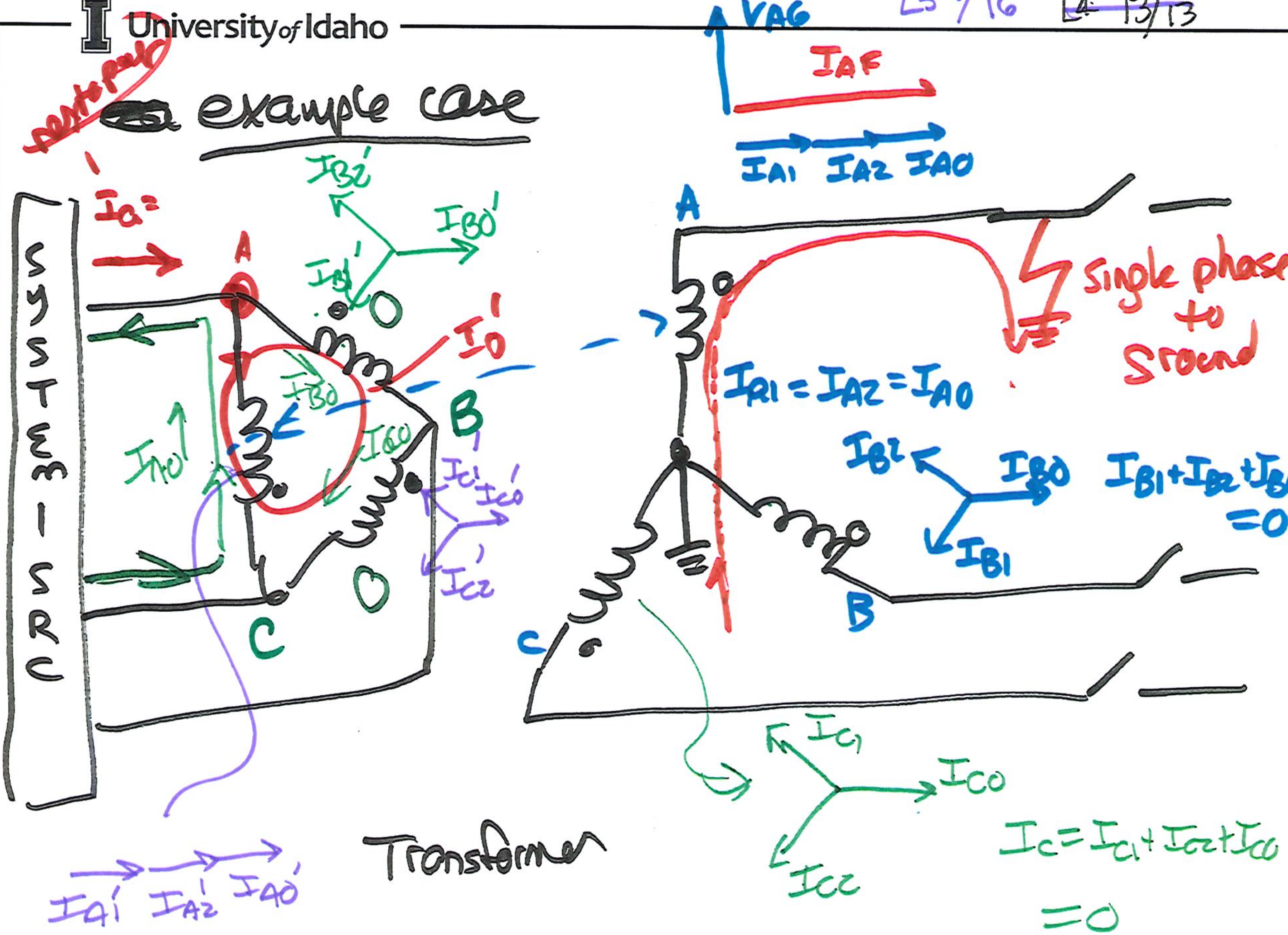
ECE 523
Symmetrical Components

Session 5

example case



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$\bar{I}_a \rightarrow$ a

$I_{aG} = I_{B0}' + I_{B1}' + I_{B2}'$

$\bar{I}_{ca} = I_{A0}' + I_{A1}' + I_{A2}'$

$I_{A0}' = I_{B0}'$

$\bar{I}_a = I_{B1}' - I_{A1}' + I_{B2}' - I_{A2}'$

$I_{A0}' + I_{A1}' + I_{A2}'$

I_0

$I_{C0}' + I_{C1}' + I_{C2}'$

$\bar{I}_c \rightarrow$ c

$= I_{A1}' - I_{C1}' + I_{A2}' - I_{C2}'$

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ECE 523: Lecture 3

- An unbalanced n-phase set of phasors can be represented by n-1 balanced n-phase sets of phasors and a zero phase set of phasors all added together by superposition. → V_{a0} → n + 0 behave the same

$V_a = V_{a1} + V_{a2} + V_{a3} .. V_{an}$

$V_b = V_{b1} + V_{b2} + V_{b3} .. V_{bn}$ ~ V_{b0}

etc.

$V_n = V_{n1} + V_{n2} + V_{n3} .. V_{n,n}$ ~ V_{n0}

- Define a generalized phase angle shift term "a":

$a = e^{j \cdot \frac{2 \cdot \pi}{n}}$ Note for a three phase system: $a := e^{j \cdot \frac{2 \cdot \pi}{3}}$ ~ n = 3 $\arg(a) = 120 \cdot deg$

- Now make a into a function that depends on n. Leave n as a variable we can define. For now set a value

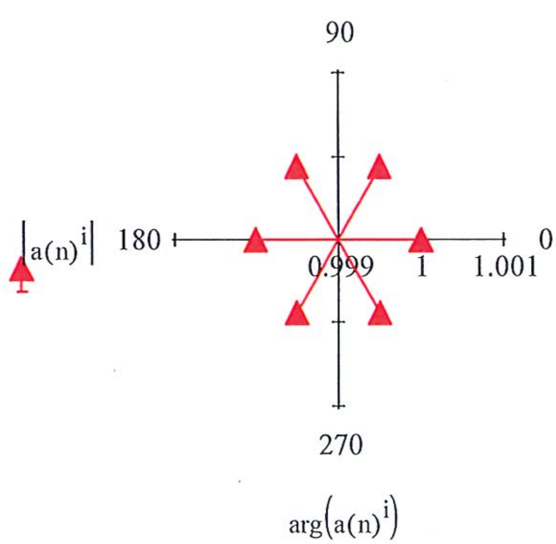
$n := 6$

$a(n) := 1e^{j \cdot \frac{2 \cdot \pi}{n}}$

We can also define an array index, and have terms that vary as this increments.

$i := 0, 1 .. (n - 1)$

- Polar plot, with magnitude anangle as "i" increments
- Set line type to "stem" in the properties
- Note that we can't tell which way it rotates as "i" increments



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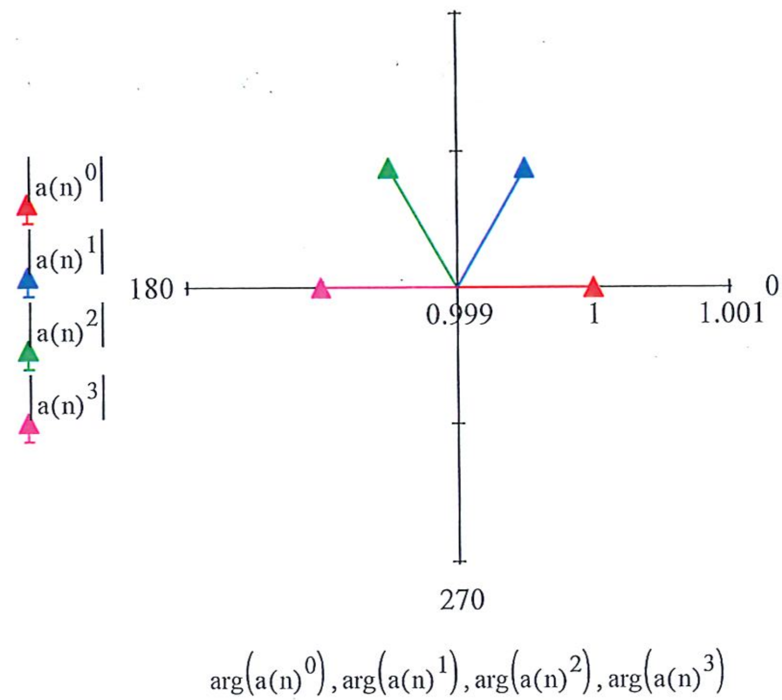
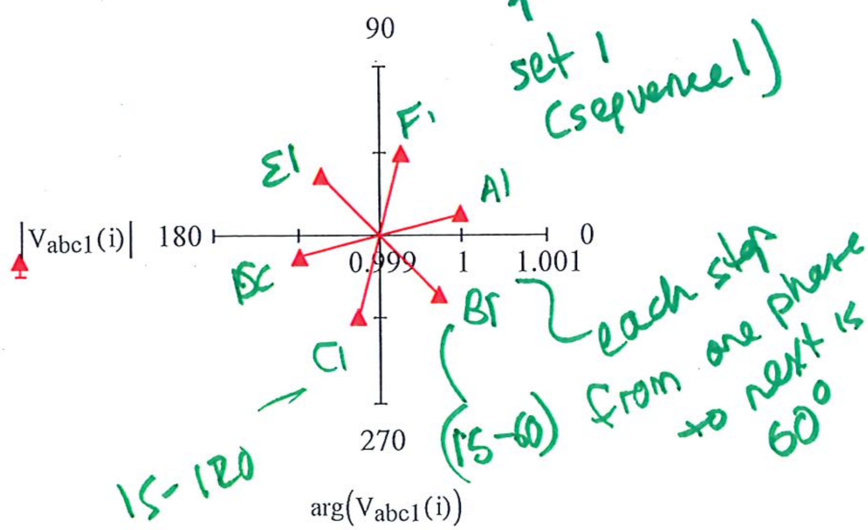
This time we will actually
show the powers of "a"



- Now look at the balanced n-phase sets:
    - Define an initial magnitude and angle reference: $V_{1ref} := 1.0e^{j\cdot 15deg}$
    - Phase relationship for phase sequence 1: $V_{abc1}(i) := a(n)^{n-i}\cdot V_{1ref}$

set 1 (sequence 1)



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- Phase relationship for phase sequence 2:

sequence 1 had a 1 here

$$V_{2ref} := 1.1e^{j\cdot 5deg} \qquad V_{abc2}(i) := a(n)^{2(n-i)}\cdot V_{2ref}$$

- each rotation from one phase to next is 120° (2·60°)

- Relationship for phase sequence "n"

$$V_{nref} := 0.5e^{j\cdot 45deg} \qquad V_{abcn}(i) := a(n)^{n(n-i)}\cdot V_{nref}$$

- Try entering different values of n

Sums of columns

- First the zero column:

$$Col\_0 := \sum_{i=1}^{n} V_{abcn}(i) \qquad |Col\_0| = 3 \qquad arg(Col\_0) = 45\cdot deg$$

$$\frac{|Col\_0|}{n} = 0.5$$

$$\left(\frac{|Col\_0|}{n}\right) - |V_{nref}| = 0$$

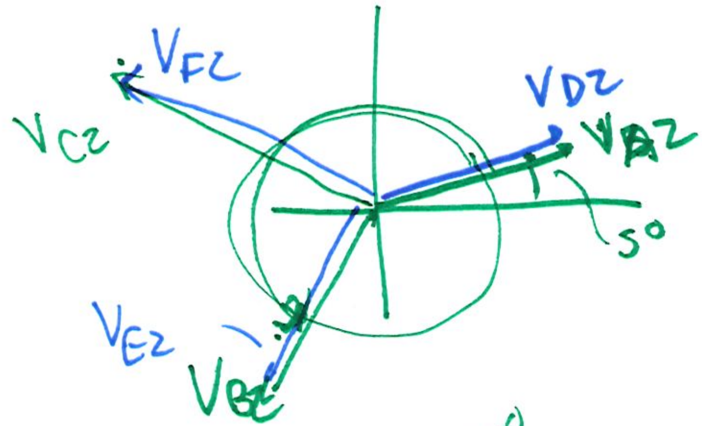
- Now the "1" column (multiply terms by a^i):

$$Col\_1 := \sum_{i=1}^{n} \left(a(n)^{i}\cdot V_{abc1}(i)\right) \qquad |Col\_1| = 6 \qquad arg(Col\_1) = 15\cdot deg$$

$$\frac{|Col\_1|}{n} = 1$$

Repeat same multiplier on column 2 and column 0

$$Col\_2_1 := \sum_{i=1}^{n} \left(a(n)^{i}\cdot V_{abc2}(i)\right) \qquad |Col\_2_1| = 0$$



~~$V_{B2} \rightarrow 5^\circ - 120^\circ$~~

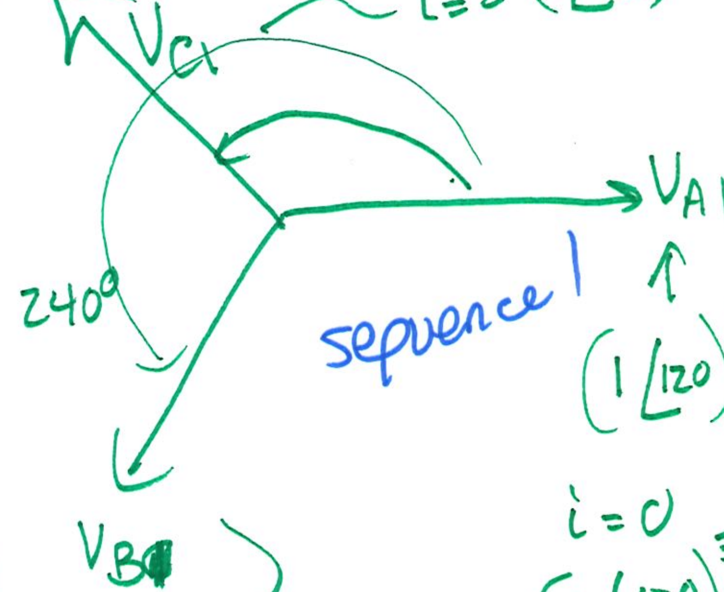
$$V_{B2} = (1\angle 60)^{2(6-1)}$$

$$(1\angle 60)^{10} V_{A2}$$

$$V_{D2} = (1\angle 60)^{2(6-3)}$$

Go back to ~~3Ø~~ 3 phase for a moment

$i=2$ $(1\angle 120)^{3-2} \cdot V_{A1}$



$$(1\angle 120)^{n-i} \cdot V_{A1}$$

$i=0$

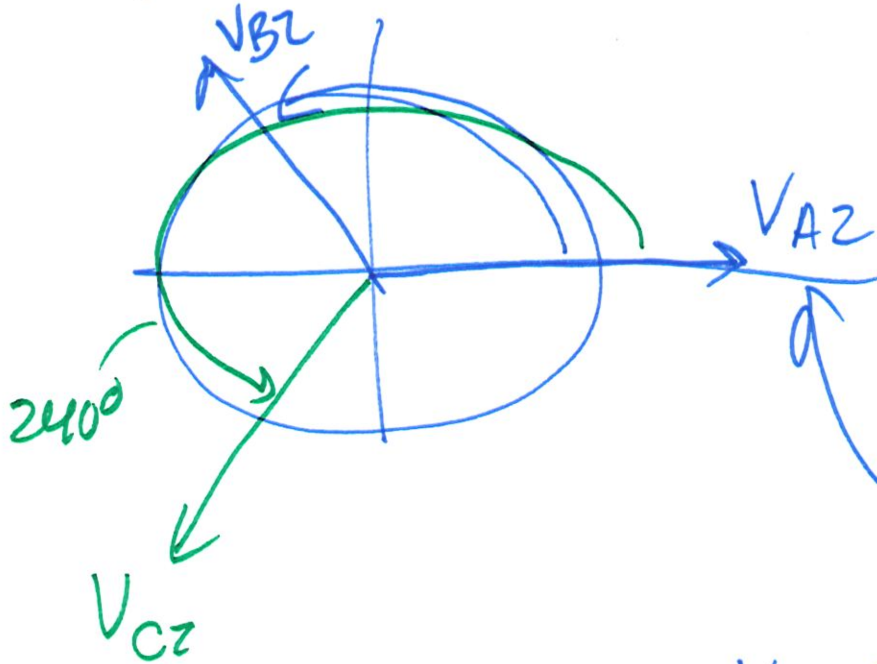
$$(1\angle 120)^3$$

$i=1$

$$(1\angle 120^\circ)^{3-1} \cdot V_{A1}$$

$$1\angle 240^\circ \cdot V_{A1}$$

Sequence 2 in 3Ø Def



$$\left(1\angle 120^\circ\right)^{2(n-i)} V_{A2}$$

$$\left(1\angle 120\right)^{2(3-0)} V_{A2}$$

$$V_{B2} = \left(1\angle 120^\circ\right)^{2(3-1)} V_{A2}$$

$$\left(1\angle 120^\circ\right)^{4}$$

$$V_{C2} = \left(1\angle 120^\circ\right)^{2(3-2)} V_{A2}$$

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$$Col\_0_1 := \sum_{i=1}^{n} \left(a(n)^{i} \cdot V_{abcn}(i) \right) \qquad \left| Col\_0_1 \right| = 0$$

- Now the "2" column (multiply terms by a^{2i}):

$$Col\_2 := \sum_{i=1}^{n} \left(a(n)^{2 \cdot i} \cdot V_{abc2}(i) \right) \qquad \left| Col\_2 \right| = 6.6 \qquad \arg(Col\_2) = 5 \cdot deg$$

$$\left(\frac{\left| Col\_2 \right|}{n} \right) - \left| V_{2ref} \right| = 0$$

Repeat same multiplier on column 1 and column 0

$$Col\_1_2 := \sum_{i=1}^{n} \left(a(n)^{2 \cdot i} \cdot V_{abc1}(i) \right) \qquad \left| Col\_1_2 \right| = 0$$

$$Col\_0_2 := \sum_{i=1}^{n} \left(a(n)^{2 \cdot i} \cdot V_{abcn}(i) \right) \qquad \left| Col\_0_2 \right| = 0$$

Matrix relation (for n = 6)

a^0

a^1

a^2

inverse transform from ABCD... to 0,1,2,3,4,5...

$$C_{012345} := \frac{1}{6} \cdot \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a(6) & a(6)^2 & a(6)^3 & a(6)^4 & a(6)^5 \\ 1 & a(6)^2 & a(6)^4 & a(6)^6 & a(6)^8 & a(6)^{10} \\ 1 & a(6)^3 & a(6)^6 & a(6)^9 & a(6)^{12} & a(6)^{15} \\ 1 & a(6)^4 & a(6)^8 & a(6)^{12} & a(6)^{16} & a(6)^{20} \\ 1 & a(6)^5 & a(6)^{10} & a(6)^{15} & a(6)^{20} & a(6)^{25} \end{pmatrix}$$

$\frac{1}{n}$

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$$6C_{012345} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.5+0.87i & -0.5+0.87i & -1 & -0.5-0.87i & 0.5-0.87i \\ 1 & -0.5+0.87i & -0.5-0.87i & 1 & -0.5+0.87i & -0.5-0.87i \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.5-0.87i & -0.5+0.87i & 1 & -0.5-0.87i & -0.5+0.87i \\ 1 & 0.5-0.87i & -0.5-0.87i & -1 & -0.5+0.87i & 0.5+0.87i \end{pmatrix}$$

$$C_{012345}^{-1} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.5-0.87i & -0.5-0.87i & -1 & -0.5+0.87i & 0.5+0.87i \\ 1 & -0.5-0.87i & -0.5+0.87i & 1 & -0.5-0.87i & -0.5+0.87i \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.5+0.87i & -0.5-0.87i & 1 & -0.5+0.87i & -0.5-0.87i \\ 1 & 0.5+0.87i & -0.5+0.87i & -1 & -0.5-0.87i & 0.5-0.87i \end{pmatrix}$$

Note that: $a(6) = 0.5 + 0.87i$ $\qquad a(6)^4 = -0.5 - 0.87i$

$a(6)^2 = -0.5 + 0.87i$ $\qquad a(6)^5 = 0.5 - 0.87i$

$a(6)^3 = -1$ $\qquad a(6)^6 = 1$

and so on.....

- Compare matrix inverse with 6 times the complex conjugate (element by element)

$$C_{012345}^{-1} - 6\cdot\overline{C_{012345}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

complex conjugate

- Set tolerance value.....

Result Format

Number Format Display Options Unit Display Tolerance

Complex threshold (10) 10

Zero threshold (15) 10

Replace the 1/n term with SQRT(n)/n = 1/SQRT(n) to give a power invariant transform:

$$C_{alt} := \frac{1}{\sqrt{6}} \cdot \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a(6) & a(6)^2 & a(6)^3 & a(6)^4 & a(6)^5 \\ 1 & a(6)^2 & a(6)^4 & a(6)^6 & a(6)^8 & a(6)^{10} \\ 1 & a(6)^3 & a(6)^6 & a(6)^9 & a(6)^{12} & a(6)^{15} \\ 1 & a(6)^4 & a(6)^8 & a(6)^{12} & a(6)^{16} & a(6)^{20} \\ 1 & a(6)^5 & a(6)^{10} & a(6)^{15} & a(6)^{20} & a(6)^{25} \end{pmatrix}$$

$$C_{alt}^{-1} - \overline{C_{alt}} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Finally, we can derive the reverse transformation:

We know that: $V_{b1} = a^{-1} \cdot V_{a1}$ This time we are trying to shift Va1 to Vb1

Note that:

$a^n = 1$ and $a^n \cdot a^{-1} = a^{n-1}$ Effectively added 360 degrees

Therefore: $a^{-1} = a^{n-1}$

- This repeats for integer multiples of n

Check with actual values:

$$a(n)^{-1} - a(n)^{n-1} = 0 \qquad a(n)^{-2} - a(n)^{n-2} = 0$$

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"1" term:	"2" term:	"n-1" term:
$V_{b1} = a^{n-1} \cdot V_{a1}$	$V_{b2} = a^{n-2} \cdot V_{a2}$	$V_{b\_nm1} = a \cdot V_{a\_nm1}$
$V_{c1} = a^{n-2} \cdot V_{a1}$	$V_{c2} = a^{n-4} \cdot V_{a2}$	$V_{c\_nm1} = a^{2} \cdot V_{a\_nm1}$
$V_{d1} = a^{n-3} \cdot V_{a1}$	$V_{d2} = a^{n-6} \cdot V_{a2}$	$V_{d\_nm1} = a^{3} \cdot V_{a\_nm1}$
etc.	etc.	etc.

$n := 6$

This what we call symmetrical components transform for a 3∅ set

$$\begin{array}{cc} & a^0 \\ a^0 & \end{array}$$

$$A_{012345} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a^{-1} & a^{-2} & a^{-3} & a^{-4} & a^{-(n-1)} \\ 1 & a^{-2} & a^{-4} & a^{-6} & a^{-8} & a^{-2(n-1)} \\ 1 & a^{-3} & a^{-6} & a^{-9} & a^{-12} & a^{-3(n-1)} \\ 1 & a^{-4} & a^{-8} & a^{-12} & a^{-16} & a^{-4(n-1)} \\ 1 & a^{-(n-1)} & a^{-2(n-1)} & a^{-3(n-1)} & a^{-4(n-1)} & a^{-5(n-1)} \end{bmatrix}$$

- Lets rewrite this a bit, realizing that $n = 6$

a^6, $a^6 = a^0$

$$A_{012345} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a^{n-1} & a^{n-2} & a^{n-3} & a^{n-4} & a^{n-5} \\ 1 & a^{n-2} & a^{n-4} & a^{2n-6} & a^{2n-8} & a^{2n-10} \\ 1 & a^{n-3} & a^{2n-6} & a^{2n-9} & a^{2n-12} & a^{3n-15} \\ 1 & a^{n-4} & a^{2n-8} & a^{2n-12} & a^{3n-16} & a^{4n-20} \\ 1 & a^{n-5} & a^{2n-10} & a^{3n-15} & a^{4n-20} & a^{4n-25} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a^5 & a^4 & a^3 & a^2 & a \\ 1 & a^4 & a^2 & a^0 & a^4 & a^2 \\ 1 & a^3 & a^0 & a^3 & a^0 & a^3 \\ 1 & a^2 & a^4 & a^0 & a^2 & a^4 \\ 1 & a & a^2 & a^3 & a^4 & a^5 \end{pmatrix}$$

3∅ set
$n = 1$

$$\begin{bmatrix} a^3 & a^3 & a^3 \\ a^3 & a^2 & a^1 \\ a^3 & a^1 & a^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

a^{3-1}, a^{3-2}, $3-2$

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$$\begin{bmatrix} V_{A0} \\ V_{A1} \\ V_{\cancel{B}A2} \end{bmatrix} = \left[A_{012}\right]^{-1} \begin{bmatrix} V_{AG} \\ V_{B\cancel{G}G} \\ V_{CG} \end{bmatrix}$$

Normal, positive sequence operation

$V_A = 1\angle 0$

$V_B = 1\angle -120$

$V_C = 1\angle +120$

$V_A \cdot \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}$

$$\begin{bmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 1\angle 0 \\ 1\angle -120^{\circ} \\ 1\angle +120 \end{bmatrix} = \begin{bmatrix} \frac{0}{3} \\ 1\angle 0 \\ 0 \end{bmatrix}$$

$$\frac{1\angle 0 + a(1\angle -120^{\circ}) + a^2(1\angle +120)}{3} = \frac{3\angle 0}{3}$$

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3 phase

$$\bar{A}_{012} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a^4 \\ 1 & a^4 & a^2 \end{bmatrix}$$

$a^4 = 1$

$$\underbrace{\begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix}}_{\text{ABC line to ground voltages}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \underbrace{\begin{bmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{bmatrix}}_{\text{phase A sequence components}}$$

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Phase A, B and C Reference: Transformations and Inverse Transformations

$a := 1 \cdot e^{j \cdot 120\text{deg}}$

Phase A symmetrical components transform	Phase B symmetrical components transform	Phase C symmetrical components transform

, Rbo

$$A_{012} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \qquad B_{012} := \begin{pmatrix} 1 & a & a^2 \\ 1 & 1 & 1 \\ 1 & a^2 & a \end{pmatrix} \qquad C_{012} := \begin{pmatrix} 1 & a^2 & a \\ 1 & a & a^2 \\ 1 & 1 & 1 \end{pmatrix}$$

from the n phase set

C_{012} $\frac{1}{n}$

$$\text{Inverse\_A}_{012} := \frac{1}{3} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \qquad \text{Inverse\_A}_{012} - A_{012}^{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{Inverse\_B}_{012} := \frac{1}{3} \cdot \begin{pmatrix} 1 & 1 & 1 \\ a^2 & 1 & a \\ a & 1 & a^2 \end{pmatrix} \qquad \text{Inverse\_B}_{012} - B_{012}^{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{Inverse\_C}_{012} := \frac{1}{3} \cdot \begin{pmatrix} 1 & 1 & 1 \\ a & a^2 & 1 \\ a^2 & a & 1 \end{pmatrix} \qquad \text{Inverse\_C}_{012} - C_{012}^{-1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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Symmetrical Components Transformations
European versus North American Notation

$a := e^{j \cdot 120deg} = 2\pi/3$

$h := e^{j \cdot 120deg}$

Phase A symmetrical components transform (A)

Phase R symmetrical components transform (H)

$$A_{012} := \begin{array}{c} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \\ \begin{matrix} 0 & 1 & 2 \end{matrix} \end{array} \qquad H_{PNZ} := \begin{array}{c} \begin{pmatrix} 1 & 1 & 1 \\ h^2 & h & 1 \\ h & h^2 & 1 \end{pmatrix} \\ \begin{matrix} P & N & Z \end{matrix} \end{array}$$

$Zaa := (14.9 + j \cdot 58.4)ohm$ $\quad$ $Zab := (4 + j \cdot 27.3)ohm$

$Zbb := (14.9 + j \cdot 58.4)ohm$ $\quad$ $Zac := (4 + j \cdot 27.3)ohm$

$Zcc := (14.9 + j \cdot 58.4)ohm$ $\quad$ $Zbc := (4 + j \cdot 27.3)ohm$

$$Z_{ABC} := \begin{pmatrix} Zaa & Zab & Zac \\ Zab & Zbb & Zbc \\ Zac & Zbc & Zcc \end{pmatrix}$$

$$Z_{012} := A_{012}^{-1} \cdot Z_{ABC} \cdot A_{012}$$

$$Z_{012} = \begin{array}{c} \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{pmatrix} 22.9 + 113i & 0 & 0 \\ 0 & 10.9 + 31.1i & 0 \\ 0 & 0 & 10.9 + 31.1i \end{pmatrix} \end{array} \Omega$$

Diagonal ⇒ perfect decoupling between pos, neg, & zero sequence

$$Z_{PNZ} := H_{PNZ}^{-1} \cdot Z_{ABC} \cdot H_{PNZ}$$

$$Z_{PNZ} = \begin{array}{c} \begin{matrix} ZP & N & Z \end{matrix} \\ \begin{pmatrix} 10.9 + 31.1i & 0 & 0 \\ 0 & 10.9 + 31.1i & 0 \\ 0 & 0 & 22.9 + 113i \end{pmatrix} \end{array} \Omega$$

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$$I_a := 100A \cdot e^{j \cdot 0deg}$$

$$I_b := 2000A \cdot e^{j \cdot 30deg}$$

$$I_c := 1900A \cdot e^{j \cdot 210deg}$$

$$I_{012} := A_{012}^{-1} \cdot \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \qquad \overrightarrow{\left|I_{012}\right|} = \begin{pmatrix} 64.4 \\ 1109.23 \\ 1142.56 \end{pmatrix} A \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} \qquad \overrightarrow{\arg\left(I_{012}\right)} = \begin{pmatrix} 15 \\ 119.37 \\ -59.39 \end{pmatrix} \cdot deg$$

$$I_{PNZ} := H_{PNZ}^{-1} \cdot \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \qquad \overrightarrow{\left|I_{PNZ}\right|} = \begin{pmatrix} 1109.23 \\ 1142.56 \\ 64.4 \end{pmatrix} A \begin{matrix} P \\ N \\ Z \end{matrix} \qquad \overrightarrow{\arg\left(I_{PNZ}\right)} = \begin{pmatrix} 119.37 \\ -59.39 \\ 15 \end{pmatrix} \cdot deg$$