ECE 523: Symmetrical Components Examples

If a load is unbalanced, its neutral, m, will not be at the same potential as the source neutral, n. Derive the relationship between the neutral shift $V_{mn}$ and the zero sequence voltage $V_{am0}$ for the system shown below. Similar to problem 2.11. Hint, consider the line to ground voltages, line to neutral voltages and neutral to ground voltages.

\[ V_{mn} = V_{m\text{ground}} - V_{n\text{ground}} \]
\[ V_{an} = V_{a\text{ground}} - V_{n\text{ground}} \]
and
\[ V_{am} = V_{a\text{ground}} - V_{m\text{ground}} \]

so, substituting
\[ V_{mn} = V_{m\text{ground}} - V_{n\text{ground}} = (V_{a\text{ground}} - V_{am}) - (V_{a\text{ground}} - V_{an}) \]

Then rearranging terms:
\[ V_{mn} = V_{an} - V_{am} \]
similarly
\[ V_{mn} = V_{bn} - V_{bm} \]
\[ V_{mn} = V_{cn} - V_{cm} \]
- Adding these three expressions, results in: 
\[ 3V_{m-n} = (V_{an} + V_{bn} + V_{cn}) - (V_{am} + V_{bm} + V_{cm}) \]

- Applying the Symmetrical Components transformation:
\[ 3V_{an0} = (V_{an} + V_{bn} + V_{cn}) = 0 \quad \text{Since the source is still balanced} \]

Similarly
\[ 3V_{am0} = (V_{am} + V_{bm} + V_{cm}) \quad \text{This does not sum to 0, since the load is unbalanced} \]

Therefore
\[ 3V_{m-n} = 3V_{an0} - 3V_{am0} = -3V_{am0} \quad \Rightarrow \quad V_{m-n} = -V_{am0} \]

As a check, the circuit was simulated with ATPDraw, and then ATPAnalyzer was used to determine the symmetrical components using instantaneous quantities.

- First we see \( V_{ag}, V_{bg}, V_{cg}, V_{ng} \) and \( V_{mg} \) (brown line)
Next plot instantaneous values of $V_{mn}$ and $V_0$ (note that $V_0$ is simply $1/3*(V_{am} + V_{bm} + V_{cm})$. Note the 180 degree phase difference.

ATPDraw Schematic

- The capacitors were added to provide a ground reference. The capacitive reactances are quite large and don't impact the results otherwise.
- Control modelling language TACS used to calculate $V_0$