ECE 523
Symmetrical Components

Session 28

Final Exam

- 3 day take home
- Available evening Dec 8 up to Dec 15.

HW 6

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ECE 523: Homework #6

Due Session 30 (December 7)

1. A cylindrical rotor, synchronous machine with the machine parameters given below is operating at rated current (1.0 pu) and 85% lagging power factor when a 3 fault occurs at the machine terminals. Compute:

   (a) The steady-state voltage E_q behind the synchronous impedance. Plot a phasor diagram showing E_q, V_a (terminal voltage), and I_a

   (b) The voltage E" behind the synchronous impedance

   (c) The initial symmetrical fault current

   (d) The peak symmetrical current after 5 cycles and 10 cycles.

   (e) The maximum asymmetrical current after 5 cycles and 10 cycles.

$pu := 1$ $\quad X_d := 1.05pu$ $\quad X''_d := 0.12pu$ $\quad T'_{d0} := 5.6sec$

$X_q := 1.02pu$ $\quad X''_q := 0.15pu$ $\quad T'_d := 1.1sec$

$X'_d := 0.23pu$ $\quad X_2 := 0.12pu$ $\quad T''_d := 0.035sec$

$X'_q := 0.23pu$ $\quad R_a := 0.0055pu$ $\quad T_a := 0.16sec$

2. Repeat problem 1 using the data for the salient pole machine given below.

$X_d := 1.25pu$ $\quad X''_d := 0.24pu$ $\quad T'_{d0} := 5.6sec$

$X_q := 0.75pu$ $\quad X''_q := 0.34pu$ $\quad T'_d := 1.8sec$

$X'_d := 0.37pu$ $\quad X_2 := 0.24pu$ $\quad T''_d := 0.035sec$

$X'_q := 0.75pu$ $\quad R_a := 0.009pu$ $\quad T_a := 0.15sec$

3. A 2000 HP, 4160V, induction motor operates at a slip of 2%, with 93% efficiency at rated load. The machine parameters are given below. Assume a power factor of 0.85 lagging at rated conditions. Do the following:

   (a) Sketch the positive and negative sequence equivalent circuits using the machines ratings as a base.

   (b) Convert the equivalent circuits of part A to a 4160V, 100MVA base

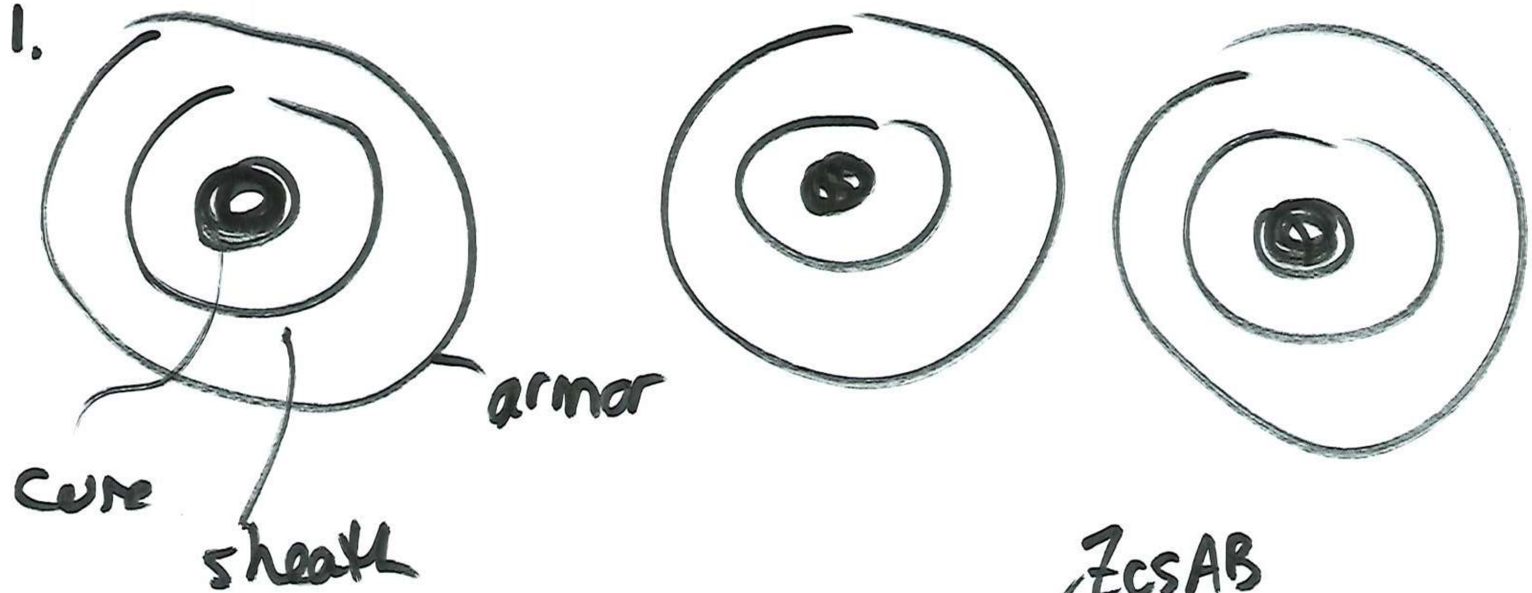
   (c) Compute the initial fault current provided by the machine to a 3 phase fault at the motor terminals (rated prefault voltage at the terminals and rated load).

   (d) Repeat part C for a LL fault

$R_s := 0.02pu$ $\quad X_s := 0.075pu$ $\quad R_r := 0.02pu$ $\quad X_r := 0.075pu$ $\quad X_m := 3.0pu$

Impedance Matrices

1.



$$Z_{ABC} = \begin{bmatrix} Z_{cAA} & Z_{cAB} & Z_{cAC} & Z_{csAA} & Z_{csAB} & Z_{cArmorAA} & Z_{cArmorAB} \cdots \\ Z_{cBA} & \cdots & & & & & \\ & & & Z_{sAA} & Z_{sAB} \cdots & & \\ & & & \cdot & Z_{sBB} & & \\ & & & & \ddots & & \end{bmatrix}$$

$$Z_{ABC} = \begin{bmatrix} Z_{cc} & Z_{cs} & Z_{C\,Armor} \\ Z_{sc} & Z_{ss} & Z_{s\,Armor} \\ Z_{Armor\,core} & Z_{Armor\,sheath} & Z_{Armor\,Armor} \end{bmatrix}$$

9x9

→ sheath & armor don't carry load current

- grounded

reduce to equivalent 3x3

- Similar to what we did with GW

Also with capacitance

If cable has crossbonding

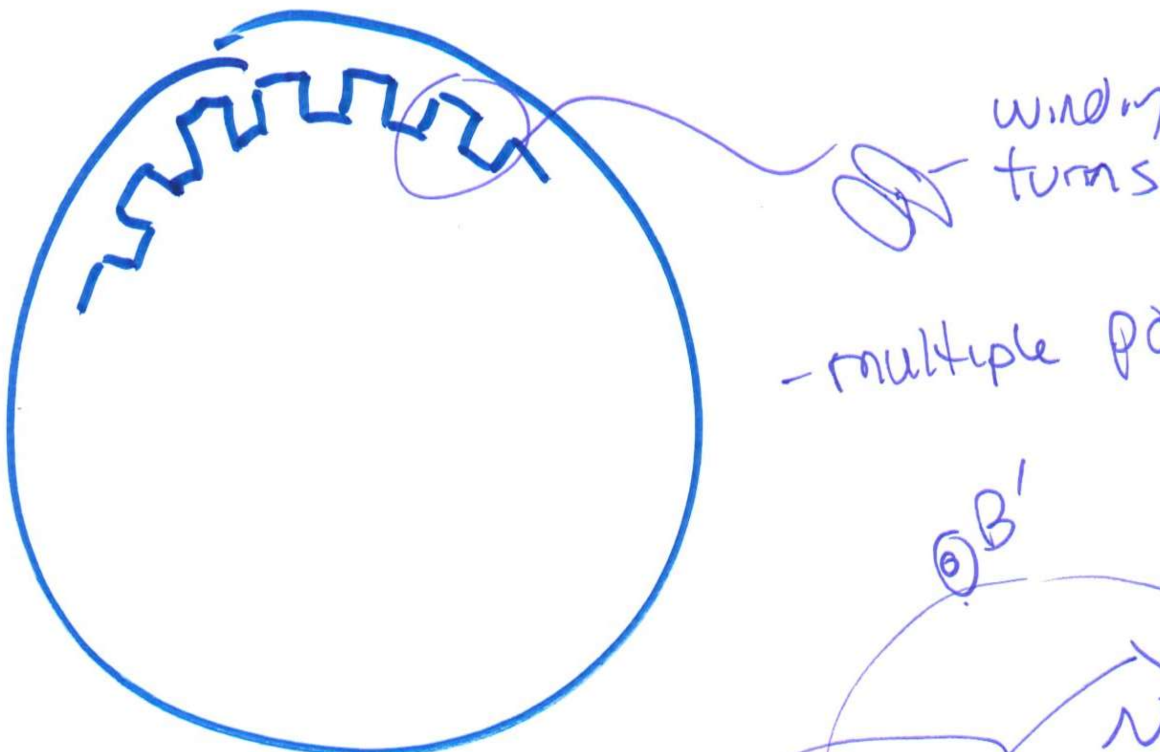
- calculate matrix for each section
  - represent change in sheath connections (rotation)
- Add the matrices (in series)
- Then reduce
- Then symmetrical components

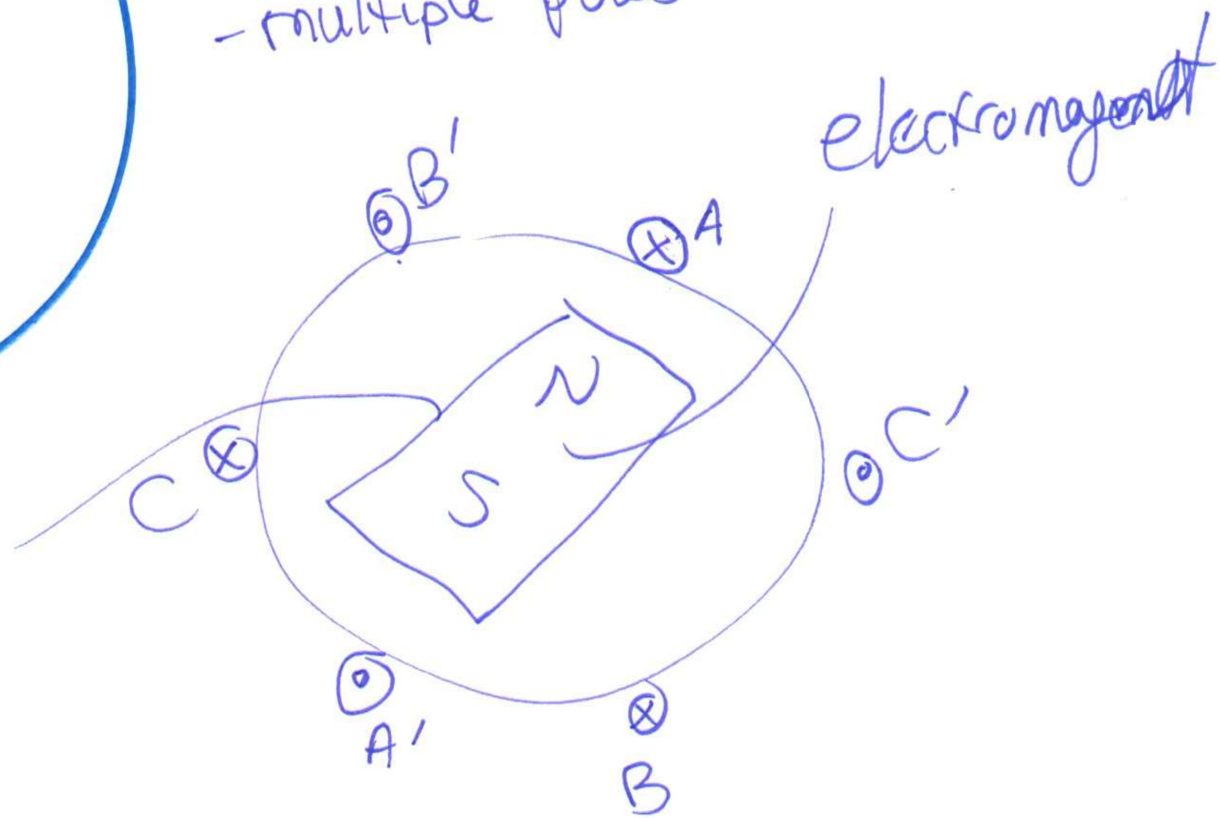
Rotating machines

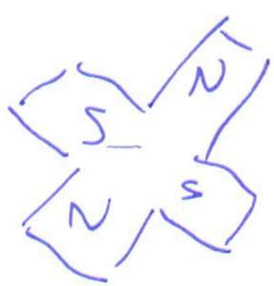
- Fault response
  - External faults

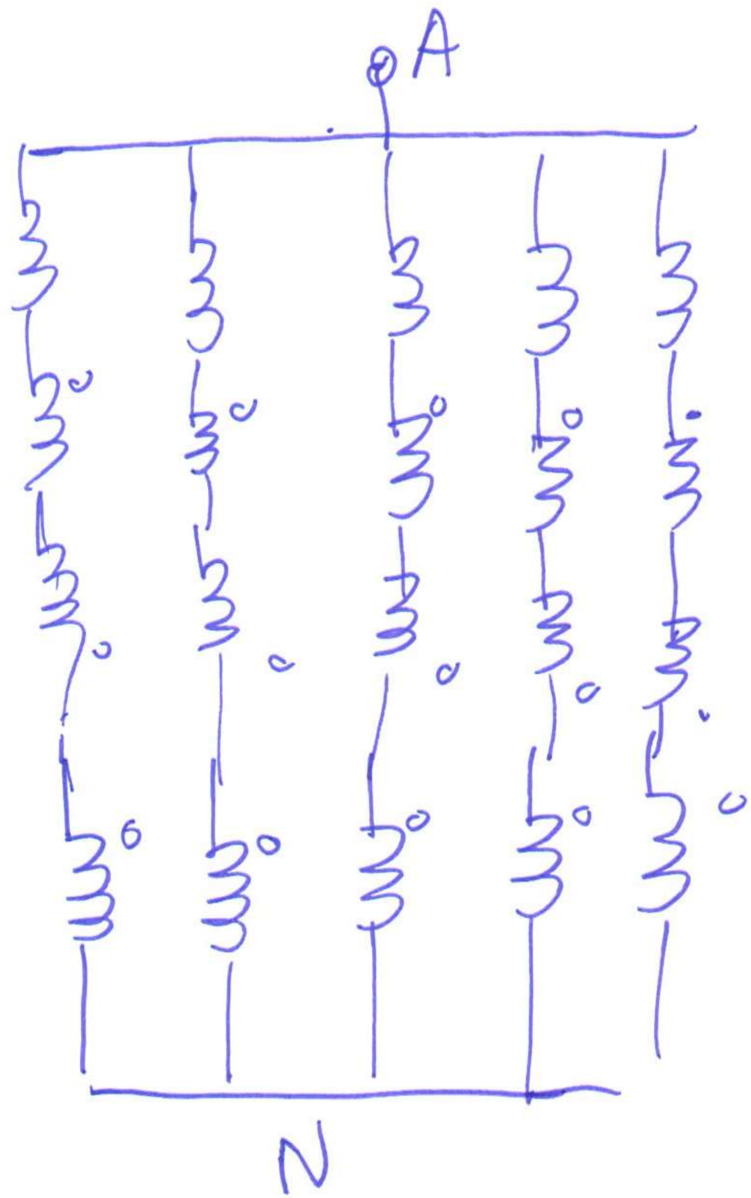
- Initially
  - Synchronous machines
    - → Generators
    - motors
    - synchronous condensers
  - Induction machines
    - → motors
    - some about generators – more with inverter based generator

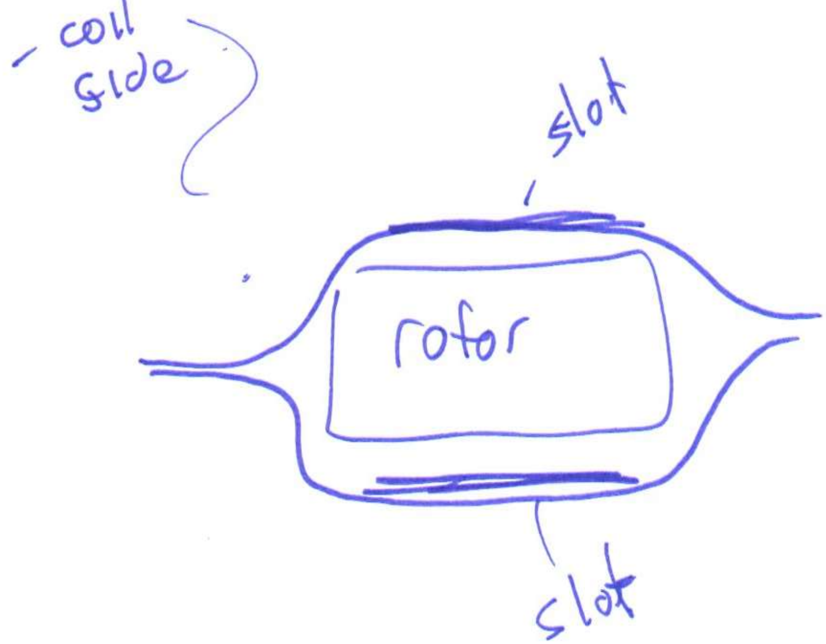
3Ø phase stator windings



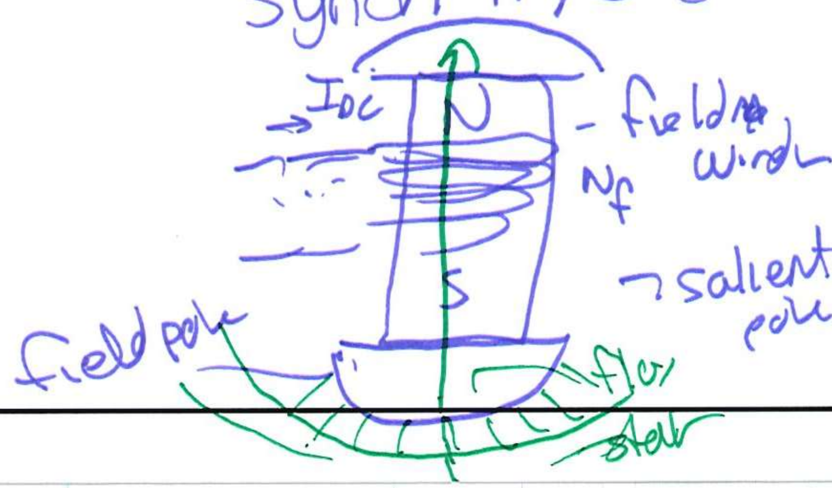




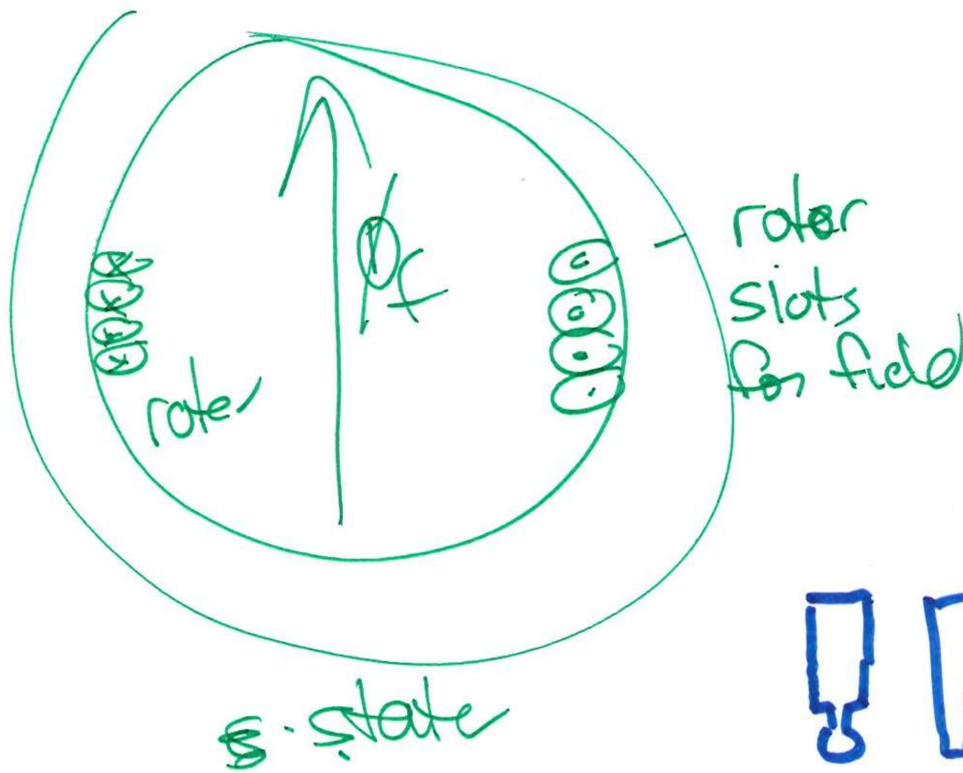




Synch m/c (machine)

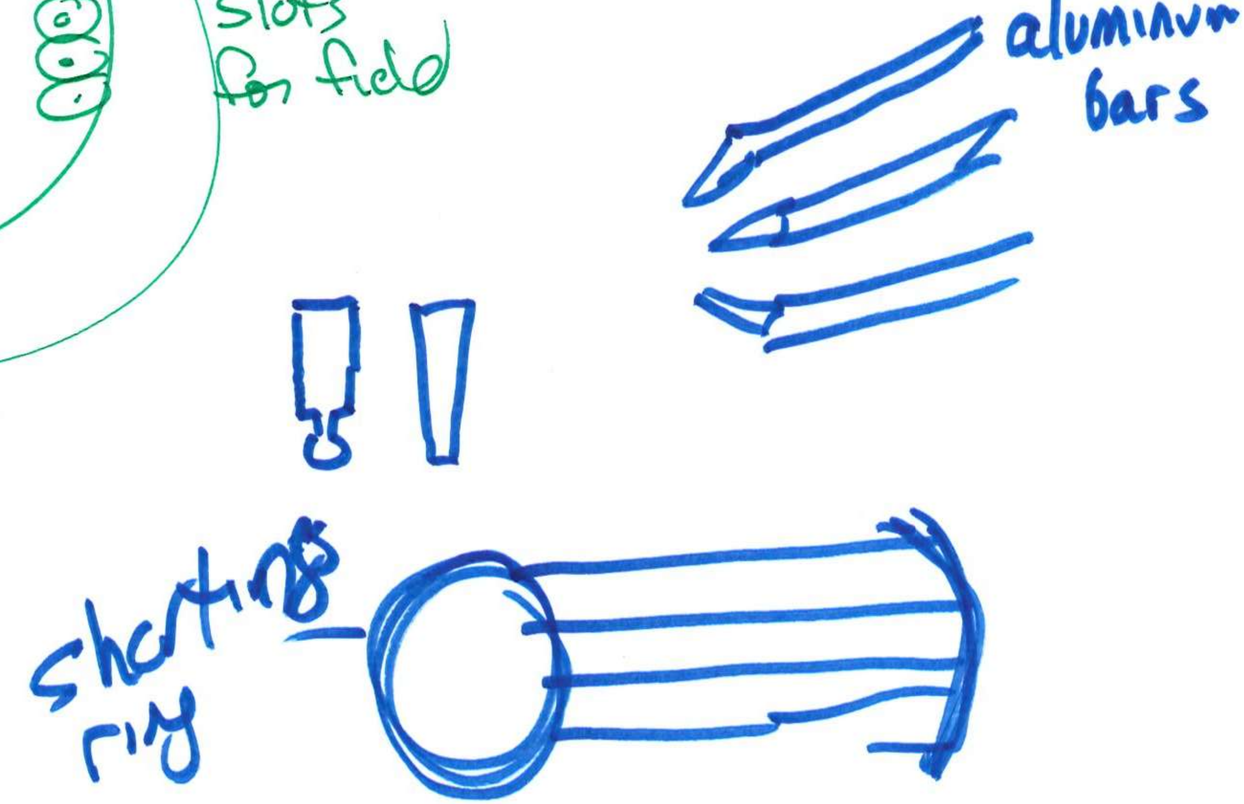


Round synch machine



Induction machine

rotor



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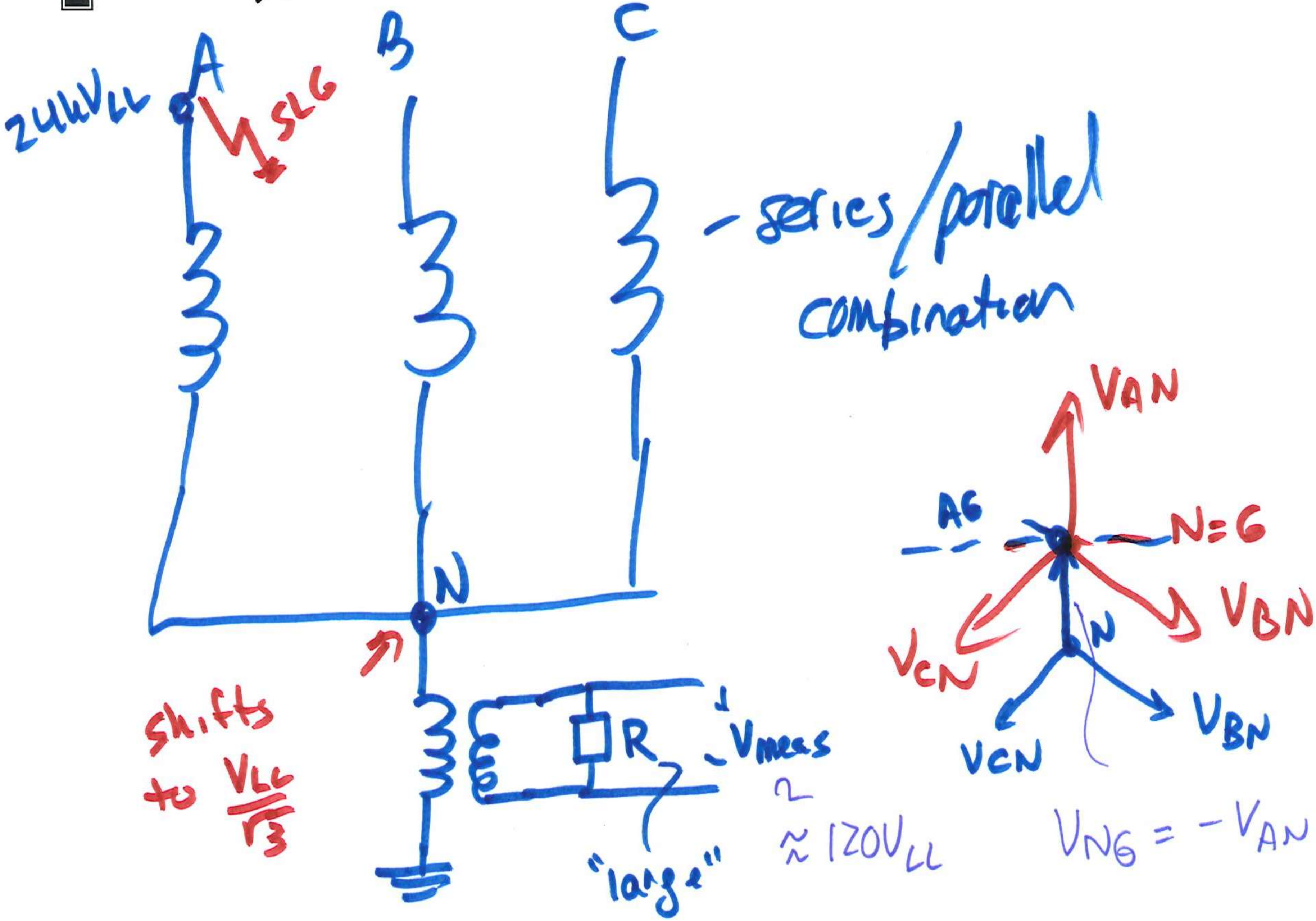

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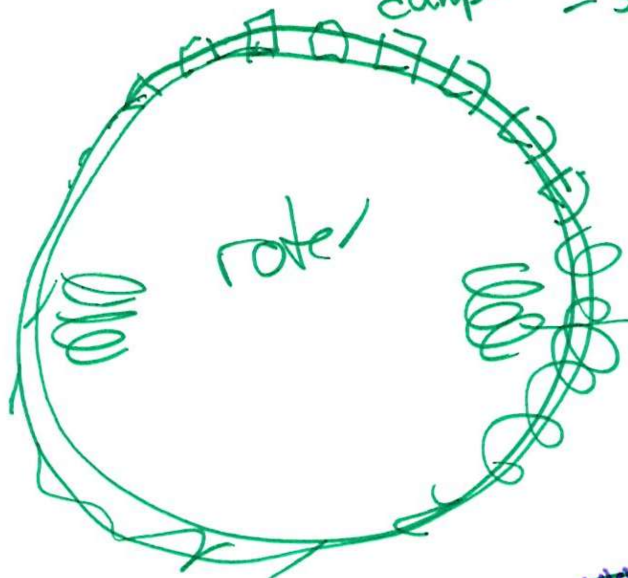
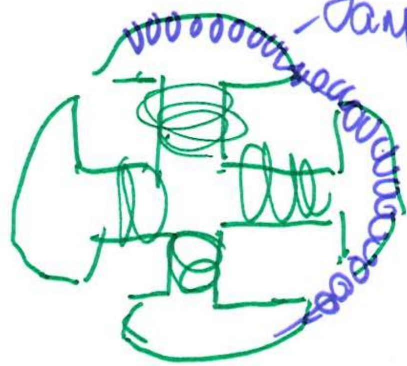
damper bars – same thing as rotor winding on induction machine



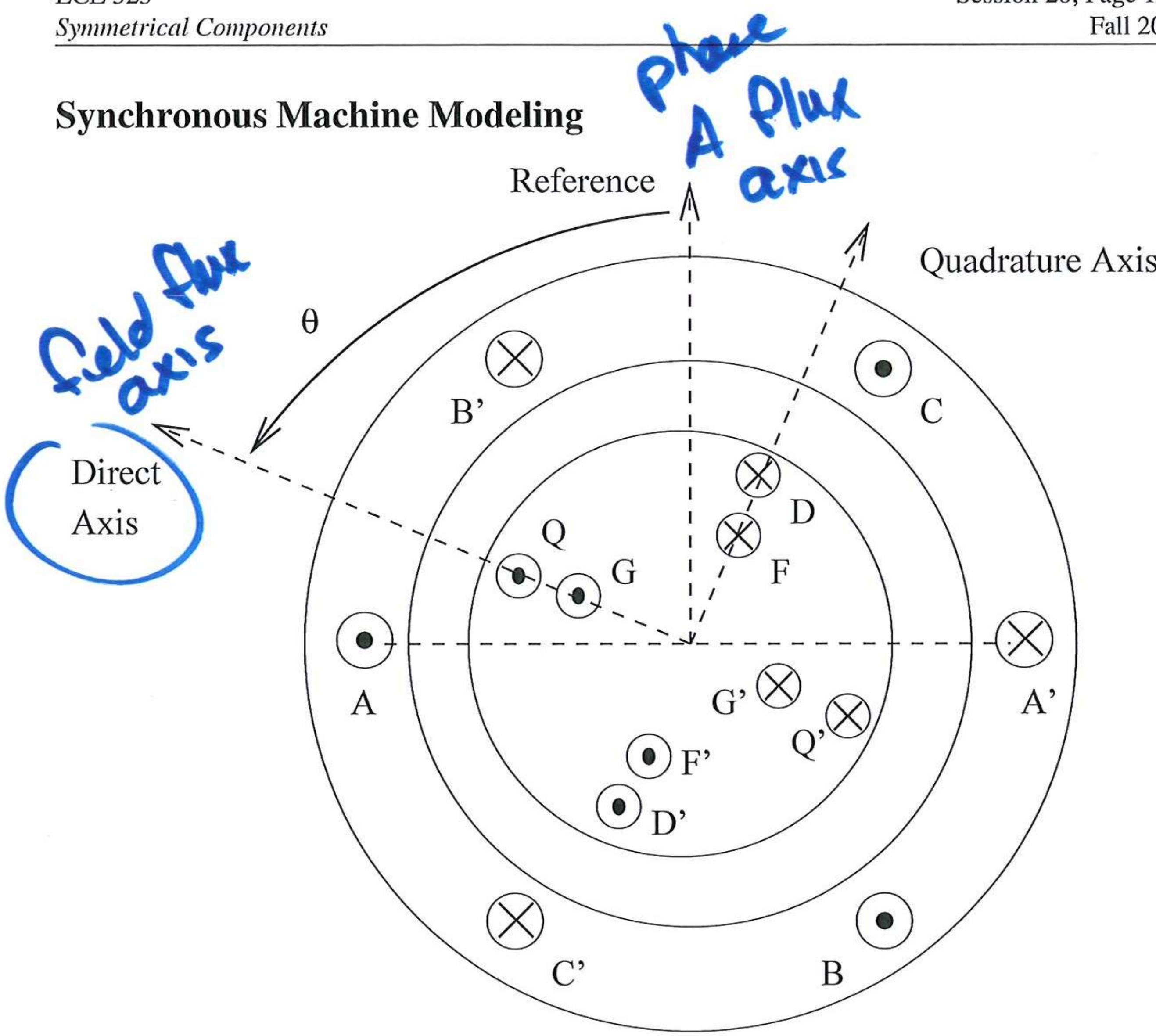
field winding

salient pole rotor



– damper bars
– in this case – cross interpole region

Synchronous Machine Modeling



Transient Model for a Synchronous Machine

Generator Convention

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Notation

f can represent i, v, λ.

$$f_\beta^\alpha$$

α	=	r	$\Rightarrow$	rotating reference frame fixed on the rotor
		s	$\Rightarrow$	stationary reference frame fixed on the stator

β	=	s	$\Rightarrow$	stator quantity
		r	$\Rightarrow$	rotor quantity
		a	$\Rightarrow$	A-Phase quantity
		b	$\Rightarrow$	B-Phase quantity
		c	$\Rightarrow$	C-phase quantity
		d	$\Rightarrow$	direct-axis quantity
		q	$\Rightarrow$	quadrature-axis quantity
		F	$\Rightarrow$	field quantity
		D	$\Rightarrow$	direct-axis damper winding quantity
		Q	$\Rightarrow$	quadrature-axis damped winding quantity
		g	$\Rightarrow$	eddy current winding quantity
		l	$\Rightarrow$	leakage quantity
		m	$\Rightarrow$	mutual quantity

$$\mathbf{f}_{abc} = \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}; \qquad \mathbf{f}^r_{dq0} = \begin{bmatrix} f^r_d \\ f^r_q \\ f^r_0 \end{bmatrix}$$

Inductances

Stator Flux Linkage Equations:

stator — rotor currents

$$\boldsymbol{\lambda}_{abc} = \mathbf{L}_s \mathbf{i}_{abc} + \mathbf{L}_{sr} \mathbf{i}_{FDgQ}$$

Rotor Flux Linkage Equations:

$$\boldsymbol{\lambda}_{FDgQ} = \mathbf{L}_{sr}^{\mathrm{T}}\mathbf{i}_{abc} + \mathbf{L}_r\mathbf{i}_{FDgQ}$$

Combined Stator and Rotor Flux Linkage Equations:

$$\begin{bmatrix} \boldsymbol{\lambda}_{abc} \\ \boldsymbol{\lambda}_{FDgQ} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_s & \mathbf{L}_{sr} \\ \mathbf{L}_{sr}^{\mathrm{T}} & \mathbf{L}_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abc} \\ \mathbf{i}_{FDgQ} \end{bmatrix}$$

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_D \\ \lambda_g \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aF} & L_{aD} & L_{ag} & L_{aQ} \\ L_{ba} & L_{bb} & L_{bc} & L_{bF} & L_{bD} & L_{bg} & L_{bQ} \\ L_{ca} & L_{cb} & L_{cc} & L_{cF} & L_{cD} & L_{cg} & L_{cQ} \\ L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} & L_{FD} & L_{Fg} & L_{FQ} \\ L_{Da} & L_{Db} & L_{Dc} & L_{DF} & L_{DD} & L_{Dg} & L_{DQ} \\ L_{ga} & L_{gb} & L_{gc} & L_{gF} & L_{gD} & L_{gg} & L_{gQ} \\ L_{Qa} & L_{Qb} & L_{Qc} & L_{QF} & L_{QD} & L_{Qg} & L_{QQ} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_g \\ i_Q \end{bmatrix}$$

field – damper – eddy current

where

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Stator Self-inductances

, depends on rotor angle

$$L_{aa} = L_s + L_m \cos 2\theta_d$$

$$L_{bb} = L_s + L_m \cos 2(\theta_d - 2\pi/3)$$

$$L_{cc} = L_s + L_m \cos 2(\theta_d + 2\pi/3)$$

Stator Mutual-inductances

$$L_{ab} = L_{ba} = -M_s - L_m \cos 2(\theta_d + \pi/6)$$

$$L_{bc} = L_{cb} = -M_s - L_m \cos 2(\theta_d - \pi/2)$$

$$L_{ca} = L_{ac} = -M_s - L_m \cos 2(\theta_d + 5\pi/6)$$

Rotor Self-inductances

$$L_F$$

$$L_D$$

$$L_g$$

$$L_Q$$

Rotor Mutual-inductances

$$L_{FD} = L_{DF} = M_{rd}$$

$$L_{gQ} = L_{Qg} = M_{rq}$$

$$L_{Fg} = L_{gF} = L_{Dg} = L_{Dg} = 0$$

$$L_{FQ} = L_{QF} = L_{DQ} = L_{DQ} = 0$$

Stator-rotor Mutual-inductances

$$L_{aF} = L_{Fa} = M_F \cos\theta_d$$

$$L_{bF} = L_{Fb} = M_F \cos(\theta_d - 2\pi/3)$$

$$L_{cF} = L_{Fc} = M_F \cos(\theta_d + 2\pi/3)$$

$$L_{aD} = L_{Da} = M_D \cos\theta_d$$

$$L_{bD} = L_{Db} = M_D \cos(\theta_d - 2\pi/3)$$

$$L_{cD} = L_{Dc} = M_D \cos(\theta_d + 2\pi/3)$$

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Park's Transformation

$$\mathbf{f}^r_{odq} = \mathbf{R}(\theta_r)\mathbf{P}(0)\mathbf{f}_{abc}$$

where $\theta_r = \omega_r + \frac{\pi}{2} + \delta$

Coordinate axis transformation

stationary ref. fram, $\omega = 0$

$$\mathbf{P}(0) := \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad abc \Rightarrow odq$$

Transformation to rotating reference frame

$$\mathbf{R}(\theta_r) := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_r & -\sin(\theta_r) \\ 0 & \sin\theta_r & \cos(\theta_r) \end{bmatrix}, \quad 0dq^s \Rightarrow 0dq^r, \quad \text{Rotation}$$

Combine into one step

$$\mathbf{P}(\theta_r) = \mathbf{R}(\theta_r)\mathbf{P}(0)$$

$$\mathbf{f}^r_{0dq} = \mathbf{P}(\theta_r)\mathbf{f}_{abc}$$

$$\begin{bmatrix} f^r_0 \\ f^r_d \\ f^r_q \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \sin\theta_r & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}$$

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zero sequence term

transform is time domain -instantaneous value

$$\mathbf{P}(\theta_r) := \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \sin\theta_r & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix}$$

$$\mathbf{P}^{-1}(\theta_r) = \mathbf{P}^{\mathrm{T}}(\theta_r)$$

$$\mathbf{P}^{-1}(\theta_r) = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \cos\theta_r & \sin\theta_r \\ \frac{1}{\sqrt{2}} & \cos(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r - \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \cos(\theta_r + \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix}$$

$$\mathbf{P}(\theta_r)\frac{d\,\mathbf{P}^{-1}(\theta_r)}{dt} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \omega \\ 0 & -\omega & 0 \end{bmatrix} := \overline{\omega\mathbf{x}}$$

cross couplin d⇔q

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Synchronous Machine Equations

1. Stator Voltage Equations: (Note: p = d/dt)

d λ / dt

$$\mathbf{v}_{abcs} = -r_s \mathbf{i}_{abcs} - p\boldsymbol{\lambda}_{abcs}$$

$$\mathbf{v}_{0dqs} = -r_s \mathbf{i}_{0dqs} - p\boldsymbol{\lambda}_{0dqs} - \overline{\boldsymbol{\omega}\mathbf{x}}\boldsymbol{\lambda}_{0dqs}$$

dλ/dt

- stator state this is our voltage...

2. Rotor Voltage Equation:

$$\mathbf{v}_{FDgQr} = -\mathbf{R}_r \mathbf{i}_{FDgQr} - p\boldsymbol{\lambda}_{FDgQr}$$

3. Stator Flux Linkage Equations:

$$\boldsymbol{\lambda}_{abcs} = \mathbf{L}_s \mathbf{i}_{abcs} + \mathbf{L}_{sr} \mathbf{i}_{FDgQr}$$

$$\boldsymbol{\lambda}_{odqs} = \mathbf{L}'_s \mathbf{i}_{odqs} + \mathbf{L}'_{sr} \mathbf{i}_{FDgQr}$$

4. Rotor Flux Linkage Equations:

$$\boldsymbol{\lambda}_{FDgQr} = \mathbf{L}_{sr}^{\mathrm{T}} \mathbf{i}_{abcs} + \mathbf{L}_r \mathbf{i}_{FDgQr}$$

$$\boldsymbol{\lambda}_{FDgQr} = \mathbf{L}_{sr}^{\prime\mathrm{T}} \mathbf{i}_{abcs} + \mathbf{L}_r \mathbf{i}_{FDgQr}$$

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constant

$$\begin{bmatrix} \lambda_{0s} \\ \lambda_{ds} \\ \lambda_{qs} \\ \lambda_{Fr} \\ \lambda_{Dr} \\ \lambda_{gr} \\ \lambda_{Qr} \end{bmatrix} = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & kM_D & 0 & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_g & kM_Q \\ 0 & kM_F & 0 & L_F & M_D & 0 & 0 \\ 0 & kM_D & 0 & M_D & L_D & 0 & 0 \\ 0 & 0 & kM_g & 0 & 0 & L_g & M_Q \\ 0 & 0 & kM_Q & 0 & 0 & M_Q & L_Q \end{bmatrix} \begin{bmatrix} i_{0s} \\ i_{ds} \\ i_{qs} \\ i_{Fr} \\ i_{Dr} \\ i_{gr} \\ i_{Qr} \end{bmatrix}$$

dc in steady state

$$k = \sqrt{\frac{3}{2}}$$

Torque

$$T_E = i_d \lambda_q - i_q \lambda_d$$

$$p_{3\phi}(t) = i_0 * v_0 + i_d * v_d + i_q * v_q$$

$$E_a = \frac{\omega_0 M_F i_F e^{j\delta}}{\sqrt{2}} = |E_a| \angle \delta$$

$$|E_a| = \frac{\omega_0 M_F i_F}{\sqrt{2}}$$

Synchronous Machine Parameters

X_d	direct axis reactance
X_q	quadrature axis reactance
X'_d	direct axis transient reactance
X'_q	quadrature axis transient reactance
X''_d	direct axis subtransient reactance
X''_q	quadrature axis subtransient reactance
X_2	negative sequence reactance
X_0	zero sequence reactance

r_{sdc}	stator dc resistance
r_{sac}	stator ac resistance
$r_{fr'}$	field resistance referred to the stator
r_2	negative sequence resistance

T'_{d0}	direct axis open-circuit transient time-constant
T'_d	direct axis short-circuit transient time-constant
T''_d	direct axis short-circuit subtransient time-constant
T_a	armature short-circuit (d.c.) time-constant