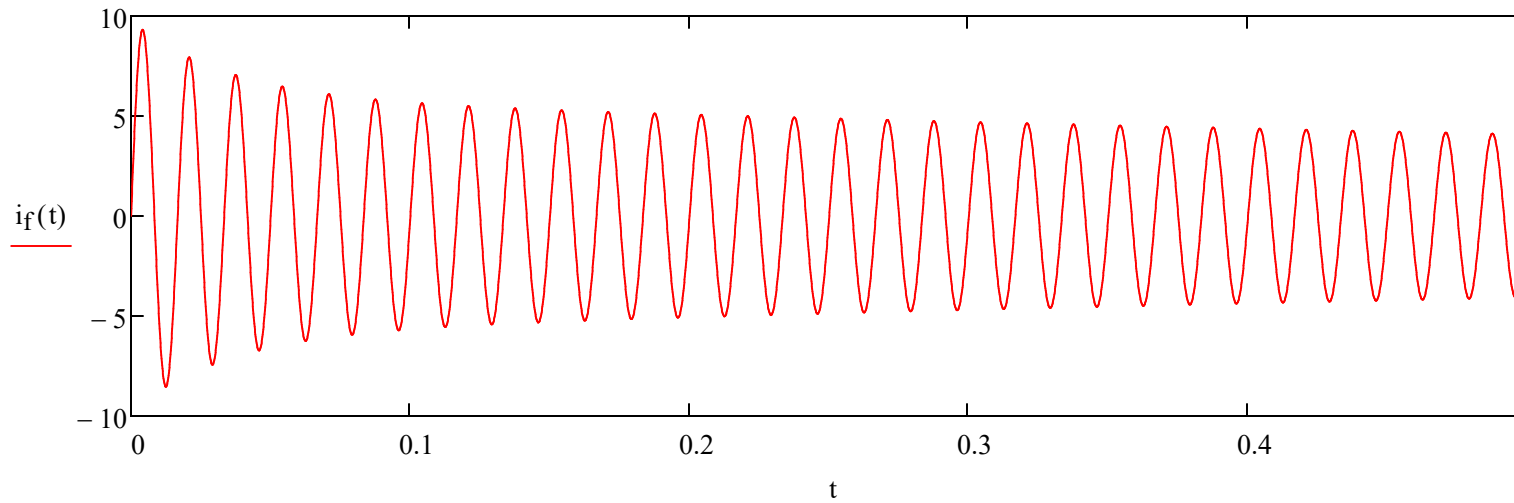
$$i_{\text{transient}}(t) := e^{\frac{-t}{T'_d}} \cdot (\sqrt{2} \cdot |I'_a| \cdot \cos(\omega \cdot t - 90\text{deg}))$$

$$i_{\text{subtransient}}(t) := e^{\frac{-t}{T''_d}} \cdot (\sqrt{2} \cdot |I''_a| \cdot \cos(\omega \cdot t - 90\text{deg}))$$

- Full symmetrical response:

$$i_f(t) := \sqrt{2} \cdot E''_a \cdot \left[ \frac{1}{X_d + X_{\text{tran}}} + \left( \frac{1}{X'_d + X_{\text{tran}}} - \frac{1}{X_d + X_{\text{tran}}} \right) \cdot e^{\frac{-t}{T'_d}} + \left( \frac{1}{X''_d + X_{\text{tran}}} - \frac{1}{X'_d + X_{\text{tran}}} \right) \cdot e^{\frac{-t}{T''_d}} \right] \cdot \cos(\omega \cdot t - 90\text{deg})$$

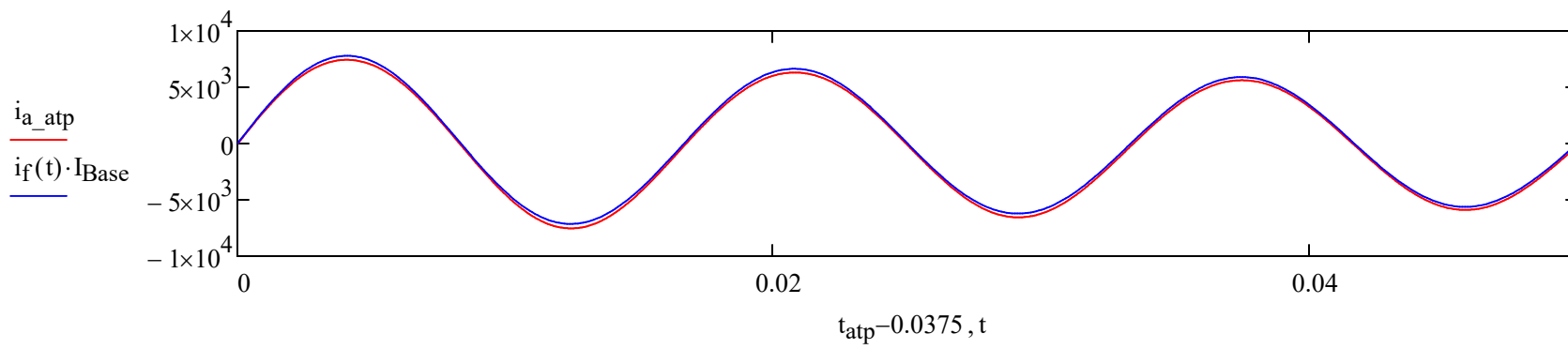
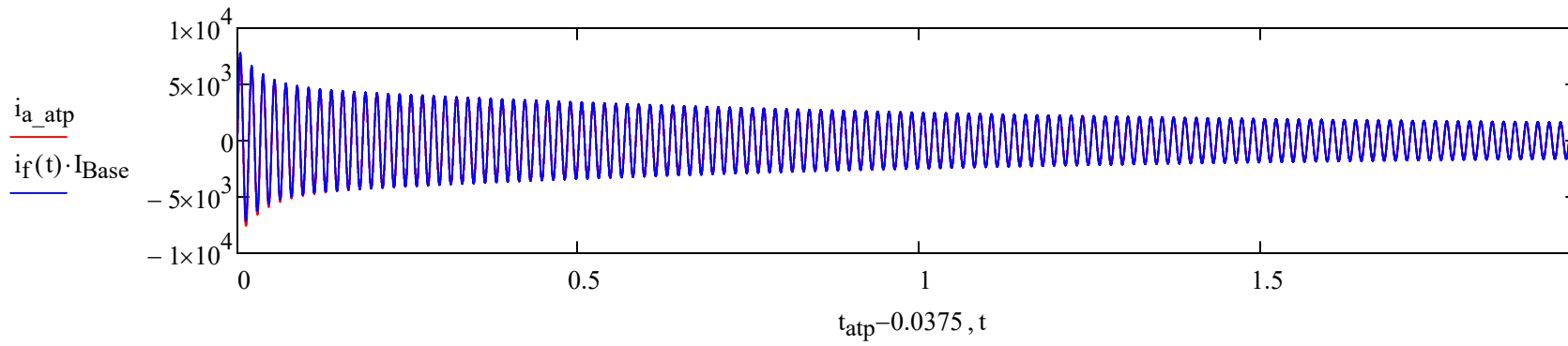




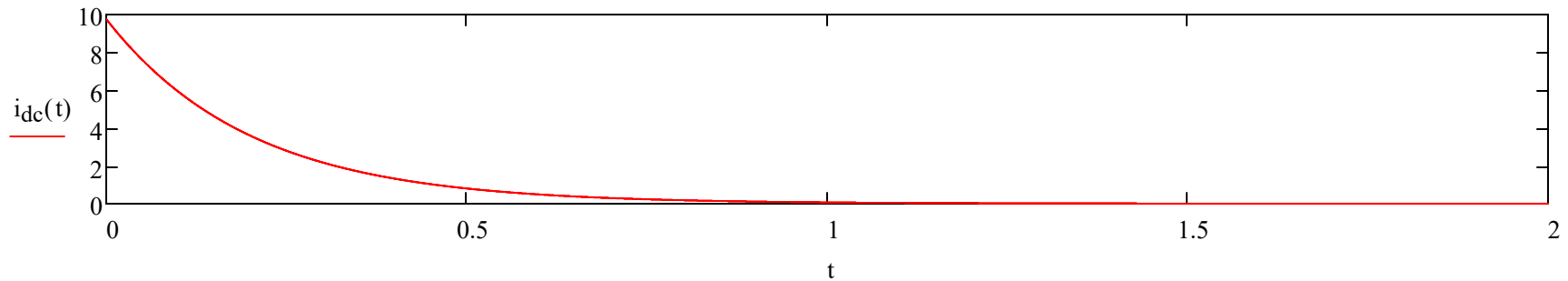
- Compare the equation results with a simulation performed with an detailed generator model in a power systems transients program. First the case with no dc offset.

ATPinput :=  
Generator.ADF

$t_{atp} := \text{ATPinput}^{(0)}$        $i_{a\_atp} := \text{ATPinput}^{(1)}$

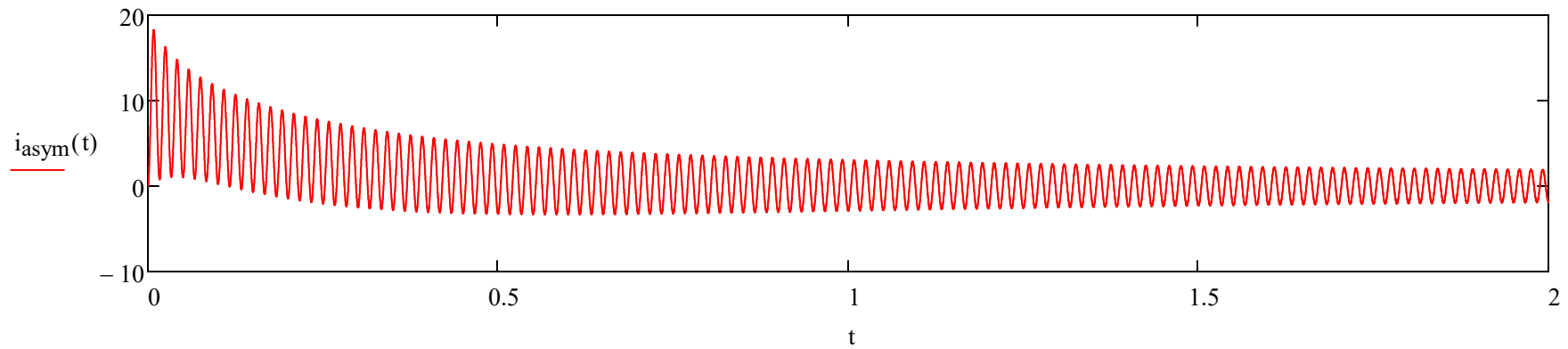


$$i_{dc}(t) := I_{dcoffsetmax} \cdot e^{-\frac{t}{T_a}}$$



$$i_{f2}(t) := \sqrt{2} \cdot E''_a \cdot \left[ \frac{1}{X_d + X_{\text{tran}}} + \left( \frac{1}{X'_d + X_{\text{tran}}} - \frac{1}{X_d + X_{\text{tran}}} \right) \cdot e^{-\frac{t}{T'_d}} + \left( \frac{1}{X''_d + X_{\text{tran}}} - \frac{1}{X'_d + X_{\text{tran}}} \right) \cdot e^{-\frac{t}{T''_d}} \right] \cdot \cos(\omega \cdot t - 180\text{deg})$$

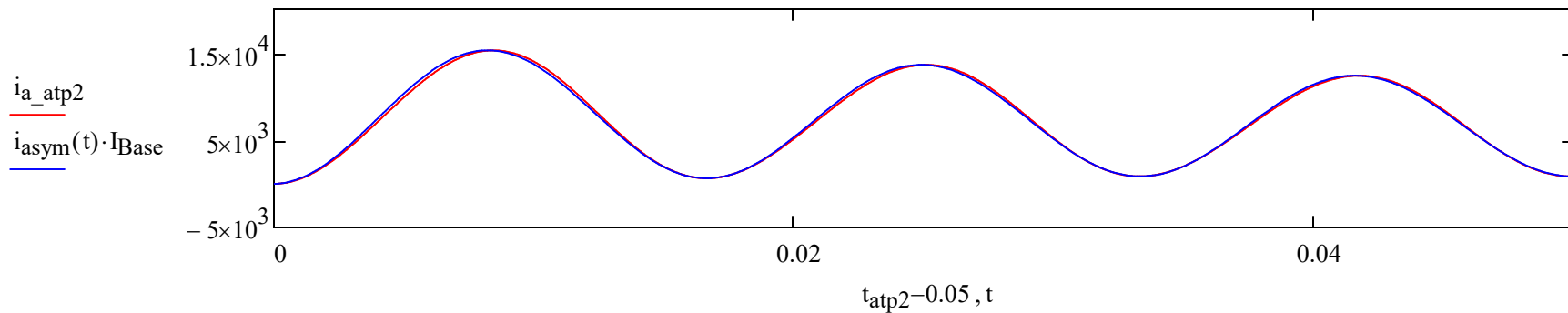
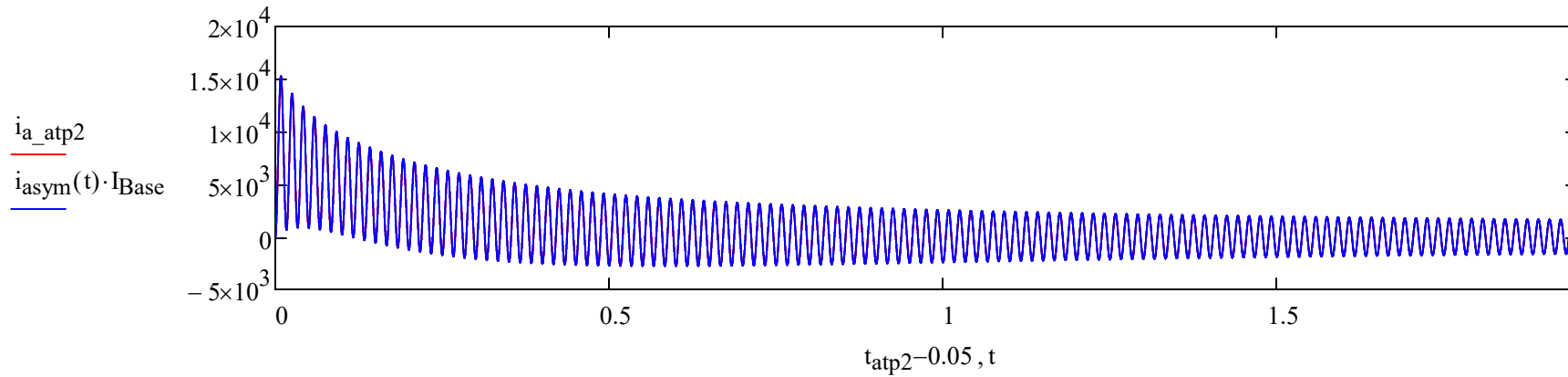
$$i_{\text{asym}}(t) := i_{f2}(t) + i_{\text{dc}}(t)$$



- Now compare with a transient simulation case with a DC offset:

ATPinput :=  
GenDConfigset.ADF

$t_{atp2} := \text{ATPinput}^{(0)}$        $i_{a\_atp2} := \text{ATPinput}^{(1)}$



**Generator Example 2: Repeat magnitude calculations with generator operating at rated full load ( $pf = 0.8$  lagging for this machine).**

$$\phi_{\text{load}} := \text{acos}(0.8) \quad \phi_{\text{load}} = 36.87 \cdot \text{deg}$$

$$S_{\text{load}} := 1 \text{ pu} \cdot e^{j \cdot \phi_{\text{load}}}$$

$$I_{\text{load}} := \left( \frac{S_{\text{load}}}{V_{\text{term}}} \right) \quad I_{\text{load}} = (0.8 - 0.6i) \cdot \text{pu} \quad |I_{\text{load}}| = 1 \cdot \text{pu}$$

$$R_{\text{load}} := \left( \frac{1}{0.8} \right) \cdot \frac{V_{\text{LL}}^2}{S_{\text{rated}}} \quad R_{\text{load}} = 11.9 \Omega$$

$$E''_{a2} := V_{\text{term}} + I_{\text{load}} \cdot j \cdot X''_d \quad |E''_{a2}| = 1.09 \cdot \text{pu} \quad \arg(E''_{a2}) = 6.09 \cdot \text{deg}$$

$$X_{\text{load}} := \left( \frac{1}{0.6} \right) \cdot \frac{V_{\text{LL}}^2}{S_{\text{rated}}}$$

$$I''_{a2} := \frac{E''_{a2}}{j \cdot X''_d + j \cdot X_{\text{tran}}} \quad |I''_{a2}| = 7.54 \cdot \text{pu} \quad \arg(I''_{a2}) = -83.91 \cdot \text{deg}$$

$$\frac{X_{\text{load}}}{\omega} = 42.1 \cdot \text{mH}$$

$$I_{\text{Base}} = 836.74 \text{ A}$$

$$E'_{a2} := V_{\text{term}} + I_{\text{load}} \cdot j \cdot X'_d \quad |E'_{a2}| = 1.16 \cdot \text{pu} \quad \arg(E'_{a2}) = 9.53 \cdot \text{deg}$$

$$\theta := \arg(I_{\text{load}})$$

$$I'_{a2} := \frac{E'_{a2}}{j \cdot X'_d + j \cdot X_{\text{tran}}} \quad |I'_{a2}| = 4.83 \cdot \text{pu} \quad \arg(I'_{a2}) = -80.47 \cdot \text{deg}$$

$$\theta = -36.87 \cdot \text{deg}$$

$$E_{a\_2} := V_{\text{term}} + I_{\text{load}} \cdot j \cdot X_d \quad |E_{a\_2}| = 1.88 \cdot \text{pu} \quad \arg(E_{a\_2}) = 27.93 \cdot \text{deg}$$

$$I_{a\_2} := \frac{E_{a\_2}}{j \cdot X_d + j \cdot X_{\text{tran}}} \quad |I_{a\_2}| = 1.71 \cdot \text{pu} \quad \arg(I_{a\_2}) = -62.07 \cdot \text{deg}$$



$$I_{d\text{offsetmax}_2} := \sqrt{2} \cdot \frac{|E''_{a2}|}{|j \cdot X''_d + j \cdot X_{\text{tran}}|} \quad I_{d\text{offsetmax}_2} = 10.66 \cdot \text{pu}$$

$$\frac{\frac{1037.6\text{A}}{I_{\text{Base}}}}{\left[ \frac{\sqrt{2} \cdot |E''_{a2}|}{(X''_d + X_{\text{tran}})} \right]} = 0.1163 \quad \text{acos}(0.1163) = 83.32 \cdot \text{deg}$$

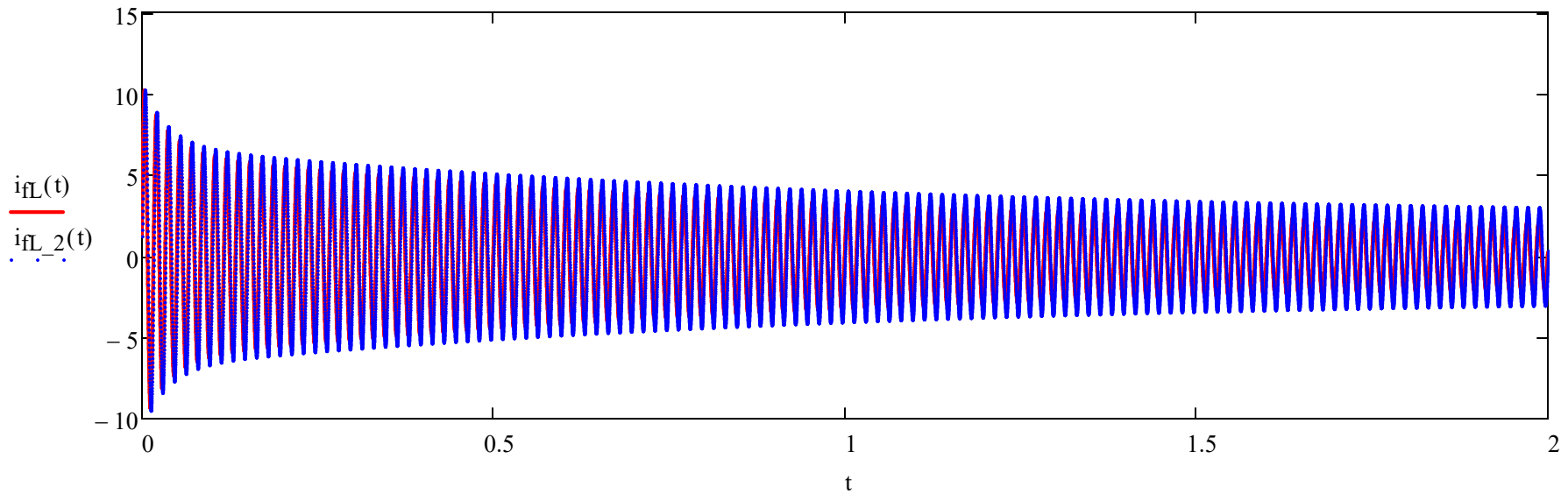
Now look at the impact of the loading on the decay exponential response:

$$i_{fL}(t) := \sqrt{2} \cdot |E''_{a2}| \cdot \left[ \left[ \frac{1}{(X''_d + X_{\text{tran}})} - \frac{1}{(X'_d + X_{\text{tran}})} \right] \cdot e^{\frac{-t}{T''_d}} + \left[ \frac{1}{(X'_d + X_{\text{tran}})} - \frac{1}{(X_d + X_{\text{tran}})} \right] \cdot e^{\frac{-t}{T'_d}} + \left[ \frac{1}{(X_d + X_{\text{tran}})} \right] \right] \cdot \cos(\omega \cdot t - 83.32 \text{deg})$$

The more accurate formula for the loaded generator case is:

$$i_{fL\_2}(t) := \sqrt{2} \cdot \left[ \left[ \frac{|E''_{a2}|}{(X''_d + X_{\text{tran}})} - \frac{|E'_{a2}|}{(X'_d + X_{\text{tran}})} \right] \cdot e^{\frac{-t}{T''_d}} + \left[ \frac{|E'_{a2}|}{(X'_d + X_{\text{tran}})} - \frac{|E_{a\_2}|}{(X_d + X_{\text{tran}})} \right] \cdot e^{\frac{-t}{T'_d}} + \left[ \frac{|E_{a\_2}|}{(X_d + X_{\text{tran}})} \right] \right] \cdot \cos((\omega \cdot t - 83.32 \text{deg}))$$

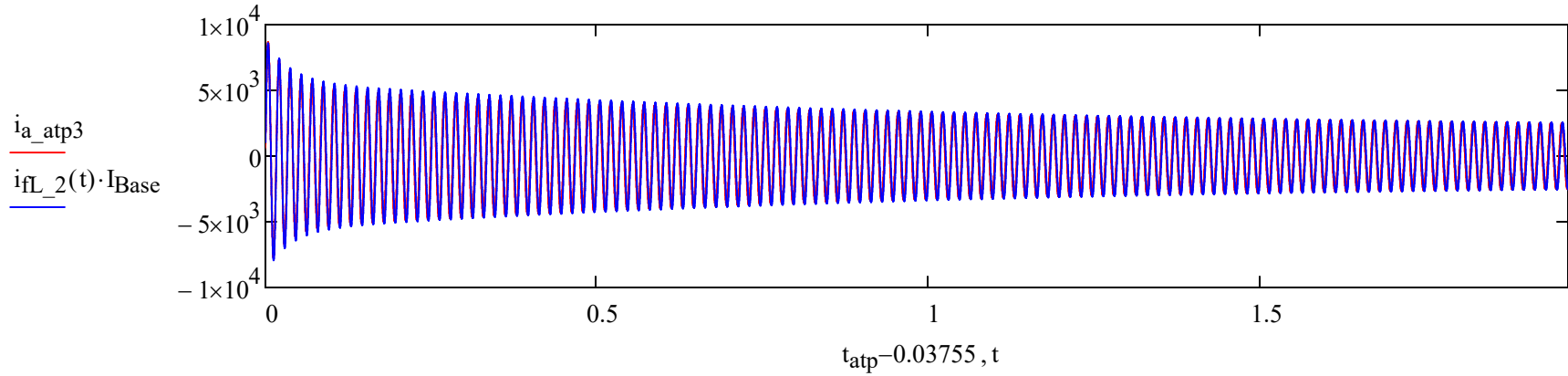
Note that the case with a single voltage value decays more quickly.



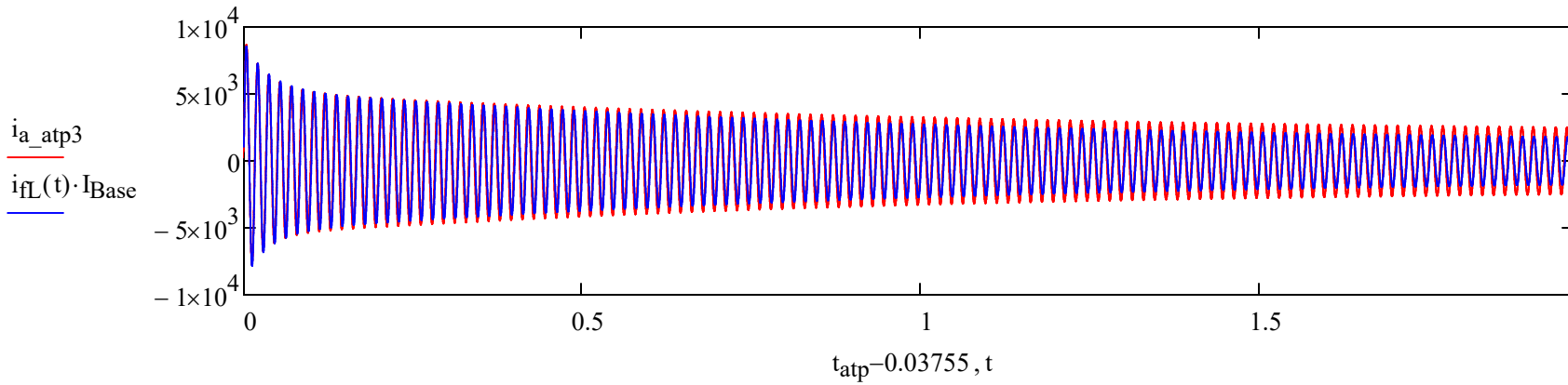
ATPinput :=  
LoadedGen.ADF

$t_{atp3} := \text{ATPinput}^{(0)}$

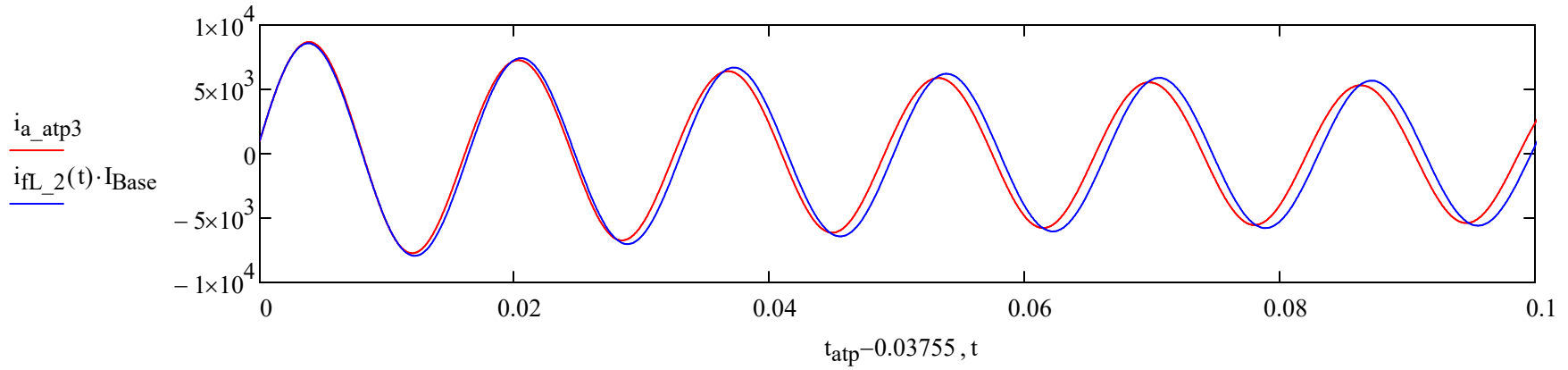
$i_{a\_atp3} := \text{ATPinput}^{(1)}$



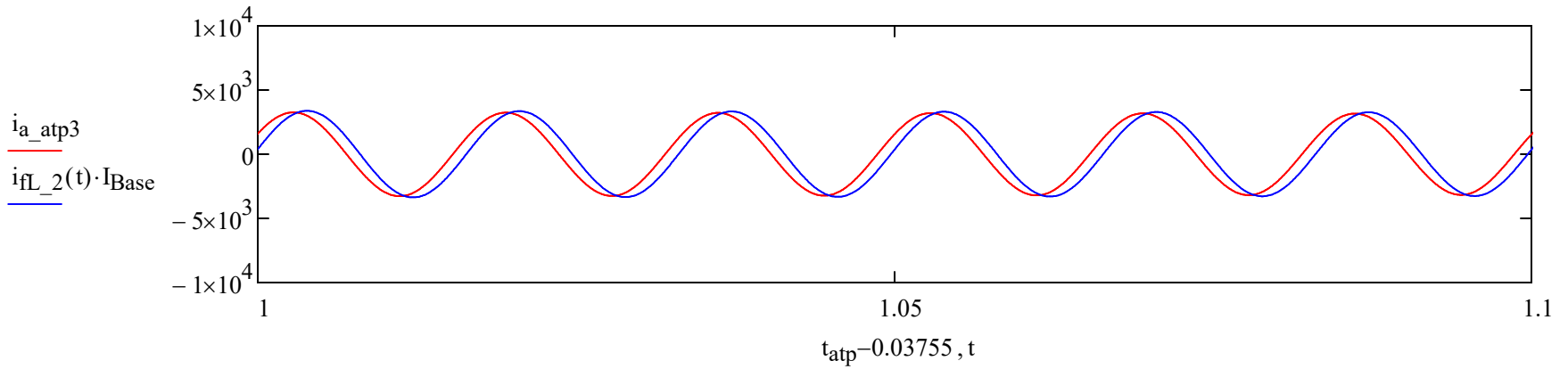
The case with the voltage values matching the impedances matches the ATP results better than the first case, which is below.



The second case with the different voltages is more accurate for amplitude



- Machine rotor is accelerating causing the phase error, which would normally occur
- Not accounted for in analytical calculations.



Notice that amplitudes line up