ECE 523
Symmetrical Components

Session 3

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Lecture 2

Define units: MVA := 1000kW   MW := MVA   pu := 1

kVA := kW   kVAR := kW

$V_{AmL} = 138kV$ $V_{ALN} := \frac{V_{Am}}{\sqrt{3}}$

$\bar{V}_A = |V_A| \angle \theta$ $\theta = 30^\circ$

$V_{A\theta} := V_{ALN} \cdot e^{j\theta}$

1j or 1i

$j := \sqrt{-1}$

1. Entering Phasors in Polar Notation in Mathcad 15

- Step 1: Go to: "Help Menu" -- "Quick Sheets"
- Step 2: Choose "Extra Math Symbols" from the list
- Step 3: Scroll down the page and copy the angle symbol to your Mathcad sheet and use it to define a function as shown below defining the complex phasor in terms of magnitude and angle, where the arguments of the function are the magnitude and angle for a complex number

$$\angle(\text{magnitude}, \text{angle}) := \text{magnitude}\cdot\cos(\text{angle}) + j\cdot\text{magnitude}\cdot\sin(\text{angle})$$

- Step 4: When you want to use this new function to enter a phasor, start typing your expression, before entering the phasor itself, go to the Evaluation Toolbar and select to enter your information as an infix number (select "xfy" from the evaluation toolbar. This will allow you to enter your function in the order you normally use for entering phasors, instead of the normal Mathcad order. You should see three placeholders in your Mathcad sheet
  - Enter the magnitude of the phasor in the first placeholder
  - Copy the angle symbol into the middle one
  - Enter the angle of the phasor in the third placeholder

$V_a := 1 \angle (45\text{deg})$ $V_a = 0.707 + 0.707i$

- A lot of people find it easiest to enter this once and then just copy it into other sheets.

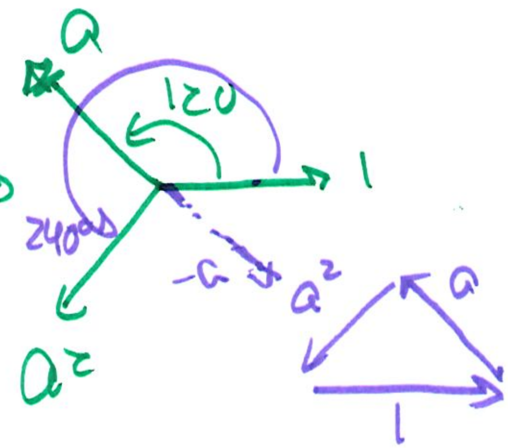
2. The "a" operator for three phase systems

$a := 1 \angle (120deg)$ $\quad a = -0.5 + 0.866i$

$a^2 = -0.5 - 0.866i$ $\quad \bar{a} = -0.5 - 0.866i = a^*$

$a^3 = 1$

$1 + a + a^2 = 0$



$1 - a = 1.5 - 0.866i$ $\quad \sqrt{3} \angle (-30deg) = 1.5 - 0.866i$

$1 - a^2 = 1.5 + 0.866i$ $\quad \sqrt{3} \angle (30deg) = 1.5 + 0.866i$

$a - 1 = -1.5 + 0.866i$ $\quad \sqrt{3} \angle (150deg) = -1.5 + 0.866i$

$a^2 - 1 = -1.5 - 0.866i$ $\quad \sqrt{3} \angle (-150deg) = -1.5 - 0.866i$

$a^2 - a = -1.732i$ $\quad \sqrt{3} \angle (-90deg) = -1.732i$

$a^2 + a = -1$ $\quad 1 \angle (180deg) = -1$

- Normal balanced three phase set:

$V_{AG} := 1 \angle (0deg)$

$V_{BG} := 1 \angle (-120deg)$ $\quad V_{BG} = -0.5 - 0.866i$

$V_{CG} := 1 \angle (120deg)$ $\quad V_{CG} = -0.5 + 0.866i$

$$\bar{V}_{ABC} = \begin{bmatrix} \bar{V}_{AG} \\ \bar{V}_{BG} \\ \bar{V}_{CG} \end{bmatrix} = \bar{V}_{AG} \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}$$

$V_{B1AG} = V\angle\theta$

$V_{B1BG} = V\angle\theta - 120°$

$V_{B1CG} = V\angle\theta + 120°$

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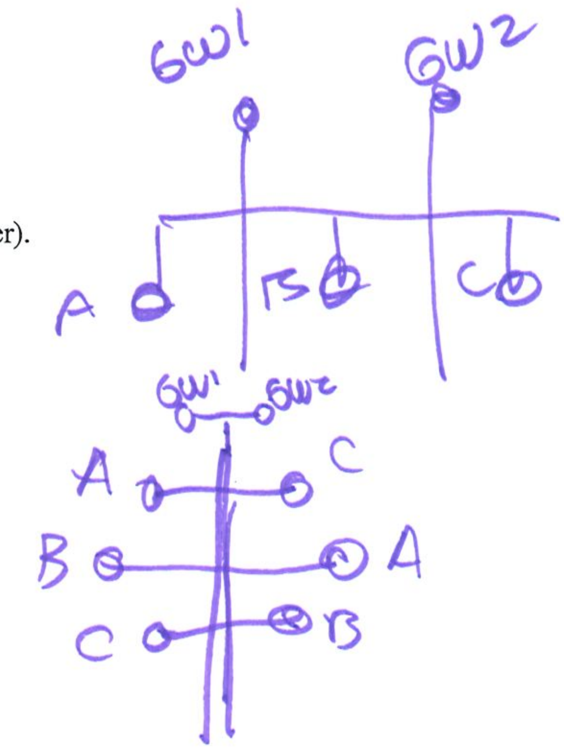
$$V_{ABC} := V_{AG} \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} \qquad V_{ABC} = \begin{pmatrix} 1 \\ -0.5 - 0.866i \\ -0.5 + 0.866i \end{pmatrix} \qquad \overrightarrow{\left|V_{ABC}\right|} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \overrightarrow{\arg\left(V_{ABC}\right)} = \begin{pmatrix} 0 \\ -120 \\ 120 \end{pmatrix} \cdot \text{deg}$$

3. Some matrix operations

- Expanding a matrix: If you have a 3x3 matrix and want to add a row and column while at the stage of entering the matrix initially, make the cell where you want to add rows and columns the active cell
- Select the add matrix/vector tool from the Matrix tool bar (or type CTRL-M) and enter the number of rows and columns you want to add (note you can add 0 rows and 1 (or more) column or vice versa. The selected number of rows appears below the active cell and the selected number of columns appears to the right of the cell.

$$A_1 := \begin{pmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 2 & 2 \\ 4 & 5 & 6 & 3 \\ 5 & 55 & 54 & 4 \end{pmatrix}$$



- Use the submatrix command if you want to pull out part of a matrix
- Pay attention to the setting for the internal ORIGIN variable (or reassign it if you prefer).

ORIGIN := 1

ORIGIN = 0 by default

$$B := \underline{\text{submatrix}}\left(A_1, \underbrace{1, 3}_{\text{rows}}, \underbrace{1, 2}_{\text{columns}}\right)$$

$$B = \begin{pmatrix} 1 & 2 \\ 1 & 3 \\ 4 & 5 \end{pmatrix}$$

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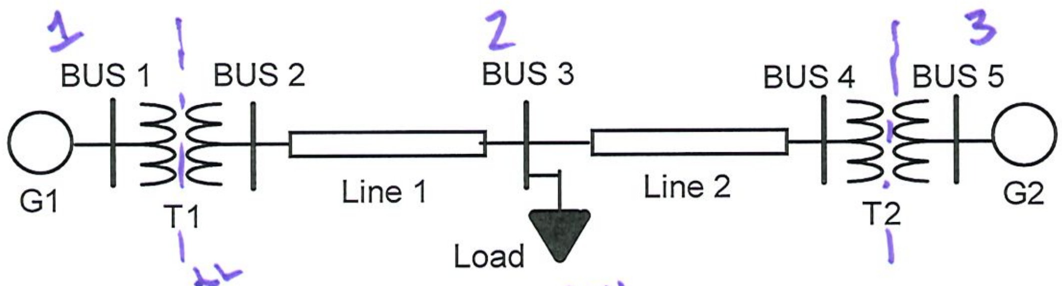
Per Unit Example: Sketch a per unit impedance diagram for the system shown below. Use a 100MVA impedance base, and the generator 1 rated voltage as your reference voltage base. Use pi models for the lines.



- G1: 50MVA, 13.8kV
- G2: 20MVA, 14.4kV
- T1: 40MVA, Δ-Y, 13.2:161kV, X = 10% → X = 0.1pu
- T2: 25MVA, Y-Δ, 161kV:13.2kV, X = 10%

- Load: 45MVA, 0.8pf lagging (Y connected, parallel impedances)
- Line 1: 100 mile, Z = 0.28 + j0.73 ohm/mi, Y = j5.9*10^{-6} mho/mi
- Line 2: 75 mile, Z = 0.28 + j0.73 ohm/mi, Y = j5.9*10^{-6} mho/mi

Positive sequence

$\frac{1}{j\omega C}$

Define Base Quantities: Section I (left of T1)

$S_B := 100MVA$

$V_{B1} := 13.8kV$ Line to line voltage

V_{LL}

$Z_{B1} := \frac{V_{B1}^2}{S_B}$ $Z_{B1} = 1.904\,\Omega$

3ϕ

$I_{B1} := \frac{S_{B \cdot 3\phi}}{\sqrt{3}\cdot V_{B1}}$ $I_{B1} = 4183.698\,A$

only magnitudes no angle

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Section II (between T1 and T2)

$V_{B2} := V_{B1} \cdot \left(\frac{161kV}{13.2kV}\right)$ $\quad V_{B2} = 168.318 \cdot kV$

$Z_{B2} := \frac{V_{B2}^{\,2}}{S_B}$ $\quad Z_{B2} = 283.31\,\Omega$

$I_{B2} := \frac{S_B}{\sqrt{3} \cdot V_{B2}}$ $\quad I_{B2} = 343.011\,A$

Section II (right of T2)

$V_{B3} := V_{B2} \cdot \left(\frac{13.2kV}{161kV}\right)$ $\quad V_{B3} = 13.8 \cdot kV$

$Z_{B3} := \frac{V_{B3}^{\,2}}{S_B}$ $\quad Z_{B3} = 1.904\,\Omega$

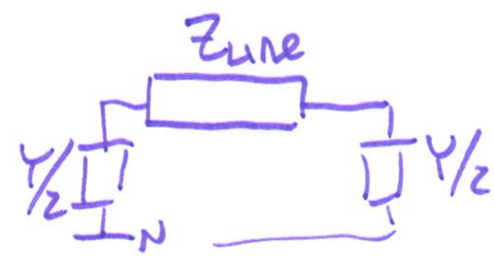
$I_{B3} := \frac{S_B}{\sqrt{3} \cdot V_{B3}}$ $\quad I_{B3} = 4183.698\,A$

Transmission Line Models:

Line 1: $\quad Length1 := 100mi$

$Z_{line1} := \left[(0.28 + j \cdot 0.73)\frac{ohm}{mi}\right] \cdot Length1$ $\quad Z_{line1} = (28 + 73i) \cdot ohm$

$Y_{line1} := j \cdot 5.9 \cdot 10^{-6} \frac{mho}{mi} \cdot Length1$ $\quad Y_{line1} = 5.9i \times 10^{-4} \cdot mho$ $\quad \frac{Y_{line1}}{2} = 2.95i \times 10^{-4} \cdot mho$



Line 1 is in section II, so use Zbase2

$Z_{line1pu} := \frac{Z_{line1}}{Z_{B2}}$ $\quad Z_{line1pu} = (0.099 + 0.258i) \cdot pu$

Note that Ybase is 1/Zbase: $\quad Y_B = 1/Z_B$

$Y_{line1pu} := Y_{line1} \cdot Z_{B2}$ $\quad Y_{line1pu} = 0.167i \cdot pu$

$$Z_{\Omega LV} = Z_{BT_{LV}} \cdot Z_{pu_{TB}} = Z_{B1} \cdot \underbrace{Z_{pu_{new}}}_{\text{on system bases}}$$

$$\frac{(V_{TLOW})^2}{S_{RATED_T}} \qquad \frac{(V_{B1})^2}{S_B}$$

$$\frac{\cancel{Z_{B1}}}{\cancel{Z_{B1}}} \cdot Z_{pu_{new}} = Z_{pu_{TB}} \left(\frac{Z_{BT_{LV}}}{Z_{B1}} \right)$$

$$= Z_{pu_{TB}} \left(\frac{\frac{(V_{TLOW})^2}{S_{RATED_T}}}{\frac{(V_{B1})^2}{S_B}} \right) = Z_{pu_{TB}} \left(\frac{V_{TLOW}}{V_{B1}} \right)^2 \left(\frac{S_{B}}{S_{RATED}} \right)$$

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$Z_B = \frac{V_{LL}^2}{S_B}$

Impedance change of base calculation

pu on system base

pu on transform base

$$X_{T1new} := X_{T1} \cdot \left(\frac{V_{T1Low}}{V_{B1}}\right)^2 \cdot \left(\frac{S_B}{S_{T1}}\right) \qquad X_{T1new} = 0.229 \cdot pu$$

$Z_{\Omega L} = Z_{BLV} \cdot Z_{puL} = Z_{BHV} \cdot Z_{puH}$ $= Z_{\Omega H}$

$Z_{\Omega L} = Z_{B_{T\,kV}} \cdot Z_{pu_T} = Z_{B1} \cdot Z_{pu_{sys}}$

Transformer Base   given pel   LV Base system

Transformer 2:

$S_{T2} := 25MVA \qquad V_{T2Low} := 13.2kV \qquad V_{T2hi} := 161kV \qquad X_{T2} := 0.10pu$

Impedance change of base calculation

$$X_{T2new} := X_{T2} \cdot \left(\frac{V_{T2Low}}{V_{B3}}\right)^2 \cdot \left(\frac{S_B}{S_{T2}}\right) \qquad X_{T2new} = 0.366 \cdot pu$$

Load Model - constant impedance

V_{rated} R jX

$magS_{load} := 45MVA \qquad pfload := 0.8 \quad \text{lagging} \qquad V_{loadrated} := 161kV \qquad \phi_{load} := acos(pfload)$

$\phi_{load} = 36.87 \cdot deg$

$S_{load} := magS_{load} \cdot e^{j \cdot \phi_{load}} \qquad S_{load} = (36 + 27i) \cdot MVA$

Since the load is wye connected with parallel impedances:

$$R_{load} := \frac{(|V_{loadrated}|)^2}{Re(S_{load})} \; P \qquad R_{load} = 720.028\,\Omega \qquad X_{load} := \frac{(|V_{loadrated}|)^2}{Im(S_{load})} \; Q \qquad X_{load} = 960.037\,\Omega$$

As a check:

$$Z_{equivload} := \left(\frac{1}{R_{load}} + \frac{1}{j \cdot X_{load}}\right)^{-1} \qquad |Z_{equivload}| = 576.022\,\Omega \qquad arg(Z_{equivload}) = 36.87 \cdot deg$$

$= \frac{(R)(jX)}{R + jX}$

$Z_{parallel} = \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)^{-1}$

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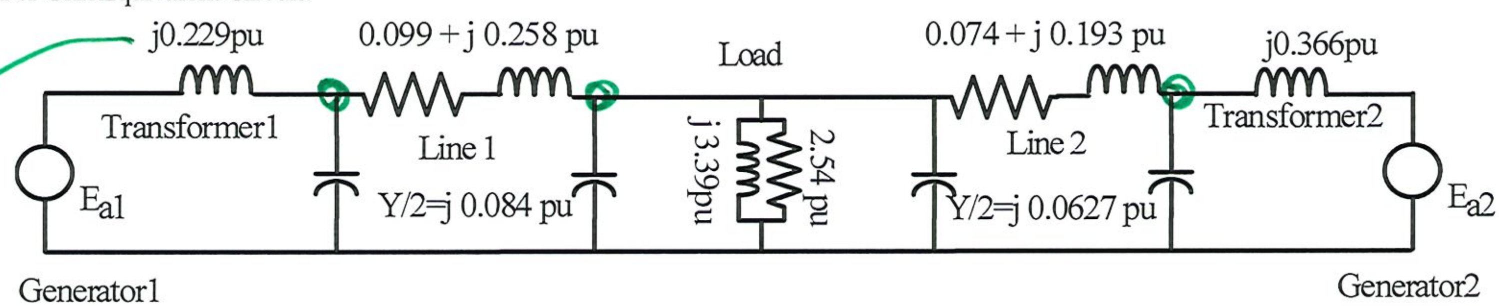
$Z_{equivload} = (460.818 + 345.613i)\,\Omega$ → series eqv

$S_{check} := \frac{\left(\left|V_{loadrated}\right|\right)^2}{\overline{Z_{equivload}}}$ $\quad S_{check} = (36 + 27i)\cdot MVA$ $\quad$ note complex conjugate in equation

Convert to per unit (load in section II):

$R_{loadpu} := \frac{R_{load}}{Z_{B2}}$ $\quad R_{loadpu} = 2.541\cdot pu$ $\quad X_{loadpu} := \frac{X_{load}}{Z_{B2}}$ $\quad X_{loadpu} = 3.389\cdot pu$

Per Unit Equivalent Circuit:



Z_{pu}

$Y_{pu} = \frac{1}{Z_{pu}}$

- per unit equivalent for power flow analysis

- for fault analysis we also need internal impedances for generators

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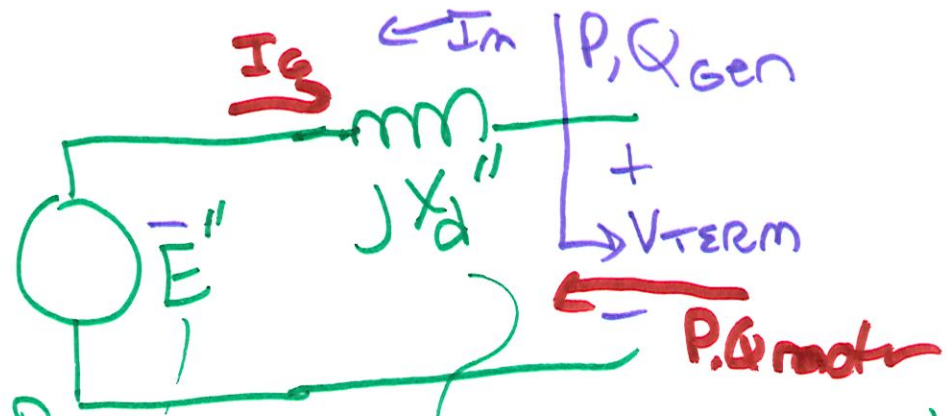
- Generator (and motor) equivalents for fault analysis
  - Simple approximation
  - Represent as a voltage behind a reactance (impedance)



without power electronic coupling/control

or E' — subtransient reactance

or E — X_d' → transient reactance

— X_d → SS reactance

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- motors or generators with power electronic interfaces

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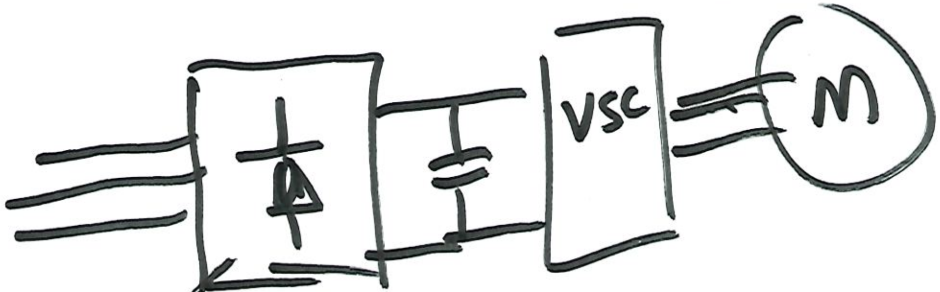
motor drives → variable frequency drive (VFD)
adjustable speed drives (ASD)

, controls frequency & amplitude & phase of <u>motor current</u>



diode rectifier