

Regulating Transformers in Sequence Domain

$$pu := 1$$

- A three-phase voltage regulating transformer has a per unit leakage reactance of 0.10, and steps the voltage from 69 kV to 345 kV. It has a MVA rating of 100 MVA and is connected in delta on the 69 kV side and Y on the 345 kV side.

$$X_L := 0.10pu$$

Tap change does not change phase shift angle. Just voltage magnitude

$$Y_L := \frac{1}{jX_L}$$

$$Y_L = -10i \cdot pu$$

$$b := \frac{69kV}{345kV}$$

$$Y_{1nominal} := \begin{pmatrix} Y_L & -Y_L \cdot e^{-j \cdot 30deg} \\ -Y_L \cdot e^{j \cdot 30deg} & Y_L \end{pmatrix}$$

$$Y_{1nominal} = \begin{pmatrix} -10i & 5 + 8.66i \\ -5 + 8.66i & -10i \end{pmatrix} \cdot pu$$

$$Y_{2nominal} := \begin{pmatrix} Y_L & -Y_L \cdot e^{j \cdot 30deg} \\ -Y_L \cdot e^{-j \cdot 30deg} & Y_L \end{pmatrix}$$

$$Y_{2nominal} = \begin{pmatrix} -10i & -5 + 8.66i \\ 5 + 8.66i & -10i \end{pmatrix} \cdot pu$$

$$Y_{0nominal} := \begin{pmatrix} 0 & 0 \\ 0 & Y_L \end{pmatrix}$$

$$Y_{0nominal} = \begin{pmatrix} 0 & 0 \\ 0 & -10i \end{pmatrix} \cdot pu$$

A. Determine the per unit equivalent circuit parameters and sketch the circuit if the high voltage winding is has a tap changer, and is regulated to be at +7.5%. Also create the two-port matrix.

$$a_{t_a} := \frac{69kV}{(1 + 0.075) \cdot (345kV)}$$

$$c_a := \frac{a_{t_a}}{b}$$

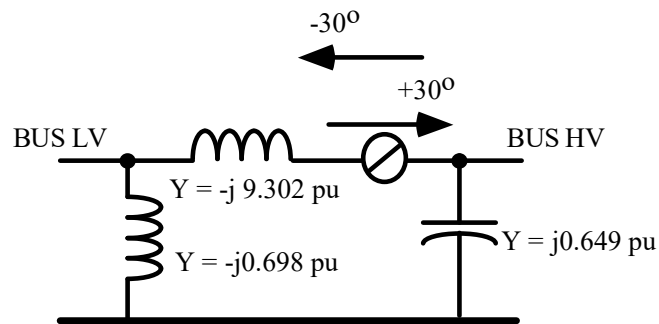
$$c_a = 0.9302$$

- π - equivalent circuit terms:

$$\text{Right leg: } \left[(|c_a|)^2 - c_a \right] \cdot Y_L = 0.649i \cdot \text{pu}$$

$$\text{Left left: } (1 - c_a) \cdot Y_L = -0.698i \cdot \text{pu}$$

$$\text{Series leg: } c_a \cdot Y_L = -9.302i \cdot \text{pu} \quad \text{same both directions since } c_a \text{ is real}$$



Note that the phase shift operator stays the same.

Matrix terms:

$$Y_{11} = c_a \cdot Y_L + (1 - c_a) \cdot Y_L = Y_L$$

$$Y_{22} = \bar{c}_a \cdot Y_L + \left[(|c_a|)^2 - c_a \right] \cdot Y_L = (|c_a|)^2 \cdot Y_L \quad \text{again, } c_a \text{ is real.}$$

$$Y_{\text{partA1}} := \begin{bmatrix} Y_L & c_a \cdot (-Y_L \cdot e^{-j \cdot 30\text{deg}}) \\ \bar{c}_a \cdot (-Y_L \cdot e^{j \cdot 30\text{deg}}) & (|c_a|)^2 \cdot Y_L \end{bmatrix}$$

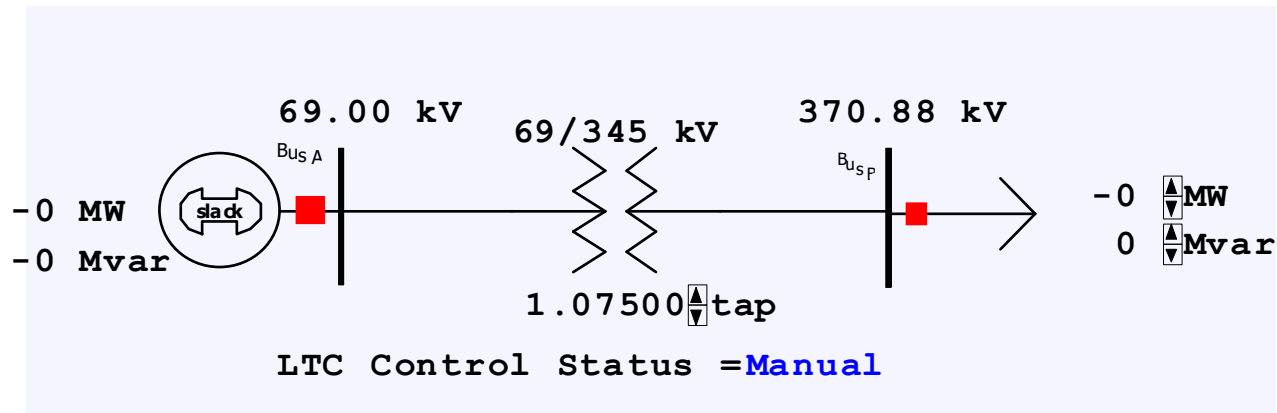
$$Y_{\text{partA1}} = \begin{pmatrix} -10i & 4.651 + 8.056i \\ -4.651 + 8.056i & -8.653i \end{pmatrix} \cdot \text{pu}$$

$$\overrightarrow{|Y_{\text{partA1}}|} = \begin{pmatrix} 10 & 9.302 \\ 9.302 & 8.653 \end{pmatrix} \cdot \text{pu} \quad \overrightarrow{\arg(Y_{\text{partA1}})} = \begin{pmatrix} -90 & 60 \\ 120 & -90 \end{pmatrix} \cdot \text{deg}$$

$$Y_{\text{partA2}} := \begin{bmatrix} Y_L & c_a \cdot (-Y_L \cdot e^{j \cdot 30 \text{deg}}) \\ -c_a \cdot (-Y_L \cdot e^{-j \cdot 30 \text{deg}}) & (|c_a|)^2 \cdot Y_L \end{bmatrix} \quad Y_{\text{partA2}} = \begin{pmatrix} -10i & -4.651 + 8.056i \\ 4.651 + 8.056i & -8.653i \end{pmatrix} \cdot \text{pu}$$

$$Y_{\text{partA0}} := \begin{bmatrix} 0 & 0 \\ 0 & (|c_a|)^2 \cdot Y_L \end{bmatrix} \quad Y_{\text{partA0}} = \begin{pmatrix} 0 & 0 \\ 0 & -8.653i \end{pmatrix} \cdot \text{pu}$$

- Powerworld file implemented to verify the results.



- Set P and Q load to zero
- Note that Powerworld puts the tap on the "From Bus" side.

Positive Sequence
 Y_{bus} matrix .

Name	Bus 1	Bus 2
Bus A	0.00 - j10.00	4.65 + j8.06
Bus B	-4.65 + j8.06	0.00 - j8.65

Matches the results calculated above.

Negative Sequence		
Name	Bus 1	Bus 2
Bus A	0.00 - j10.00	-4.65 + j8.06
Bus B	4.65 + j8.06	0.00 - j8.65

Zero Sequence		
Name	Bus 1	Bus 2
Bus A	0.00 + j0.00	0.00 + j0.00
Bus B	0.00 + j0.00	0.00 - j8.65

B. Determine the per unit equivalent circuit parameters and sketch the circuit for a phase angle regulator with a 15 degree phase shift, and a 345kV to 345kV rating. The transformer has an effective leakag reactance of 0.25pu (combining windings)

$$Z_{\text{par}} := j \cdot 0.25 \text{ pu} \quad Y_{\text{par}} := \frac{1}{Z_{\text{par}}} \quad Y_{\text{par}} = -4i \cdot \text{pu}$$

$$b_{\text{par}} := \frac{345 \text{ kV}}{345 \text{ kV}}$$

- First look at case without a phase shift

$$\phi_{\text{nom}} := 0 \text{ deg}$$

$$t_{\text{nom}} := \frac{1}{\cos(\phi_{\text{nom}})} \quad t_{\text{nom}} = 1$$

$$a_{t_par0} := \frac{345 \text{ kV}}{(t_{\text{nom}} \cdot e^{j \cdot \phi_{\text{nom}}}) \cdot (345 \text{ kV})} \quad c_{\text{par0}} := \left(\frac{a_{t_par0}}{b_{\text{par}}} \right) \quad c_{\text{par0}} = 1$$

- π - equivalent circuit terms:

$$\text{Right leg:} \quad \left[(|c_{\text{par0}}|)^2 - c_{\text{par0}} \right] \cdot Y_{\text{par}} = 0 \cdot \text{pu}$$

$$\text{Left left:} \quad (1 - c_{\text{par0}}) \cdot Y_{\text{par}} = 0 \cdot \text{pu}$$

$$\text{Series leg:} \quad c_{\text{par0}} \cdot Y_{\text{par}} = -4i \cdot \text{pu} \quad \text{same both directions since } c_a \text{ is real}$$

- Y_{bus} matrix terms:

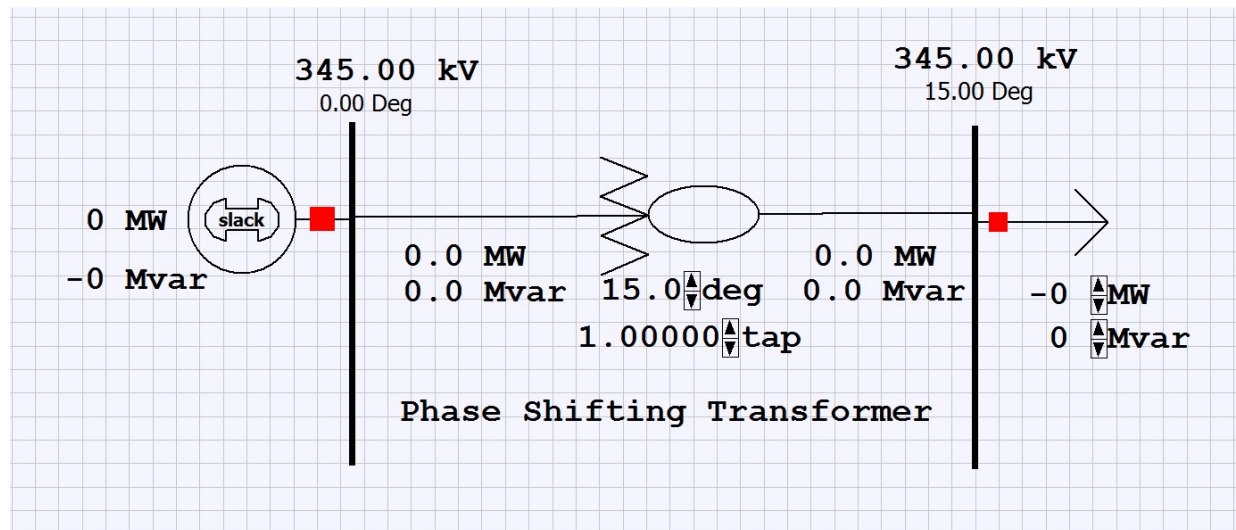
$$Y_{11} = c_{\text{par}0} \cdot Y_{\text{par}} + (1 - c_{\text{par}0}) \cdot Y_{\text{par}} = Y_{\text{par}}$$

$$Y_{22} = \overline{c_{\text{par}0}} \cdot Y_{\text{par}} + \left[(|c_{\text{par}0}|)^2 - c_{\text{par}0} \right] \cdot Y_{\text{par}} = (|c_{\text{par}0}|)^2 \cdot Y_{\text{par}}$$

Positive sequence matrix:

$$Y_{\text{par}1_0\text{deg}} := \begin{bmatrix} Y_{\text{par}} & c_{\text{par}0} \cdot (-Y_{\text{par}}) \\ c_{\text{par}0} \cdot (-Y_{\text{par}}) & (|c_{\text{par}0}|)^2 \cdot Y_{\text{par}} \end{bmatrix} \quad Y_{\text{par}1_0\text{deg}} = \begin{pmatrix} -4i & 4i \\ 4i & -4i \end{pmatrix} \cdot \text{pu}$$

- Phase Shifting Transformer in Powerworld



- Y_{bus} for 0 degree condition:

- Same positive, negative and zero (depending on transformer connections)

Positive Sequence	Bus 1	Bus 2
Name	Bus 1	Bus 2
Generator Bus	0.00 - j4.00	-0.00 + j4.00
Load Bus	-0.00 + j4.00	0.00 - j4.00

- Now look at case with +15 degree shift

$$\phi_{15} := 15\text{deg}$$

$$t_{15} := \frac{1}{\cos(\phi_{15})} \quad t_{15} = 1.035$$

$$a_{t_par15} := \frac{345\text{kV}}{\left(t_{\text{nom}} \cdot e^{j \cdot \phi_{15}}\right) \cdot (345\text{kV})} \quad c_{\text{par15}} := \frac{a_{t_par15}}{b_{\text{par}}} \quad |c_{\text{par15}}| = 1 \quad \arg(c_{\text{par15}}) = -15 \cdot \text{deg}$$

- Positive sequence π - equivalent circuit terms:

$$\text{Right leg:} \quad \left[\left(|c_{\text{par15}}| \right)^2 - c_{\text{par15}} \right] \cdot Y_{\text{par}} = (1.035 - 0.136i) \cdot \text{pu}$$

$$\text{Left left:} \quad (1 - c_{\text{par15}}) \cdot Y_{\text{par}} = (1.035 - 0.136i) \cdot \text{pu}$$

$$\text{Series leg:} \quad c_{\text{par15}} \cdot Y_{\text{par}} = (-1.035 - 3.864i) \cdot \text{pu}$$

- Y_{bus} matrix terms:

$$Y_{11} := c_{\text{par15}} \cdot Y_{\text{par}} + (1 - c_{\text{par15}}) \cdot Y_{\text{par}} \quad Y_{11} = -4i \cdot \text{pu} \quad \text{Still equal } Y_{\text{par}}$$

$$Y_{22} := \left[c_{\text{par15}} \cdot Y_{\text{par}} + \left[\left(|c_{\text{par15}}| \right)^2 - c_{\text{par15}} \right] \cdot Y_{\text{par}} \right] \quad Y_{22} = -4i \quad \left(|c_{\text{par15}}| \right)^2 \cdot Y_{\text{par}} = -4i$$

Positive sequence matrix:

$$Y_{\text{par1_15deg}} := \begin{bmatrix} Y_{\text{par}} & c_{\text{par15}} \cdot (-Y_{\text{par}}) \\ c_{\text{par15}} \cdot (-Y_{\text{par}}) & \left(|c_{\text{par15}}| \right)^2 \cdot Y_{\text{par}} \end{bmatrix} \quad Y_{\text{par1_15deg}} = \begin{pmatrix} -4i & 1.035 + 3.864i \\ -1.035 + 3.864i & -4i \end{pmatrix} \cdot \text{pu}$$

$$\overrightarrow{|Y_{\text{par1}_15\text{deg}}|} = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$$

$$\overrightarrow{\arg(Y_{\text{par1}_15\text{deg}})} = \begin{pmatrix} -90 & 75 \\ 105 & -90 \end{pmatrix} \cdot \text{deg}$$

Negative sequence pi equivalent terms:

$$\text{Right leg: } \left[(|c_{\text{par15}}|)^2 - \overline{c_{\text{par15}}} \right] \cdot Y_{\text{par}} = (-1.035 - 0.136i) \cdot \text{pu}$$

$$\text{Left left: } (1 - \overline{c_{\text{par15}}}) \cdot Y_{\text{par}} = (-1.035 - 0.136i) \cdot \text{pu}$$

$$\text{Series leg: } \overline{c_{\text{par15}}} \cdot Y_{\text{par}} = (1.035 - 3.864i) \cdot \text{pu}$$

Negative sequence matrix:

$$Y_{11_2} := \overline{c_{\text{par15}}} \cdot Y_{\text{par}} + (1 - \overline{c_{\text{par15}}}) \cdot Y_{\text{par}}$$

$$Y_{11_2} = -4i \cdot \text{pu} \quad \text{Still equal } Y_{\text{par}}$$

$$Y_{22_2} := \left[\overline{c_{\text{par15}}} \cdot Y_{\text{par}} + \left[(|c_{\text{par15}}|)^2 - \overline{c_{\text{par15}}} \right] \cdot Y_{\text{par}} \right]$$

$$Y_{22_2} = -4i \quad (|c_{\text{par15}}|)^2 \cdot Y_{\text{par}} = -4i$$

$$Y_{\text{par2}_15\text{deg}} := \begin{bmatrix} Y_{\text{par}} & \overline{c_{\text{par15}}} \cdot (-Y_{\text{par}}) \\ \overline{c_{\text{par15}}} \cdot (-Y_{\text{par}}) & (|c_{\text{par15}}|)^2 \cdot Y_{\text{par}} \end{bmatrix}$$

$$Y_{\text{par2}_15\text{deg}} = \begin{pmatrix} -4i & -1.035 + 3.864i \\ 1.035 + 3.864i & -4i \end{pmatrix} \cdot \text{pu}$$

$$\overrightarrow{|Y_{\text{par2}_15\text{deg}}|} = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$$

$$\overrightarrow{\arg(Y_{\text{par2}_15\text{deg}})} = \begin{pmatrix} -90 & 105 \\ 75 & -90 \end{pmatrix} \cdot \text{deg}$$

- The zero sequence impedance will depend on the connections of the transformers that constitute the phase angle regulator, and will require test data on the zero sequence behavior of the different windings.
- Powerworld result with +15 degree shift

Positive Sequence		
Name	Bus 1	Bus 2
Generator Bus	$0.00 - j4.00$	$1.04 + j3.86$
Load Bus	$-1.04 + j3.86$	$0.00 - j4.00$

Negative Sequence		
Name	Bus 1	Bus 2
Generator Bus	$0.00 - j4.00$	$-1.04 + j3.86$
Load Bus	$1.04 + j3.86$	$0.00 - j4.00$

Both match the results above.