ECE 523
Symmetrical Components

Session 29

L28 — 16/20

Park's Transformation

$$\mathbf{f}^{r}_{odq} = \mathbf{R}(\theta_r)\mathbf{P}(0)\mathbf{f}_{abc}$$

where $\theta_r = \omega_r + \frac{\pi}{2} + \delta$

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Coordinate axis transformation

stationary ref fram, $\omega = 0$

$$\mathbf{P}(0) := \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \qquad abc \Rightarrow odq$$

Transformation to rotating reference frame

$$\mathbf{R}(\theta_r) := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_r & -\sin(\theta_r) \\ 0 & \sin\theta_r & \cos(\theta_r) \end{bmatrix}, \qquad 0dq^s \Rightarrow 0dq^r, \quad \text{Rotation}$$

Combine into one step

$$\mathbf{P}(\theta_r) = \mathbf{R}(\theta_r)\mathbf{P}(0)$$

$$\mathbf{f}^{r}_{0dq} = \mathbf{P}(\theta_r)\mathbf{f}_{abc}$$

$$\begin{bmatrix} f^r_0 \\ f^r_d \\ f^r_q \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \sin\theta_r & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}$$

$\theta_r = \omega_r t$

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Synchronous Machine Equations

1. Stator Voltage Equations: (Note: p = d/dt)

$$\mathbf{v}_{abcs} = -r_s \mathbf{i}_{abcs} - p\boldsymbol{\lambda}_{abcs}$$

$$\mathbf{v}_{0dqs} = -r_s \mathbf{i}_{0dqs} - p\boldsymbol{\lambda}_{0dqs} - \overline{\omega \mathbf{x}}\boldsymbol{\lambda}_{0dqs}$$

d λ / dt

- steady state this is our voltage...

L matrix is constant

dλ/dt

2. Rotor Voltage Equation:

$$\mathbf{v}_{FDgQr} = -\mathbf{R}_r \mathbf{i}_{FDgQr} - p\boldsymbol{\lambda}_{FDgQr}$$

3. Stator Flux Linkage Equations:

$$\boldsymbol{\lambda}_{abcs} = \mathbf{L}_s \mathbf{i}_{abcs} + \mathbf{L}_{sr}\mathbf{i}_{FDgQr}$$

$$\boldsymbol{\lambda}_{odqs} = \mathbf{L}'_s \mathbf{i}_{odqs} + \mathbf{L}'_{sr}\mathbf{i}_{FDgQr}$$

4. Rotor Flux Linkage Equations:

$$\boldsymbol{\lambda}_{FDgQr} = \mathbf{L}_{sr}^{\mathbf{T}} \mathbf{i}_{abcs} + \mathbf{L}_r \mathbf{i}_{FDgQr}$$

$$\boldsymbol{\lambda}_{FDgQr} = \mathbf{L}_{sr}^{\prime\mathbf{T}} \mathbf{i}_{abcs} + \mathbf{L}_r \mathbf{i}_{FDgQr}$$

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constant

$$\begin{bmatrix} \lambda_{0s} \\ \lambda_{ds} \\ \lambda_{qs} \\ \lambda_{Fr} \\ \lambda_{Dr} \\ \lambda_{gr} \\ \lambda_{Qr} \end{bmatrix} = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & kM_D & 0 & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_g & kM_Q \\ 0 & kM_F & 0 & L_F & M_D & 0 & 0 \\ 0 & kM_D & 0 & M_D & L_D & 0 & 0 \\ 0 & 0 & kM_g & 0 & 0 & L_g & M_Q \\ 0 & 0 & kM_Q & 0 & 0 & M_Q & L_Q \end{bmatrix} \begin{bmatrix} i_{0s} \\ i_{ds} \\ i_{qs} \\ i_{Fr} \\ i_{Dr} \\ i_{gr} \\ i_{Qr} \end{bmatrix}$$

dc in steady state

$$k = \sqrt{\frac{3}{2}}$$

Torque

$$T_E = i_d \lambda_q - i_q \lambda_d$$

$$p_{3\phi}(t) = i_0 * v_0 + i_d * v_d + i_q * v_q$$

$$E_a = \frac{\omega_0 M_F i_F e^{j\delta}}{\sqrt{2}} = |E_a| \angle \delta$$

$$|E_a| = \frac{\omega_0 M_F i_F}{\sqrt{2}}$$

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short circuit

Synchronous Machine Parameters

3s steady state

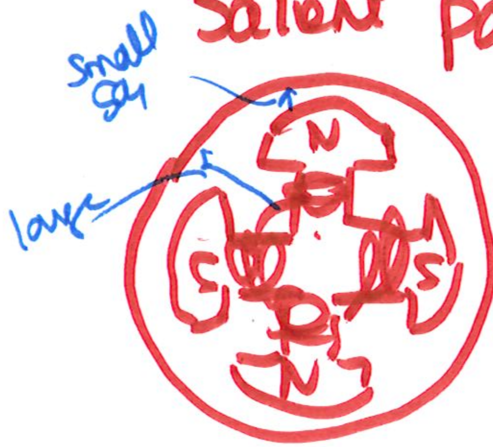
damper 60s

X_d	direct axis reactance
X_q	quadrature axis reactance
X'_d	direct axis transient reactance
X'_q	quadrature axis transient reactance
X''_d	direct axis subtransient reactance
X''_q	quadrature axis subtransient reactance
X_2	negative sequence reactance
X_0	zero sequence reactance
r_{sdc}	stator dc resistance
r_{sac}	stator ac resistance
$r_{f_{r'}}$	field resistance referred to the stator
r_2	negative sequence resistance
T'_{d0}	direct axis open-circuit transient time-constant
T'_d	direct axis short-circuit transient time-constant
T''_d	direct axis short-circuit subtransient time-constant
T_a	armature short-circuit (d.c.) time-constant

Round Rotor

$X_d \approx X_q$

Salient pole



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Typical values

Round Roto

salient pole

Machine Constant	Turbo Generator	Hydro Generator	Synchronous Condensor	Synchronous Motor
X_d	1.1	1.15	1.80	1.20
X_q	1.08	0.75	1.15	0.90
X'_d	0.23	0.37	0.40	0.35
X'_q	0.23	0.75	1.15	0.90
X''_d	0.12	0.24	0.25	0.30
X''_q	0.15	0.34	0.30	0.40
X_2	0.13	0.29	0.27	0.35
X_0	0.05	0.11	0.09	0.16
r_{sdc}	0.003	0.012	0.008	0.01
r_{sac}	0.005	0.012	0.008	0.01
r_2	0.035	0.100	0.05	0.06
T'_{d0}	5.6	5.6	9.0	6.0
T'_d	1.1	1.8	2.0	1.4
T''_d	0.035	0.035	0.035	0.035
T_a	0.16	0.15	0.17	0.15

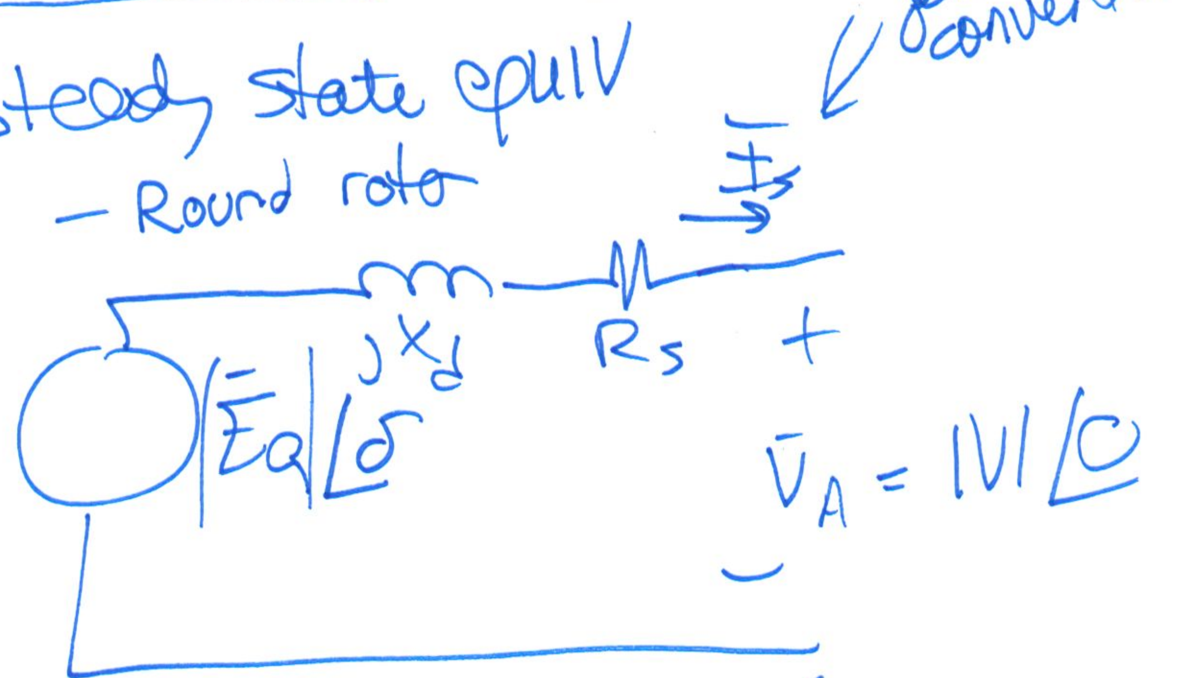
stater effective

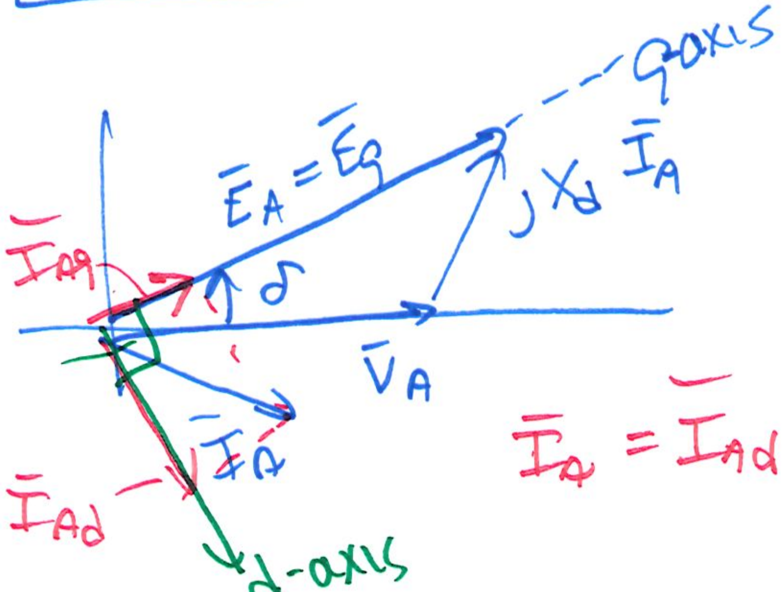
dominated by field windin constant

Response of machine

Steady state equiv

- Round rotor





$\bar{I}_A = \bar{I}_{Ad} + j\bar{I}_{Aq}$

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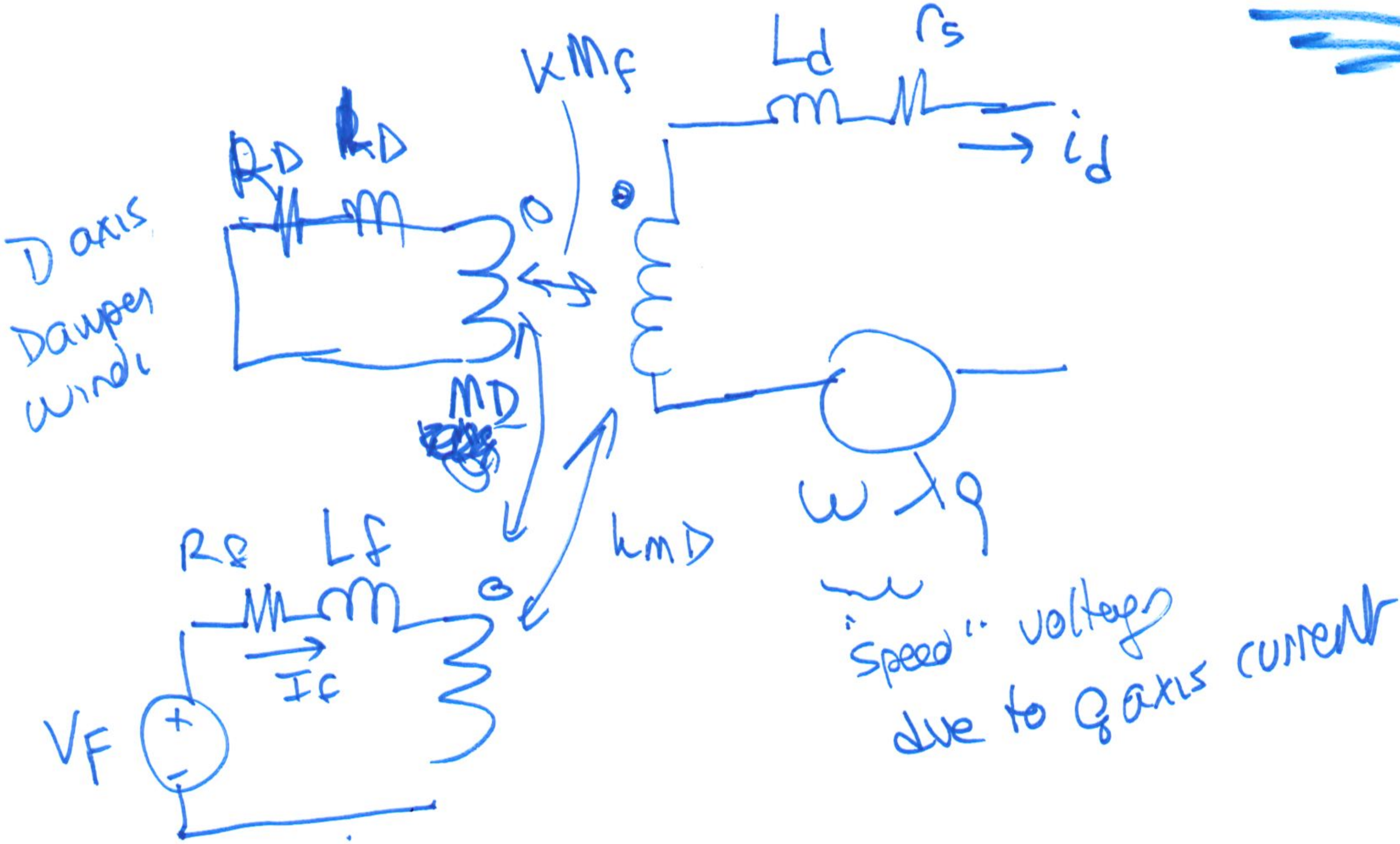
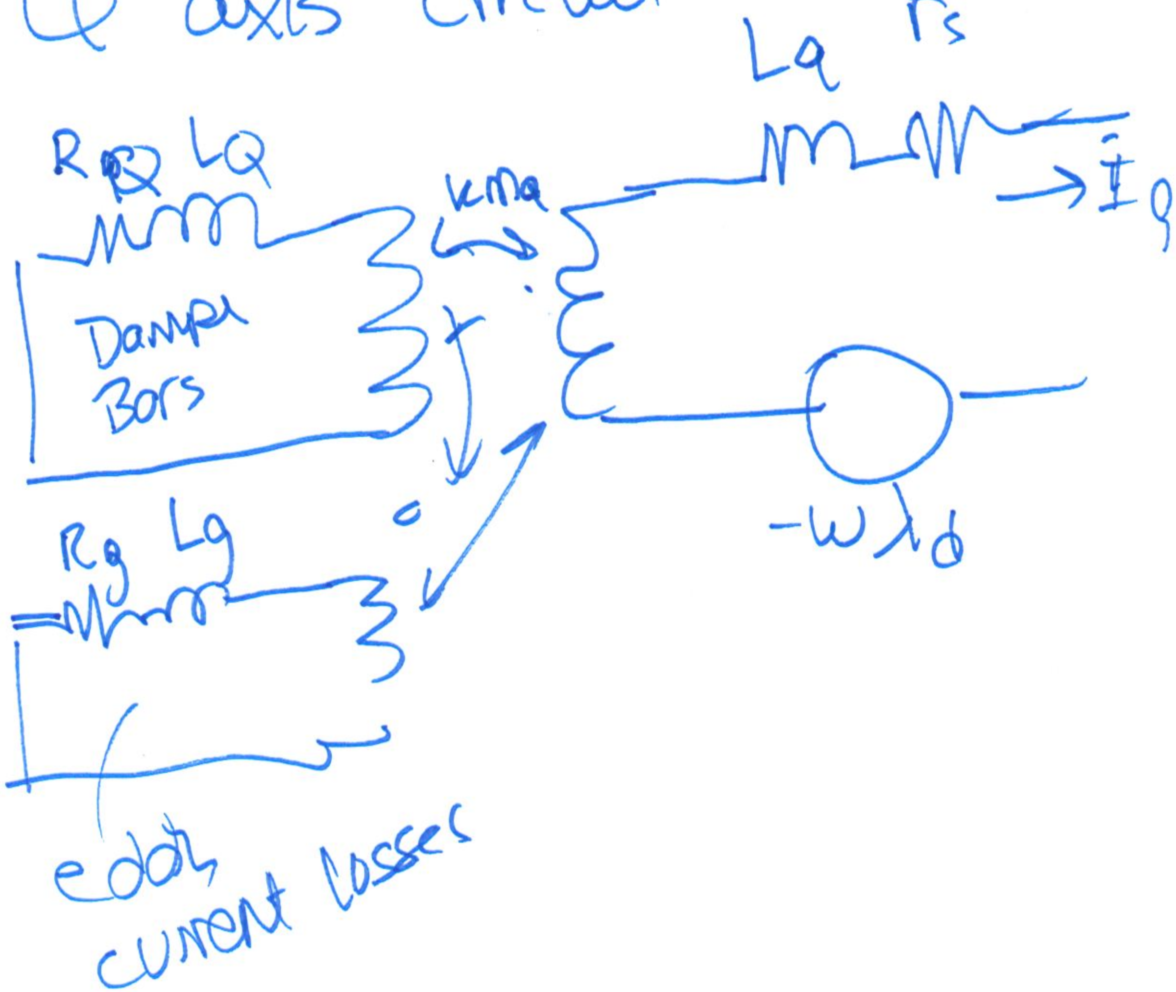
Transient/Dynamic Equivalent

D-axis



Q axis circuit



Short circuit response

(50 Hz)
→ 60Hz response
  → Positive } → time varying amplitude
    Negative }
    Zero - depend on grounding

→ decaying dc offset
- double frequency

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Positive sequence

Three Phase Short Circuit of a Synchronous Machine

Internal volt

$$i_{as}(t) \approx \sqrt{2}|\tilde{E}_a| \left[\frac{1}{X_d} + \left(\frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-\frac{t}{T_d'}} + \left(\frac{1}{X_d''} - \frac{1}{X_d'} \right) e^{-\frac{t}{T_d''}} \right] \sin(\omega_e t + \alpha)$$

α: angle on volt waveform for fault initiation

$$-\sqrt{2}|\tilde{E}_a| \frac{1}{2} \left(\frac{1}{X_d''} + \frac{1}{X_q''} \right) e^{-\frac{t}{T_a}} \sin(\alpha)$$

$$-\sqrt{2}|\tilde{E}_a| \frac{1}{2} \left(\frac{1}{X_d''} - \frac{1}{X_q''} \right) e^{-\frac{t}{T_a}} \sin(2\omega_e t + \alpha)$$

$R << X$
- fault at terminals
- $R_f = 0$

Name	Magnitude	Frequency	T
Steady	$E_a \frac{1}{X_d}$	Fundamental	∞
Transient	$E_a \left(\frac{1}{X_d'} - \frac{1}{X_d} \right)$	Fundamental	T_d'
Subtransient	$E_a \left(\frac{1}{X_d''} - \frac{1}{X_d'} \right)$	Fundamental	T_d''
Asymmetrical	$\frac{E_a}{2} \left(\frac{1}{X_d''} + \frac{1}{X_q''} \right) \sin(\alpha)$	Zero	T_a
Second Harmonic	$\frac{E_a}{2} \left(\frac{1}{X_d''} - \frac{1}{X_q''} \right)$	Double Fundamental	T_a

symmetrical

dc offset

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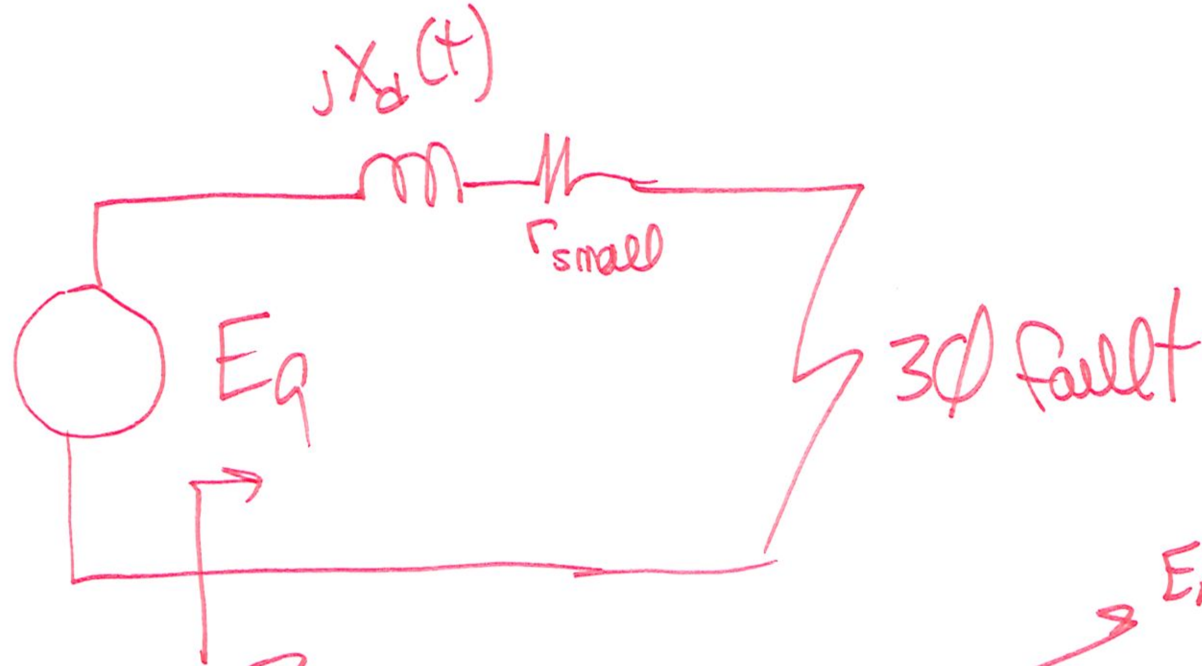
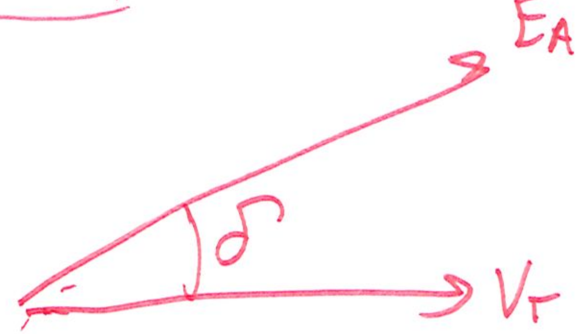
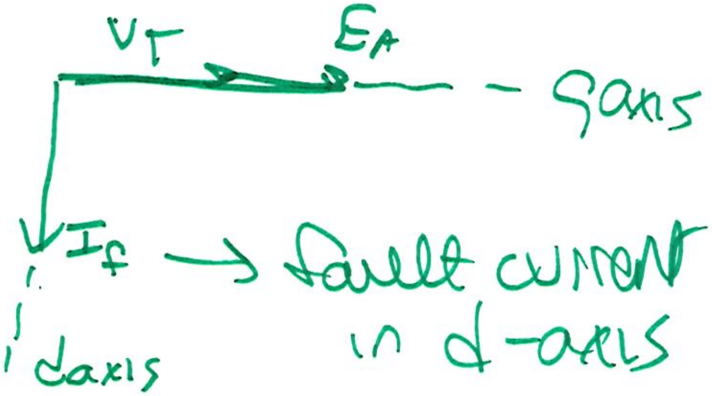


$P = ?$: $3|I|^2 r_{small}$



$$P = \frac{|E_A||V_T|}{X_S}\sin(\delta)$$

$\delta = 0$ for $P = 0$



$I_f \rightarrow$ fault current in d-axis

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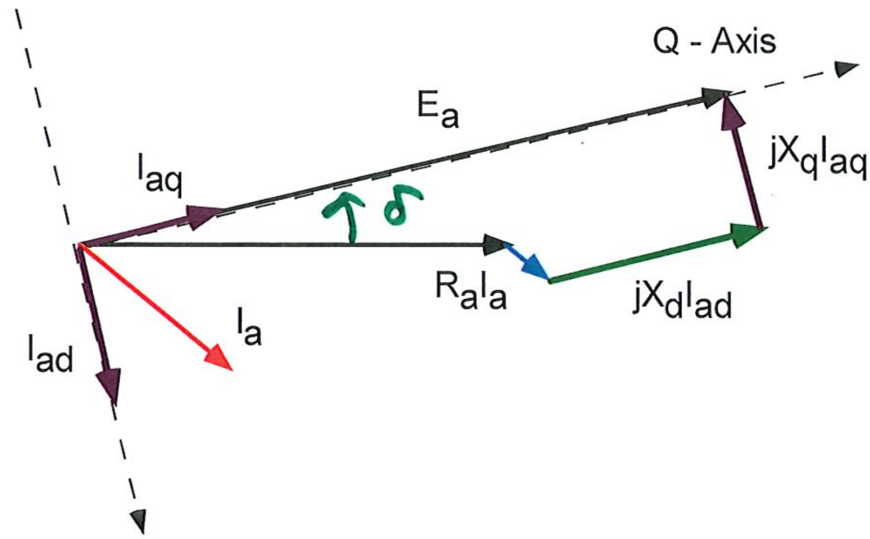
Phasor Diagram of a Salient Pole Synchronous Generator



- Define q-axis

  $a1 = V_a + R_a \cdot I_a + j \cdot X_q \cdot I_a \qquad \theta_q = \arg(a1)$

- Angle of the back EMF, E_a

  $E_a = V_{at} + R_a \cdot I_a + j \cdot X_d \cdot I_{ad} + j \cdot X_q \cdot I_{aq} \qquad \theta_q = \delta_a = \arg(E_a)$

- Show that the angle of E_a is the same as the angle of a1

  $E_a - a1 = \left(V_{at} + R_a \cdot I_a + j \cdot X_d \cdot I_{ad} + j \cdot X_q \cdot I_{aq}\right) - \left(V_a + R_a \cdot I_a + j \cdot X_q \cdot I_a\right)$

- Which simplified to

  $E_a - a1 = \left(j \cdot X_d \cdot I_{ad} + j \cdot X_q \cdot I_{aq}\right) - \left(j \cdot X_q \cdot I_a\right)$

- By defininition we know that:

  $I_a = I_{ad} + I_{aq}$

- Substituting

  $E_a - a1 = \left(j \cdot X_d \cdot I_{ad} + j \cdot X_q \cdot I_{aq}\right) - \left(j \cdot X_q \cdot I_{ad} + j \cdot X_q \cdot I_{aq}\right)$

- Which simplified to:

  $E_a - a1 = I_{ad} \cdot \left(j \cdot X_d - j \cdot X_q\right)$

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$$S_{load} := 1pu \cdot e^{j \cdot \phi_{load}}$$

$$I_{load} := \overline{\left(\frac{S_{load}}{V_{term}}\right)} \qquad I_{load} = (0.8 - 0.6i) \cdot pu \qquad |I_{load}| = 1 \cdot pu$$

$$E''_{a2} := V_{term} + I_{load} \cdot j \cdot X''_d \qquad |E''_{a2}| = 1.093 \cdot pu \qquad \arg(E''_{a2}) = 6.091 \cdot deg$$

$$Iapp\_2 := \frac{E''_{a2}}{j \cdot X''_d + j \cdot X_{tran}} \qquad |Iapp\_2| = 4.462 \cdot pu \qquad \arg(Iapp\_2) = -83.909 \cdot deg$$

$$E'_{a2} := V_{term} + I_{load} \cdot j \cdot X'_d \qquad |E'_{a2}| = 1.16 \cdot pu \qquad \arg(E'_{a2}) = 9.527 \cdot deg$$

$$I'_{a2} := \frac{E'_{a2}}{j \cdot X'_d + j \cdot X_{tran}} \qquad |I'_{a2}| = 3.412 \cdot pu \qquad \arg(I'_{a2}) = -80.473 \cdot deg$$

$$E_{a\_2} := V_{term} + I_{load} \cdot j \cdot X_d \qquad |E_{a\_2}| = 1.879 \cdot pu \qquad \arg(E_{a\_2}) = 27.929 \cdot deg$$

$$I_{a\_2} := \frac{E_{a\_2}}{j \cdot X_d + j \cdot X_{tran}} \qquad |I_{a\_2}| = 1.566 \cdot pu \qquad \arg(I_{a\_2}) = -62.071 \cdot deg$$

$$I_{dcoffsetmax\_2} := \sqrt{2} \cdot \frac{|E''_{a2}|}{|j \cdot X''_d + j \cdot X_{tran}|} \qquad I_{dcoffsetmax\_2} = 6.31 \cdot pu$$

Example 3: A three phase fault occurs at the generator terminals, determine the symmetrical fault current at the instant of the fault, after 1/2 cycle, after 3 cycles, after 30 cycles and after 300 cycles. Assume generator is loaded as in the previous example

$$T''_d := 0.035sec$$

$$T'_d := 0.730sec$$

$$t := 0sec$$

$$I_a(t) := E''_{a2} \cdot \left[\frac{1}{X_d} + \left(\frac{1}{X'_d} - \frac{1}{X_d}\right) \cdot e^{\frac{-t}{T'_d}} + \left(\frac{1}{X''_d} - \frac{1}{X'_d}\right) \cdot e^{\frac{-t}{T''_d}}\right]$$

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Instantaneous fault current: $\left|I_a(t)\right| = 7.539 \cdot pu$

After 1/2 cycle $t1 := \frac{0.5}{60Hz}$ $t1 = 8.333 \times 10^{-3}\,s$ $\left|I_a(t1)\right| = 6.866 \cdot pu$

After 3 cycles $t2 := \frac{3}{60Hz}$ $\left|I_a(t2)\right| = 5.034 \cdot pu$

After 30 cycles $t3 := \frac{30}{60Hz}$ $t3 = 0.5\,s$ $\left|I_a(t3)\right| = 2.789 \cdot pu$

- Should use E'_a for 30 cycles, not E''_a

After 300 cycles $t4 := \frac{300}{60Hz}$ $\left|I_a(t4)\right| = 0.998 \cdot pu$

$I_{ss2} := \frac{E_{a\_2}}{X_d}$ $\left|I_{ss2}\right| = 1.708 \cdot pu$

- Note the difference, should use E_a not E_a'' now.

- Calculate momentary and interrupting ratings for generator breakers

$I_{dppMax} := \sqrt{3} \cdot \frac{E''_a}{X''_d}$ $I_{dppMax} = 11.945 \cdot pu$

- Accounts dc offset as well as fundamental component

$I_{dppsymm} := \frac{E''_a}{X''_d}$

$Momentary := 1.6 \cdot I_{dppsymm}$ $Momentary = 11.034 \cdot pu$ at the mimimum

M is based on the number of cycles react in: r Andersen

$M := 1.1$ Assumes 5 cycle breakers (see Table 6.2)

$Interrupting := M \cdot I_{dppsymm}$ $Interrupting = 7.586 \cdot pu$

Determine these currents in Amps if:

1) The generator is rated at 13.2kV and 150 MVA

$VB1 := 13.2kV$ $MVA := 1000kW$ $SB1 := 150MVA$

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$$Ibase := \frac{SB1}{\sqrt{3} \cdot VB1} \qquad Ibase = 6.561 \cdot kA$$

$$I_{moment1} := Momentary \cdot Ibase \qquad |I_{moment1}| = 72.395 \cdot kA$$

$$I_{interrupt1} := Interrupting \cdot Ibase \qquad |I_{interrupt1}| = 49.772 \cdot kA$$

2) The generator is rated at 13.2kV and 600 MVA (M=M+0.1)

$$Sb2 := 600MVA \qquad Ibase2 := \frac{Sb2}{\sqrt{3} \cdot VB1}$$

$$I_{moment2} := Momentary \cdot Ibase2 \qquad |I_{moment2}| = 289.58 \cdot kA$$

$$I_{nterrupting2} := (M + 0.1) \cdot I_{dppsymm}$$

$$I_{interrupt2} := I_{nterrupting2} \cdot Ibase2 \qquad |I_{interrupt2}| = 217.185 \cdot kA$$

Example 5 for a salient pole motor. Terminal voltage is 1.0pu, drawing rated curent, 0.9 pf leading.

$$I_{tmag} := 1.0 \qquad \phi := -acos(0.9) \qquad \phi = -25.842 \cdot deg$$

$$I_{tpu} := I_{tmag} \cdot e^{-j \cdot \phi} \qquad V_{tpu} := 1.0$$

Machine constants (average values from table 6.1):

$$X_d := 1.20pu \qquad X''_d := 0.30pu \qquad T'_{d0} := 6.0sec$$

$$X_q := 0.90pu \qquad X''_q := 0.40pu \qquad T'_d := 1.4sec$$

$$X'_d := 0.35pu$$

$$X'_q := 0.90pu \qquad X_2 := 0.35pu \qquad T''_d := 0.035sec$$

$$T_a := 0.15sec$$

$R_a := 0.01pu$ AC resistance

(a) The voltage E behind the synchronous impedance

We need to include the d and q components now in the expression to account for the small saliency and include the resistance:

- Define angle of the q-axis

$$a1 := V_{tpu} - \left(R_a \cdot I_{tpu} + j \cdot X_q \cdot I_{tpu}\right) \qquad \theta_q := arg(a1) \qquad \theta_q = -30.486 \cdot deg$$

motor current directi

I_A to $I_d + I_q$

$$\text{magIad} := \left|I_{tpu}\right| \cdot \sin(\phi + \theta_q) \qquad \text{magIad} = -0.832 \cdot pu$$

$$\text{magIaq} := \left|I_{tpu}\right| \cdot \cos(\phi + \theta_q) \qquad \text{magIaq} = 0.554 \cdot pu$$

$$I_{aq} := \text{magIaq} \cdot e^{j\theta_q}$$

$$I_{ad} := \text{magIad} \cdot e^{j \cdot (\theta_q - 90deg)}$$

$$E_a := V_{tpu} - \left(R_a \cdot I_{tpu} + j \cdot X_d \cdot I_{ad} + j \cdot X_q I_{aq}\right)$$

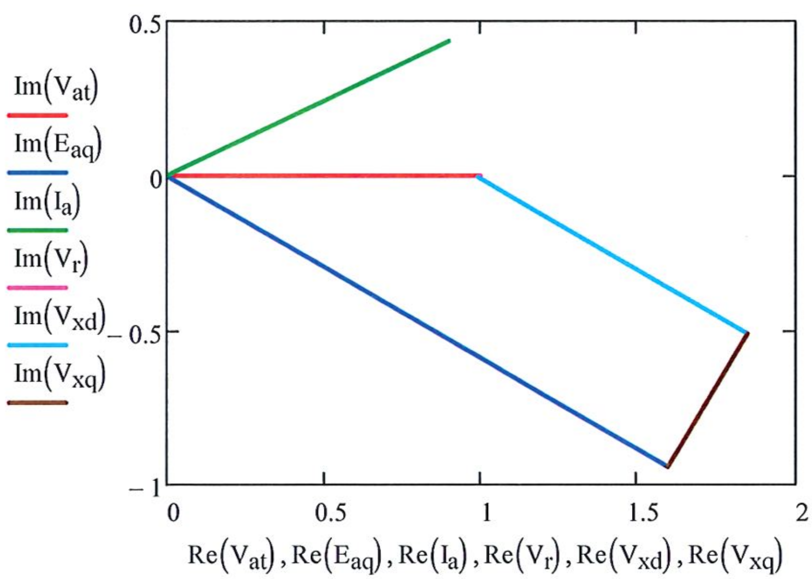
$$\left|E_a\right| = 1.855 \cdot pu \qquad \arg(E_a) = -30.486 \cdot deg$$

Steady-state voltage. Not good for fault current calculations.

- Phasor diagram

$$V_{at} := \begin{pmatrix} 0 \\ V_{tpu} \end{pmatrix} \qquad E_{aq} := \begin{pmatrix} 0 \\ E_a \end{pmatrix} \qquad I_a := \begin{pmatrix} 0 \\ I_{tpu} \end{pmatrix} \qquad V_r := \begin{pmatrix} V_{tpu} \\ V_{tpu} - R_a \cdot I_{tpu} \end{pmatrix}$$

$$V_{xd} := \begin{pmatrix} V_{tpu} - R_a \cdot I_{tpu} \\ V_{tpu} - R_a \cdot I_{tpu} - jX_d \cdot I_{ad} \end{pmatrix} \qquad V_{xq} := \begin{pmatrix} V_{tpu} - R_a \cdot I_{tpu} - jX_d \cdot I_{ad} \\ V_{tpu} - R_a \cdot I_{tpu} - jX_d \cdot I_{ad} - j \cdot X_q \cdot I_{aq} \end{pmatrix}$$

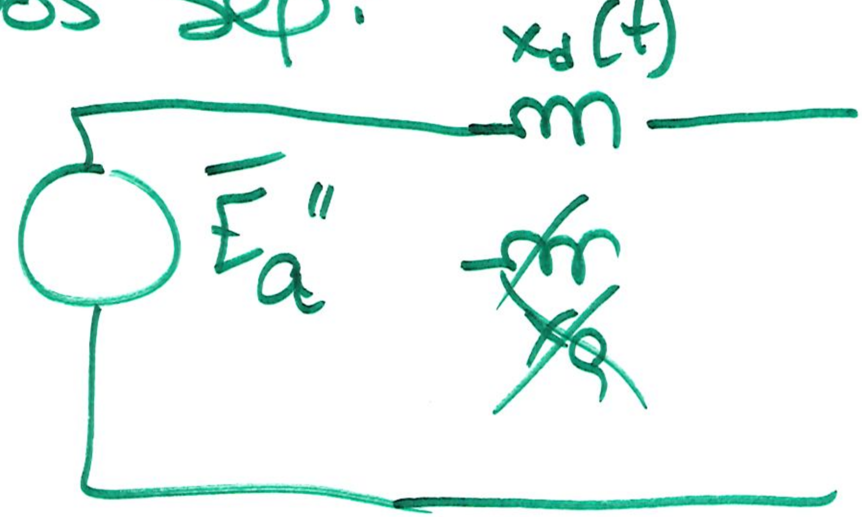


One could also use alternation equation to calculate same thing in equation

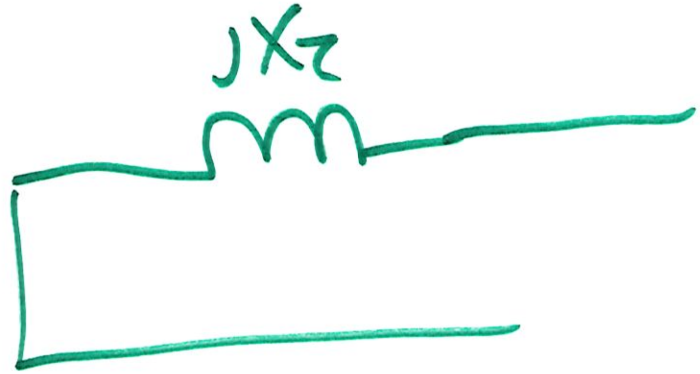
$$E_q := V_{tpu} - \left[R_a \cdot I_{tpu} + j \cdot X_q \cdot I_{tpu} + j \cdot (X_d - X_q) \cdot I_{ad}\right]$$

sequence models

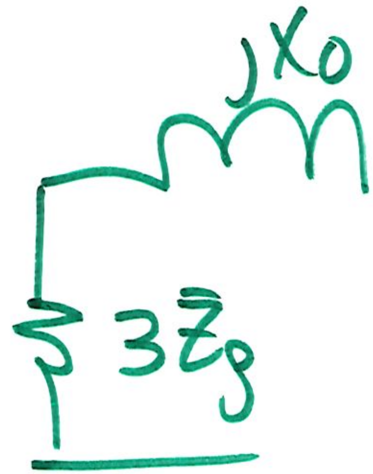
pos seq:



neg



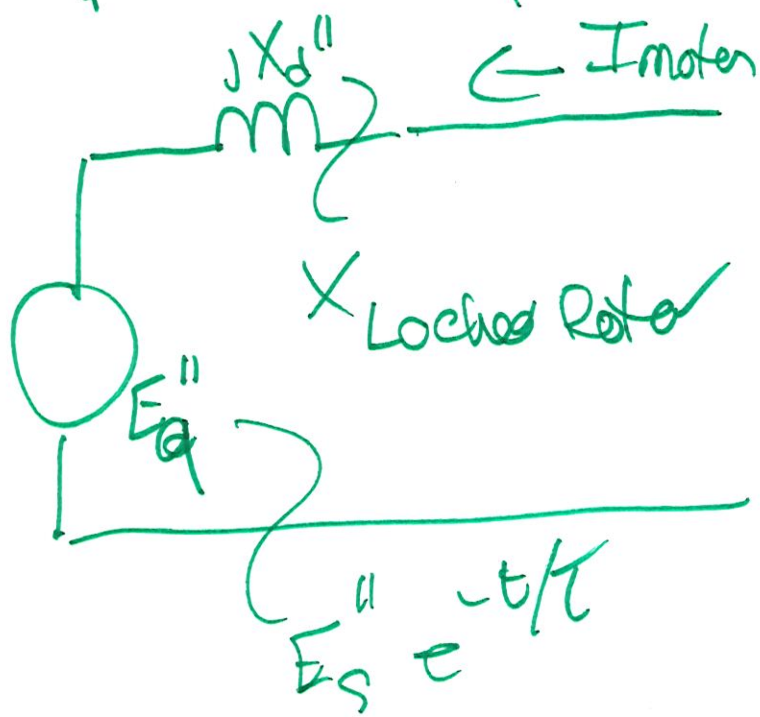
zero



Induction Machines

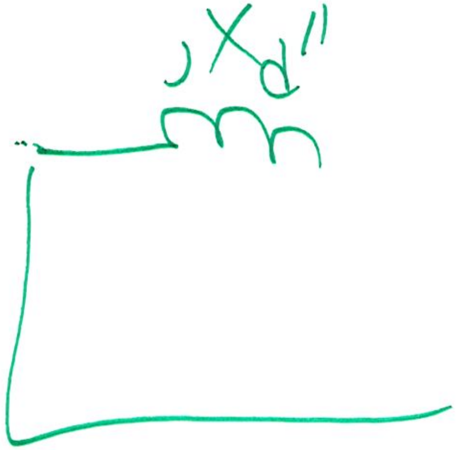
fault studies

positive seq

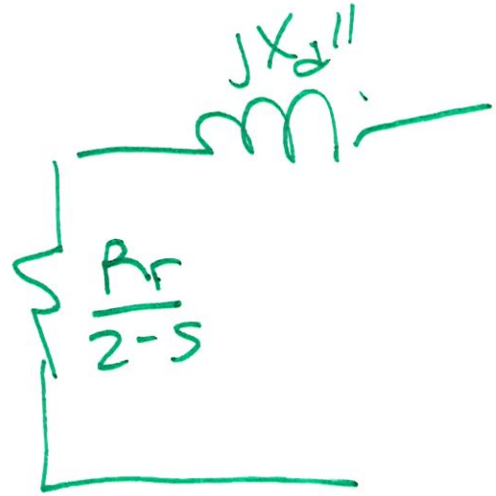


Negative sequence

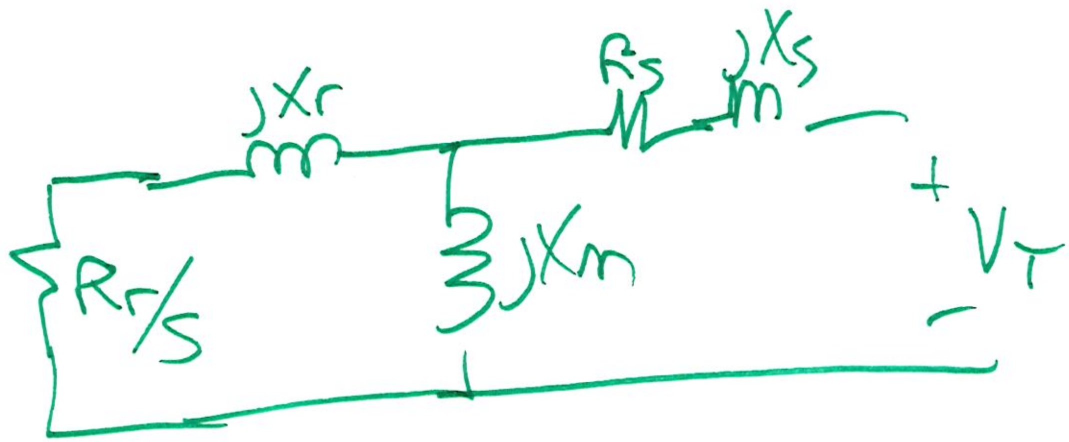
approx.



more accurate
- calculate prefault slip



SS pos sequence model



Therefore we connect the positive and negative sequence circuits in series and impose the voltage V1 - V2 across them (derived from the value of Vbc)

Original voltage:

$V_{ap} := 1pu\,e^{j\cdot 0deg}$ $\quad V_{bp} := 1pu\,e^{-j\cdot 120deg}$ $\quad V_{cp} := 1pu\,e^{j\cdot 120deg}$

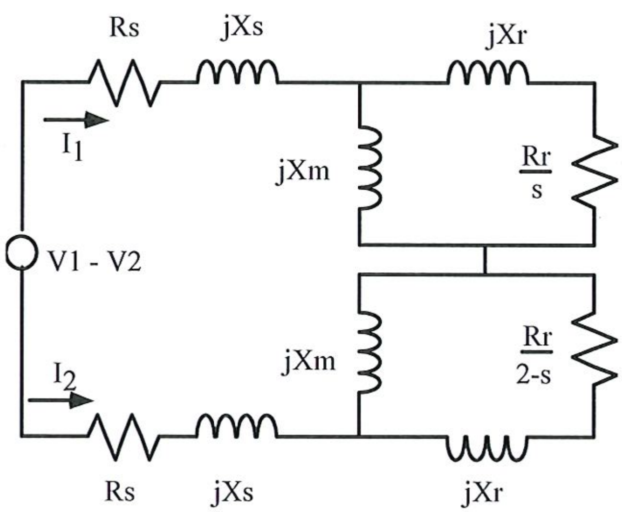
$V_{bc} := V_{bp} - V_{cp}$ $\quad \left|V_{bc}\right| = 1.732\cdot pu$ $\quad \arg\left(V_{bc}\right) = -90\cdot deg$

- ***This is 1.0 per unit in the line to line voltage......***

Then V1 - V2 is:

$V_{1\_2} := j\cdot\frac{V_{bc}}{\sqrt{3}}$ $\quad V_{1\_2} = 1\cdot pu$ $\quad$ the problem was set up to use nice numbers

Now we can solve the circuit below, where the positive and negative sequence circuits are connected in



SS positive

- First come up with the equivalent circuit for the parallel combination in the two rotor circuits...

$$z_{pos\_rot} := \left(\frac{1}{j\cdot X_m} + \frac{1}{j\cdot X_r + \frac{R_r}{slip\_rated}}\right)^{-1} \qquad z_{pos\_rot} = (0.858 + 0.357i)\cdot pu$$

$$z_{neg\_rot} := \left(\frac{1}{jX_m} + \frac{1}{j\cdot X_r + \frac{R_r}{2 - slip\_rated}}\right)^{-1} \qquad z_{neg\_rot} = \left(9.583\times 10^{-3} + 0.078i\right)\cdot pu$$